

# Large Vocabulary Continuous Speech Recognition

Karel Beneš

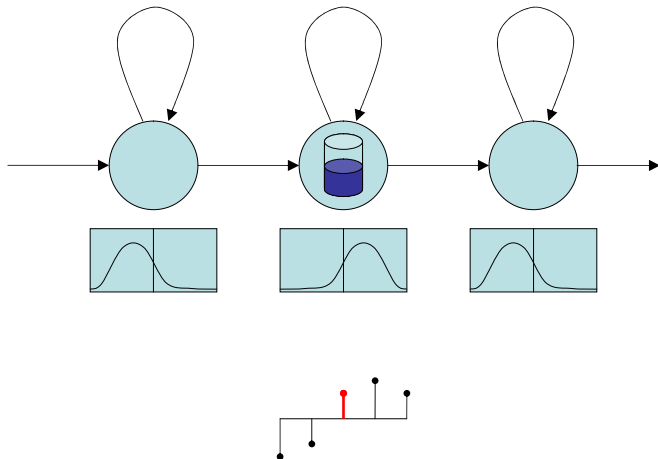
Slides obtained from Mirko Hannemann,  
as based on other slides from Daniel Povey  
and some slides from T. Schultz, M. Mohri, M. Riley and S. Renals

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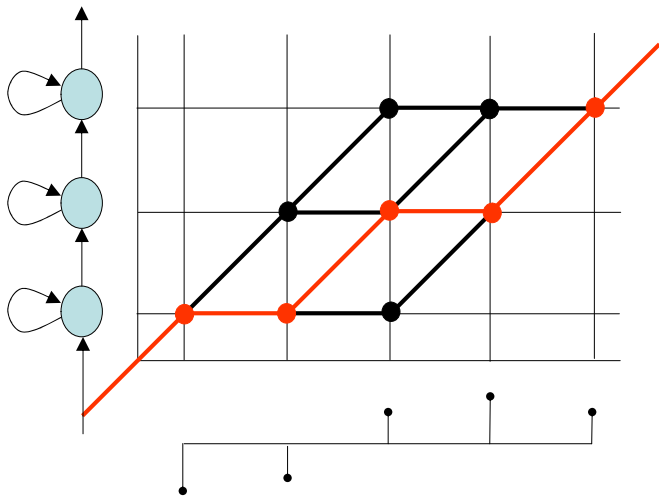
# Roadmap of the Lecture

- 1 Reminder: HMMs, ASR, Viterbi
- 2 Reminder: FSA, FST, WFST
- 3 Encoding Knowledge in WFSTs
- 4 Decoding with WFSTs

