Seznam rovnic pro zkoušku ISS, leden 2020

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Uvedení rovnice v tomto seznamu je bez jakékoliv záruky a neznamená, že rovnice bude přímo aplikovatelná v kterémkoliv příkladu na zkoušce.

$$p(t) = |x(t)|^2 \quad p[n] = |x[n]|^2$$

$$E_\infty = \int_{-\infty}^{+\infty} |x(t)|^2 dt \qquad E_\infty = \sum_{-\infty}^{\infty} |x[n]|^2$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} |x[n]|^2$$

$$f_1 = \frac{1}{T_1} \quad \omega_1 = \frac{2\pi}{T_1}, \qquad f_1' = \frac{1}{N_1} \quad \omega_1' = \frac{2\pi}{N_1}$$

$$x(t) = C_1 \cos(\omega_1 t + \phi_1) = \frac{C_1}{2} e^{j\phi_1} e^{j\omega_1 t} + \frac{C_1}{2} e^{-j\phi_1} e^{-j\omega_1 t}$$

$$x[n] = C_1 \cos(\omega_1 n + \phi_1) = \frac{C_1}{2} e^{j\phi_1} e^{j\omega_1 n} + \frac{C_1}{2} e^{-j\phi_1} e^{-j\omega_1 n}$$

$$c_k = \frac{1}{T_1} \int_{T_1} x(t) e^{-jk\omega_1 t} dt$$

$$x(t) = \sum_{k=-\infty}^{+\infty} c_k e^{jk\omega_1 t}$$

$$c_k = c_{-k}^\star \quad x(t) \to x(t-\tau) \Rightarrow c_k \to c_k e^{-jk\omega_1 \tau}$$

$$x(t) = C_1 \cos(\omega_1 t + \phi_1):$$

$$c_1 = \frac{C_1}{2} e^{j\phi_1} \qquad c_{-1} = \frac{C_1}{2} e^{-j\phi_1}$$

$$\int_{-b}^{b} e^{\pm jxy} dy = 2b \operatorname{sinc}(bx)$$

$$c_k = D \frac{\vartheta}{T_1} \operatorname{sinc}\left(\frac{\vartheta}{2} k\omega_1\right)$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{+j\omega t} d\omega$$

$$X(j\omega) = X^\star(-j\omega).$$

$$x(t) \to x(t-\tau) \Rightarrow X(j\omega) \to X(j\omega) e^{-j\omega\tau}$$

$$x(t) \to x(mt) \Rightarrow X(j\omega) \to \frac{1}{m} X\left(\frac{j\omega}{m}\right)$$

$$x(t) = \delta(t-\tau) \;\Rightarrow\; X(j\omega) = e^{-j\omega\tau}$$

$$X(j\omega) = D\vartheta \operatorname{sinc}\left(\frac{\vartheta}{2}\omega\right)$$

$$F_s = \frac{1}{T_s} \quad \Omega_s = \frac{2\pi}{T_s}$$

$$x_s(t) = x(t)s(t) \quad X_s(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(\omega - k\omega_1).$$

$$\Omega_s > 2\omega_{max} \quad F_s > 2f_{max}$$

$$n = \frac{nT}{T} \quad f' = \frac{f}{F_s}, \quad \omega' = \frac{\omega}{F_s}$$

$$R_N[n] = \begin{cases} 1 & \text{pro } \; n \in [0, N-1] \\ 0 & \text{jinde} \end{cases}$$

$$\tilde{x}[n] = x[\text{mod}_N n]$$

$$x[n] \longrightarrow x[\text{mod}_N(n-m)]$$

$$x[n] \longrightarrow R_N[n] x[\text{mod}_N(n-m)]$$

$$x[n] \star y[n] = \sum_{k=-\infty}^{\infty} x[k] y[n-k]$$

$$x[n] \tilde{\star} y[n] = \sum_{k=0}^{N-1} x[k] y[\text{mod}_N(n-k)]$$

$$x[n] \circledN y[n] = R_N[n] \sum_{k=0}^{N-1} x[k] y[\text{mod}_N(n-k)]$$

$$\tilde{X}(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \tilde{X}(e^{j\omega}) e^{+j\omega n} d\omega$$

$$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n] e^{-j\frac{2\pi}{N}kn}$$

$$\tilde{x}[n] = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}[k] e^{j\frac{2\pi}{N}kn}$$

$$k\omega_1 \;\; \text{kde} \;\; \omega_1 = \frac{2\pi}{N}$$

$$\tilde{X}[k] = \tilde{X}^\star[-k], \quad \tilde{X}[k] = \tilde{X}[k+gN]$$

$$x[n] = C_1 \cos(\frac{2\pi}{N} n + \phi_1):$$

$$|\tilde{X}[1]| = |\tilde{X}[N-1]| = \frac{NC_1}{2} \qquad \arg \tilde{X}[1] = -\arg \tilde{X}[N-1] = \phi$$