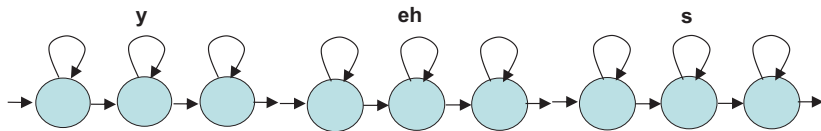
# Hidden Markov Model, Token passing



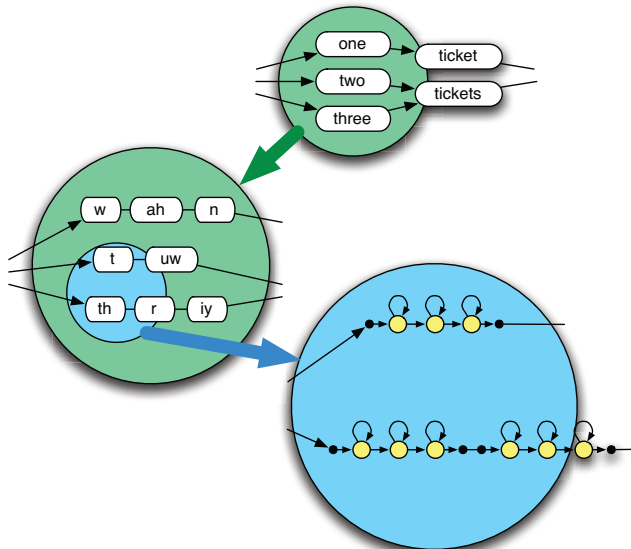
# Viterbi algorithm, trellis



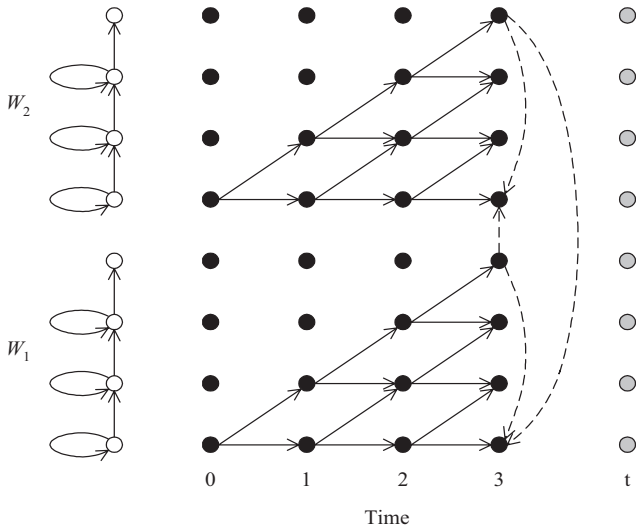
# Phoneme based models — re-usable acoustic units



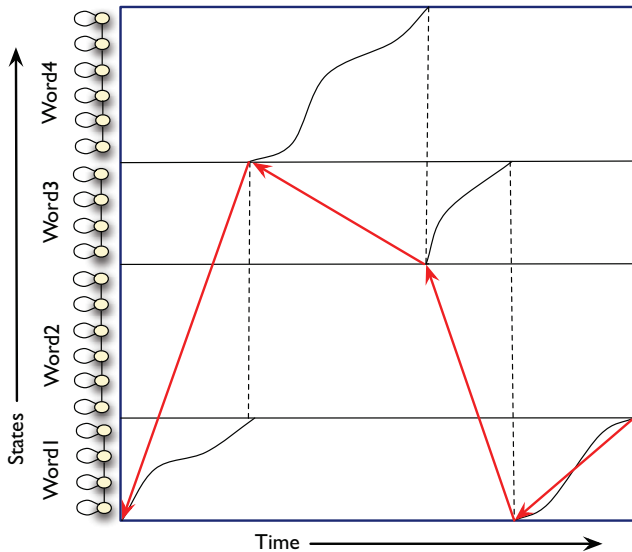
# Decoding graph/recognition network



# Viterbi path with complex models



# Viterbi path with back-tracking





# Roadmap of the Lecture

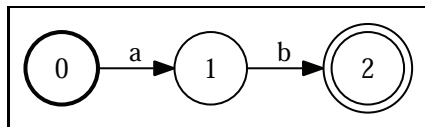
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# Why Finite State Transducers?

## Motivation:

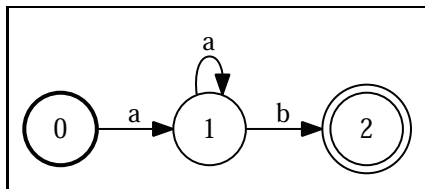
- most components (LM, lexicon, lattice) are finite-state
  - unified framework for describing models
  - integrate different models into a single model via composition operations
  - improve search efficiency via optimization algorithms
  - flexibility to extend (add new models)
- *speed*: pre-compiled search space, near realtime performance on embedded systems
- *flexibility*: same decoder used for hand-held devices and LVCSR

# Finite State Acceptor (FSA)



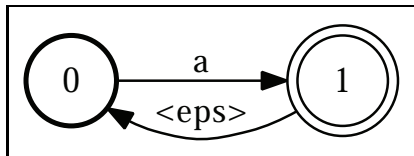
- ▶ An FSA “accepts” a set of strings
- ▶ (a string is a sequence of symbols).
- ▶ View FSA as a representation of a possibly infinite set of strings.
- ▶ This FSA accepts just the string  $ab$ , i.e. the set  $\{ab\}$
- ▶ Numbers in circles are state labels (not really important).
- ▶ Labels on arcs are the symbols.
- ▶ Start state(s) bold; final/accepting states have extra circle.
  - ▶ Note: it is sometimes assumed there is just one start state.