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{+j\frac{2\pi}{N}kn}$$

$\frac{k}{N}$ do $\frac{N-1}{N}$, $\quad 2\pi\frac{k}{N}$ do $2\pi\frac{N-1}{N}$

$\frac{k}{N}F_s$ do $\frac{N-1}{N}F_s$, $\quad \frac{k}{N}2\pi F_s$ do $\frac{N-1}{N}2\pi F_s$

$$X[k] = X^\star[N-k]$$

$$x[n] \longrightarrow R_N x[\text{mod}_N(n-m)] \Rightarrow X[k] \longrightarrow X[k] e^{-j\frac{2\pi}{N}mk}$$

$$y[n] = x[n] \star h[n] = \sum_{k=-\infty}^{+\infty} x[k] h[n-k]$$

$$y(t) = x(t) \star h(t) = \int_{-\infty}^{+\infty} x(\tau) h(t-\tau) d\tau.$$

$$h[n] = 0 \text{ pro } n < 0 \qquad h(t) = 0 \text{ pro } t < 0$$

$$H(j\omega) = \int_{-\infty}^{+\infty} h(t) e^{-j\omega t} dt.$$

$$H(j\omega) = H^\star(-j\omega)$$

$$x(t) = C_1 \cos(\omega_1 t + \phi_1):$$

$$y(t) = |H(j\omega_1)| C_1 \cos\left[\omega_1 t + \phi_1 + \arg H(j\omega_1)\right].$$

$$x(t) \xrightarrow{\mathcal{F}} X(j\omega) \quad Y(j\omega) = H(j\omega)X(j\omega) \quad Y(j\omega) \xrightarrow{\mathcal{F}^{-1}} y(t).$$

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

$$X(s) = \int_{-\infty}^{+\infty} x(t) e^{-st} dt,$$

$$\frac{dx(t)}{dt} \longrightarrow sX(s).$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k},$$

$$H(j\omega) = H(s)|_{s=j\omega}$$

$$H(s) = \frac{b_M}{a_N} \frac{\prod_{k=1}^{M}(s-n_k)}{\prod_{k=1}^{N}(s-p_k)}.$$

$$\Re\{p_k\} < 0.$$

$$H(e^{j\omega}) = \sum_{k=0}^{\infty} h[k] e^{-j\omega k}$$

$$H(e^{j\omega}) = H^\star(e^{-j\omega})$$

$$x[n] = C_1 \cos(\omega_1 n + \phi_1):$$

$$y[n] = C_1 |H(e^{j\omega_1})| \cos(\omega_1 n + \phi_1 + \arg H(e^{j\omega_1}))$$

$$y[n] = \sum_{k=0}^{Q} b_k x[n-k] - \sum_{k=1}^{P} a_k y[n-k],$$

$$h[n] = \begin{cases} 0 & \text{pro } \ n<0 \ \text{ a pro } \ n>Q \\ b_n & \text{pro } \ 0 \le n \le Q \end{cases}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n},$$

$$x[n-k] \longrightarrow z^{-k} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{Q} b_k z^{-k}}{1 + \sum_{k=1}^{P} a_k z^{-k}} = \frac{B(z)}{A(z)},$$

$$H(e^{j\omega}) = H(z)|_{z=e^{j\omega}}$$

$$H(z) = \frac{B(z)}{A(z)} = b_0 z^{P-Q} \frac{\prod_{k=1}^{Q}(z-n_k)}{\prod_{k=1}^{P}(z-p_k)},$$

$$|p_k| < 1$$

$$F(x,t) = \mathcal{P}\{\xi(t) < x\}, \quad F(x,n) = \mathcal{P}\{\xi[n] < x\},$$