# A less trivial FSA



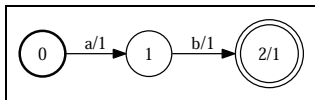
- ▶ The previous example doesn't show the power of FSAs because we could represent the set of strings finitely.
- ▶ This example represents the infinite set  $\{ab, aab, aaab, \dots\}$
- ▶ Note: a string is "accepted" (included in the set) if:
  - ▶ There is a path with that sequence of symbols on it.
  - ▶ That path is "successful" (starts at an initial state, ends at a final state).

# The epsilon symbol



- ▶ The symbol  $\epsilon$  has a special meaning in FSAs (and FSTs)
- ▶ It means “no symbol is there”.
- ▶ This example represents the set of strings  $\{a, aa, aaa, \dots\}$
- ▶ If  $\epsilon$  were treated as a normal symbol, this would be  $\{a, a\epsilon a, a\epsilon a\epsilon a, \dots\}$
- ▶ In text form,  $\epsilon$  is sometimes written as `<eps>`
- ▶ Toolkits implementing FSAs/FSTs generally assume `<eps>` is the symbol numbered zero

# Weighted finite state acceptors



- ▶ Like a normal FSA but with costs on the arcs and final-states
- ▶ Note: cost comes after “/”. For final-state, “2/1” means final-cost 1 on state 2.
- ▶ View WFA as a function from a string to a cost.
- ▶ In this view, unweighted FSA is  $f : \text{string} \rightarrow \{0, \infty\}$ .
- ▶ If multiple paths have the same string, take the one with the lowest cost.
- ▶ This example maps  $ab$  to  $(3 = 1 + 1 + 1)$ , all else to  $\infty$ .

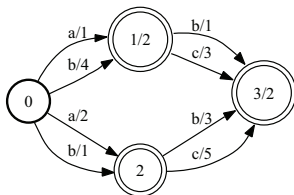
- ▶ The semiring concept makes WFSAs more general.
- ▶ A semiring is
  - ▶ A set of elements (e.g.  $\mathbb{R}$ )
  - ▶ Two special elements  $\bar{1}$  and  $\bar{0}$  (the identity element and zero)
  - ▶ Two operations,  $\oplus$  (plus) and  $\otimes$  (times) satisfying certain axioms.

*Semiring examples.*  $\oplus_{\log}$  is defined by:  $x \oplus_{\log} y = -\log(e^{-x} + e^{-y})$ .

SEMIRING	SET	$\oplus$	$\otimes$	$\bar{0}$	$\bar{1}$
Boolean	$\{0, 1\}$	$\vee$	$\wedge$	0	1
Probability	$\mathbb{R}_+$	+	$\times$	0	1
Log	$\mathbb{R} \cup \{-\infty, +\infty\}$	$\oplus_{\log}$	+	$+\infty$	0
Tropical	$\mathbb{R} \cup \{-\infty, +\infty\}$	min	+	$+\infty$	0

- In WFSAs, weights are multiplied along paths
- summed over paths with identical symbol-sequences

# Probability or tropical semiring



Probability semiring  $(\mathbb{R}_+, +, \times, 0, 1)$

$$[A](ab) = 14$$

$$(1 \times 1 \times 2 + 2 \times 3 \times 2 = 14)$$

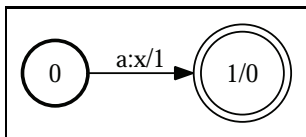
Tropical semiring  $(\mathbb{R}_+ \cup \{\infty\}, \min, +, \infty, 0)$

$$[A](ab) = 4$$

$$(\min(1 + 1 + 2, 3 + 2 + 2) = 4)$$

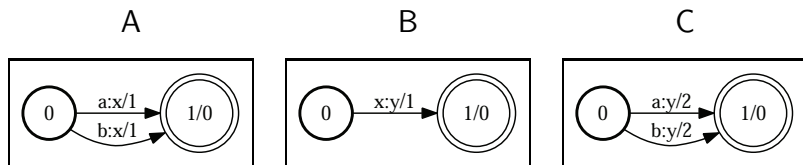


# Weighted finite state transducers (WFST)



- ▶ Like a WFSA except with two labels on each arc.
- ▶ View it as a function from a (pair of strings) to a weight
- ▶ This one maps  $(a, x)$  to 1 and all else to  $\infty$
- ▶ Note: view 1 and  $\infty$  as costs.  $\infty$  is  $\bar{0}$  in semiring.
- ▶ Symbols on the left and right are termed “input” and “output” symbols.

# Composition of WFSTs



- ▶ Notation:  $C = A \circ B$  means,  $C$  is  $A$  composed with  $B$ .
- ▶ In special cases, composition is similar to function composition
- ▶ Composition algorithm “matches up” the “inner symbols”
  - ▶ i.e. those on the output (right) of  $A$  and input (left) of  $B$

# Composition algorithm

- ▶ Ignoring  $\epsilon$  symbols, algorithm is quite simple.
- ▶ States in  $C$  correspond to tuples of (state in  $A$ , state in  $B$ ).
  - ▶ But some of these may be inaccessible and pruned away.
- ▶ Maintain queue of pairs, initially the single pair  $(0, 0)$  (start states).
- ▶ When processing a pair  $(s, t)$ :
  - ▶ Consider each pair of (arc  $a$  from  $s$ ), (arc  $b$  from  $t$ ).
  - ▶ If these have matching symbols (output of  $a$ , input of  $b$ ):
    - ▶ Create transition to state in  $C$  corresponding to (next-state of  $a$ , next-state of  $b$ )
    - ▶ If not seen before, add this pair to queue.
- ▶ With  $\epsilon$  involved, need to be careful to avoid redundant paths...

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# Construction of decoding network

- WFST approach [Mohri et al.]
- exploit several knowledge sources (lexicon, grammar, phonetics) to find most likely spoken word sequence

$$HCLG = H \circ C \circ L \circ G \quad (1)$$

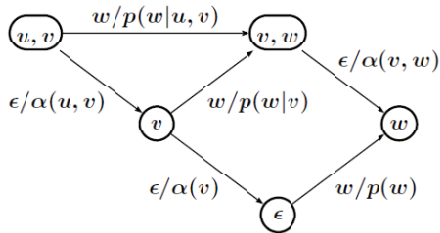
- G probabilistic grammar or language model acceptor (word)
- L lexicon (phones to words)
- C context-dependent relabeling (ctx-dep-phone to phone)
- H HMM structure (PDF labels to context-dependent phones)

Create H, C, L, G separately and compose them together

- **G:** Grammar Transducer  
Backing-off language model:

$$p(w|h) = \begin{cases} f(w|h) & : \text{if } N(w, h) > 0 \\ \alpha(h) \cdot f(w|\bar{h}) & : \text{if } N(w, h) = 0 \end{cases}$$

- **Input:** word
- **Weight:** history dependent word probability



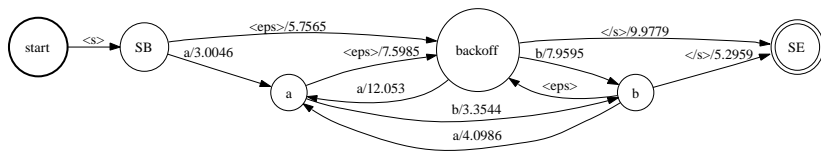
# Language models (ARPA back-off)

\1-grams:

-5.2347 a -3.3  
-3.4568 b  
0.0000 <s> -2.5  
-4.3333 </s>

\2-grams:

-1.4568 a b  
-1.3049 <s> a  
-1.78 b a  
-2.30 b </s>



A	ax
ABERDEEN	ae b er d iy n
ABOARD	ax b r ao dd
ADD	ae dd
ABOVE	ax b ah v

Non-determinism: the same phone sequence can output different words (“I scream for ice cream.”)



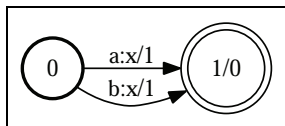
# Pronunciation lexicon L with disambiguation symbols