$$p(x,t) = \frac{\delta F(x,t)}{\delta x}, \quad p(x,n) = \frac{\delta F(x,n)}{\delta x}$$

$$\int_{-\infty}^{+\infty} p(x,t) dx = 1$$

$$\mathcal{P}\{a < \xi(t) < b\} = F(b,t) - F(a,t) = \int_a^b p(x,t) dx$$

$$a(t) = E\{\xi(t)\} = \int_{-\infty}^{+\infty} x p(x,t) dx$$

$$a[n] = E\{\xi[n]\} = \int_{-\infty}^{+\infty} x p(x,n) dx$$

$$D(t) = E\{[\xi(t) - a(t)]^2\} = \int_{-\infty}^{+\infty} [x - a(t)]^2 p(x,t) dx$$

$$D[n] = E\{[\xi[n] - a[n]]^2\} = \int_{-\infty}^{+\infty} [x - a[n]]^2 p(x,n) dx$$

$$\sigma(t) = \sqrt{D(t)} \qquad \sigma[n] = \sqrt{D[n]}$$

$$\hat{D}(t) = \frac{1}{\Omega} \sum_{\omega=1}^{\Omega} [\xi_\omega(t) - \hat{a}(t)]^2, \quad \hat{\sigma}(t) = \sqrt{\hat{D}(t)}, \quad \hat{D}[n], \sigma[n] = \dots$$

$$R(t_1,t_2) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x_1 x_2 p(x_1,x_2,t_1,t_2) dx_1 dx_2,$$

$$R(n_1,n_2) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x_1 x_2 p(x_1,x_2,n_1,n_2) dx_1 dx_2,$$

$$\hat{a} = \frac{1}{T} \int_0^T x(t) dt \quad \hat{D} = \frac{1}{T} \int_0^T [x(t) - \hat{a}]^2 dt \quad \hat{\sigma} = \sqrt{\hat{D}}$$

$$\hat{a} = \frac{1}{N} \sum_{n=0}^{N-1} x[n] \quad \hat{D} = \frac{1}{N} \sum_{n=0}^{N-1} [x[n] - \hat{a}]^2 \quad \hat{\sigma} = \sqrt{\hat{D}}$$

$$\hat{R}(\tau) = \frac{1}{T} \int_0^T x(t) x(t+\tau) dt$$

$$\hat{R}_{vych}[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] x[n+k],$$

$$\hat{R}_{nevych}[k] = \frac{1}{N-|k|} \sum_{n=0}^{N-1} x[n] x[n+k],$$

$$G(j\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} R(\tau) e^{-j\omega\tau} d\tau \quad R(\tau) = \int_{-\infty}^{+\infty} G(j\omega) e^{+j\omega\tau} d\omega$$

$$G(e^{j\omega}) = \sum_{k=-\infty}^{\infty} R[k] e^{-j\omega k} \quad R[k] = \frac{1}{2\pi} \int_{-\pi}^{\pi} G(e^{j\omega}) e^{+j\omega k} d\omega$$

$$\text{pro } \ \omega_k = \frac{2\pi}{N} k \quad \hat{G}(e^{j\omega_k}) = \frac{1}{N} |X[k]|^2.$$

$$G_y(j\omega) = |H(j\omega)|^2 G_x(j\omega)$$

$$G_y(e^{j\omega}) = |H(e^{j\omega})|^2 G_x(e^{j\omega})$$

$$P = R[0] = a^2 + D$$

$$\Delta = \frac{x_{max} - x_{min}}{L-1} \approx \frac{x_{max} - x_{min}}{L}.$$

$$x[n] \to x_q[n] \quad e[n] = x[n] - x_q[n].$$

$$P_e = \frac{\Delta^2}{12} \qquad SNR = 10 \log_{10} \frac{P_s}{P_e} \quad [\text{dB}].$$

$$SNR = 1.76 + 6\,b \ \text{ dB}.$$

$$X[m,n] = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} x[k,l] e^{-j2\pi\left(\frac{mk}{M} + \frac{nl}{N}\right)}.$$

$$y[k,l] = x[k,l] \star h[k,l] = \sum_{i=-\frac{I}{2}}^{\frac{I}{2}} \sum_{j=-\frac{J}{2}}^{\frac{J}{2}} h[i,j] x[k-i, l-j]$$