A	ax #1
ABERDEEN	ae b er d iy n
ABOARD	ax b r ao dd
ADD	ae dd #1
ABOVE	ax b ah v

## Added disambiguation symbols:

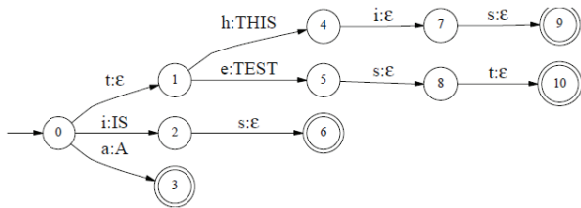
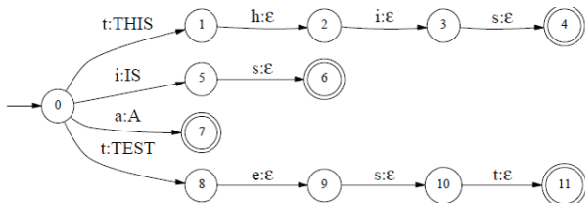
- if a phone sequences can output different words (“I scream for ice cream.”)
- non-determinism: introduce disambiguation symbols, remove at last stage

# Deterministic WFSTs

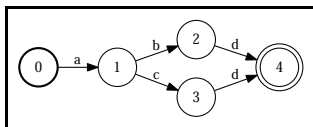
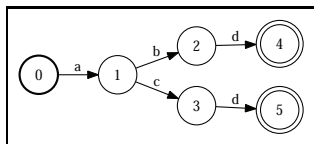


- ▶ Taken to mean “deterministic on the input symbol”
- ▶ I.e., no state can have  $> 1$  arc out of it with the same input symbol.
- ▶ Some interpretations (e.g. Mohri/AT&T/OpenFst) allow  $\epsilon$  input symbols (i.e. being  $\epsilon$ -free is a separate issue).
- ▶ I prefer a definition that disallows epsilons, except as necessary to encode a string of output symbols on an arc.
- ▶ Regardless of definition, not all WFSTs can be determinized.

# Determinization (like making tree-structured lexicon)

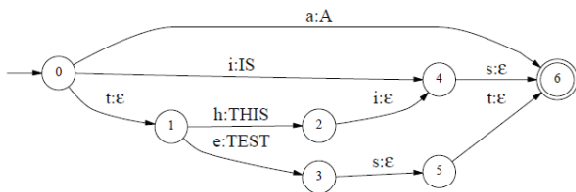
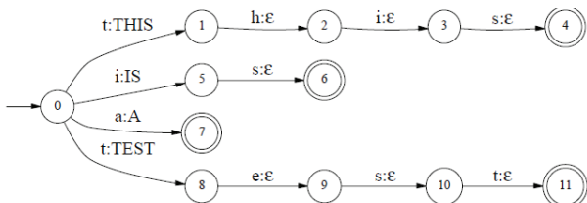


# Minimal deterministic WFSTs

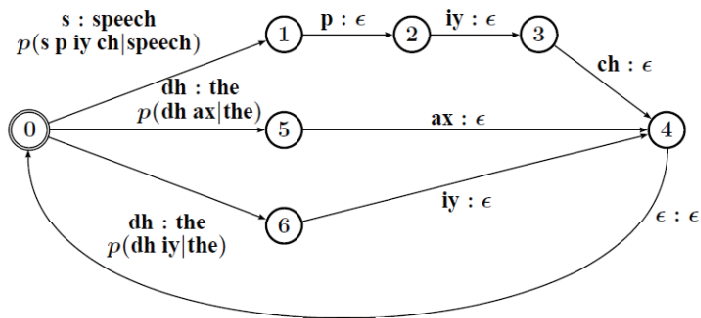


- ▶ Here, the left FSA is not minimal but the right one is.
- ▶ “Minimal” is normally only applied to *deterministic* FSAs.
- ▶ Think of it as suffix sharing, or combining redundant states.
- ▶ It's useful to save space (but not as crucial as determinization, for ASR).

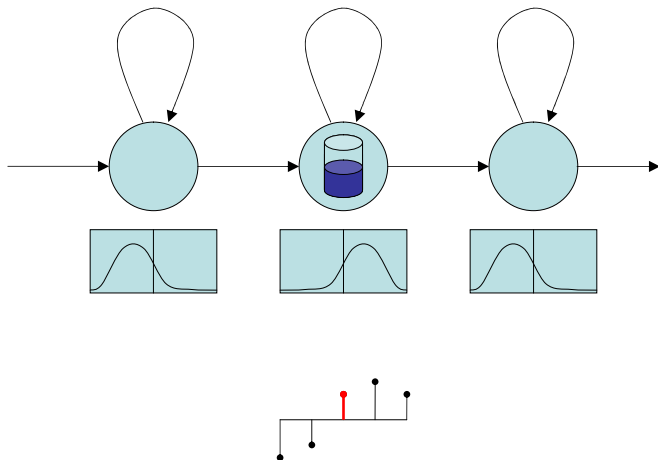
# Minimization (like suffix sharing)



- **L:** Context-Dependency Transducer
  - **Input:** context-independent phone (phoneme)
  - **Output:** word
  - **Weight:** pronunciation probability

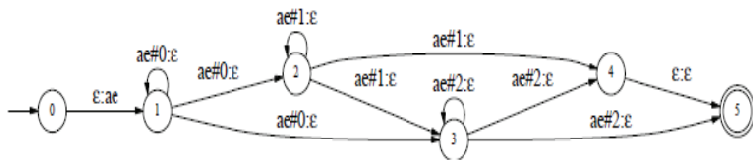


# HMM as transducer



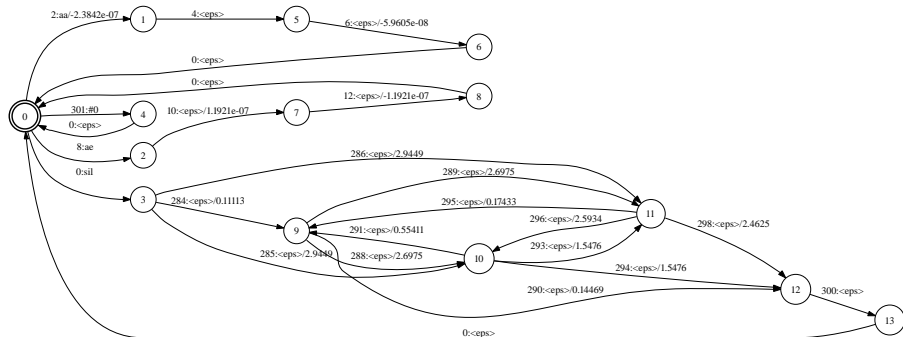
# HMM as transducer (monophone)

- H: HMM Topology Transducer (maps states to phonemes)
  - **Input:** state
  - **Output:** context-dependent phone (triphone)
  - **Weight:** HMM transition probability





# HMM transducer $H_a$



Let's put all together:

$$HCLG = H \circ C \circ L \circ G \quad (2)$$

- H HMM: input PDF labels, output context-dependent phones
- C context-dependency: input ctx-dep-phones, output phones
- L lexicon: input phones, output words
- G language model: input/output words

WFST approach by [Mohri et al.]

$$HCLG = rds(\min(\det(H \circ \det(C \circ \det(L \circ G)))))) \quad (3)$$

rds — remove disambiguation symbols

min — minimization, includes weight pushing

det — determinization

Kaldi toolkit [Povey et al.]

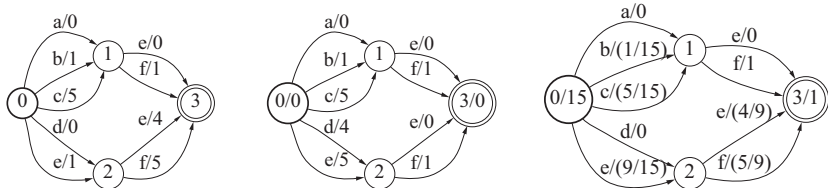
$$HCLG = asl(\min(rds(\det(H_a \circ \min(\det(C \circ \min(\det(L \circ G)))))))) \quad (4)$$

asl — add self loops

rds — remove disambiguation symbols

# Weight and label pushing

- two WFSA's are equal, if they accept the same label sequences with the same weights
- local distribution of weights along the path can be different
- same holds for output labels in WFSTs



- for pruning: apply costs as early as possible
  - make outgoing arcs stochastic distribution
- output labels not synchronized anymore in WFST

# Decoding graph construction (complexities)

- Have to do things in a careful order or algorithms “blow up”
- Determinization for WFSTs can fail
  - need to insert “disambiguation symbols” into the lexicon.
  - need to “propagate these through” H and C.
- Need to guarantee that final HCLG is stochastic:
  - i.e. sums to one, like a properly normalized HMM
  - needed for optimal pruning (discard unlikely paths)
  - usually done by weight-pushing, but standard algorithm can fail, because FST representation of back-off LMs is non-stochastic
- We want to recover the phone sequence from the recognized path (words)
  - sometimes also the model-indices (PDF-ids) and the HCLG arcs that were used in best path

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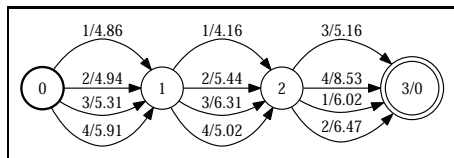
- Solve

$$W' = \operatorname{argmax}_W P(X|W)P(W)$$

- Compose recognizer as  $(H \circ C \circ L \circ G)$  which maps states to word sequences
- Decode by aligning the feature vectors  $X$  with HCLG "

i.e.,

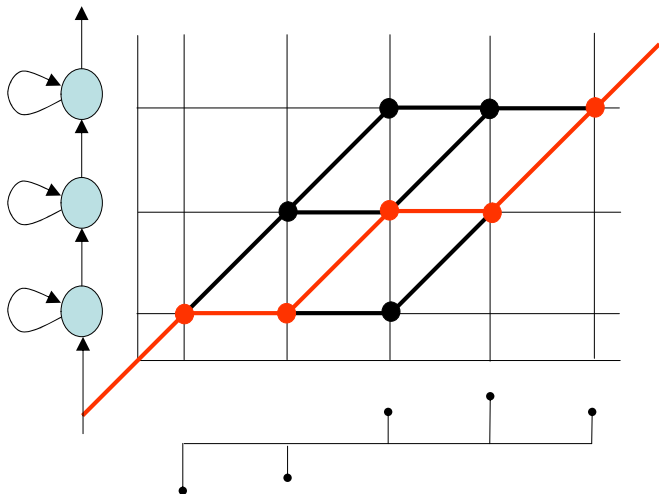
$$W' = \operatorname{argmax}_W X \circ (H \circ C \circ L \circ G)$$



- ▶ First– a “WFST definition” of the decoding problem.
- ▶ Let  $U$  be an FST that encodes the acoustic scores of an utterance (as above).
- ▶ Let  $S = U \circ HCLG$  be called the search graph for an utterance.
- ▶ Note: if  $U$  has  $N$  frames (3, above), then
  - ▶ #states in  $S$  is  $\leq (N + 1)$  times #states in  $HCLG$ .
  - ▶ Like  $N + 1$  copies of  $HCLG$ .

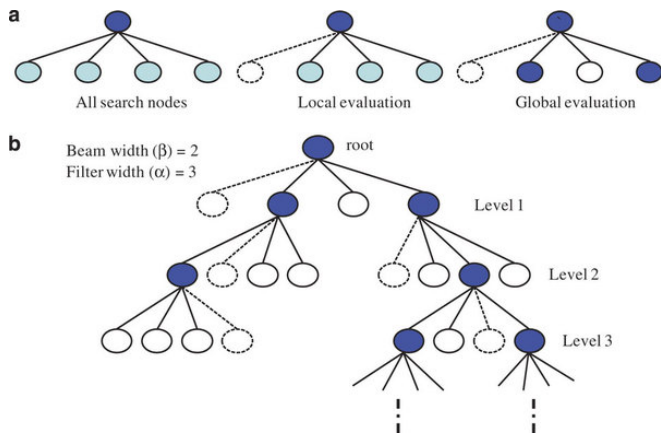


# Viterbi algorithm, trellis



This seems easy, but it only applies for training!

# Beam search



“Beam size” is in practice often a limit on value (13.0), with another hard limit on number of tokens (200).

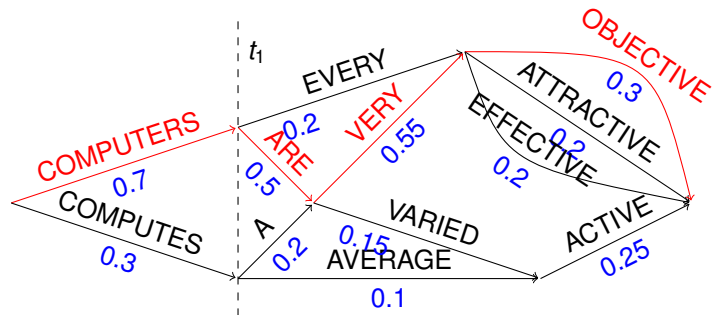
- ▶ With beam pruning, we search a subgraph of  $S$ .
- ▶ The set of “active states” on all frames, with arcs linking them as appropriate, is a subgraph of  $S$ .
- ▶ Let this be called the *beam-pruned subgraph* of  $S$ ; call it  $B$ .
- ▶ A standard speech recognition decoder finds the best path through  $B$ .
- ▶ In our case, the output of the decoder is a linear WFST that consists of this best path.
- ▶ This contains the following useful information:
  - ▶ The word sequence, as output symbols.
  - ▶ The state alignment, as input symbols.
  - ▶ The cost of the path, as the total weight.

# Decoding output

```
utt1 [ 2 6 6 6 6 10 ] [ 614 613 613 613 711 ] [ 122 123  
utt1 SIL th ax  
utt1 <s> THE
```

# Word Lattice / Word Graph

Word Lattice: a compact representation of the search space



The word “lattice” is used in the ASR literature as:

- Some kind of compact representation of the alternate word hypotheses for an utterance.
- Like an N-best list but with less redundancy.
- Usually has time information, sometimes state or word alignment information.
- Generally a directed acyclic graph with only one start node and only one end node.

# Finite State Transducers for ASR

Pro's:

**Fast:** compact/minimal search space due to combined minimization of lexicon, phonemes, HMM's

**Simple:** easy construction of recognizer by composition from states, HMMs, phonemes, lexicon, grammar

**Flexible:** whatever new knowledge sources, the compose/optimize/search remains the same

Con's:

- composition of complex models generates a huge WFST
- search space increases, and huge memory is required
- esp. how to deal with huge language models

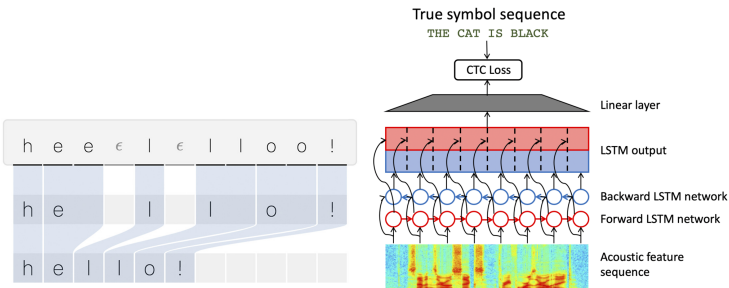
Compared to what?

Compared to only composing  $H \circ C \circ L$  and keeping LM separate. Done e.g. in RASR.

- OpenFST** <http://www.openfst.org>  
Library, developed at Google Research (M. Riley, J. Schalkwyk, W. Skut) and NYU's Courant Institute (C. Allauzen, M. Mohri)
- Mohri08** M. Mohri et al., "*Speech Recognition with weighted finite state transducers.*"
- Kaldi** <http://kaldi.sourceforge.net>  
Open source toolkit in C++ with recipes (D. Povey and others)
- Povey11** D. Povey et al., "*The Kaldi Speech Recognition Toolkit.*"
- Povey12** D. Povey et al., "*Generating exact lattices in the WFST framework.*"
- RASR** <https://github.com/rwth-i6/rasr>



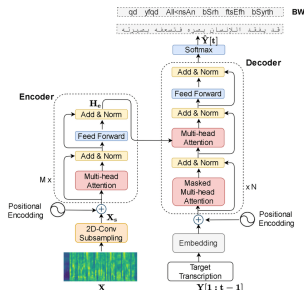
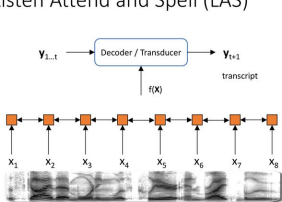
# Alternatives to WFST: CTC



- Independent softmax for each frame
- Produces actual output or a *blank*
- Greedy decoding super fast, usually little search error

# Alternatives to WFST: Listen-attend-spell

## Listen Attend and Spell (LAS)



- Autoregressive factorization
- => slower
- => more accurate, LM included