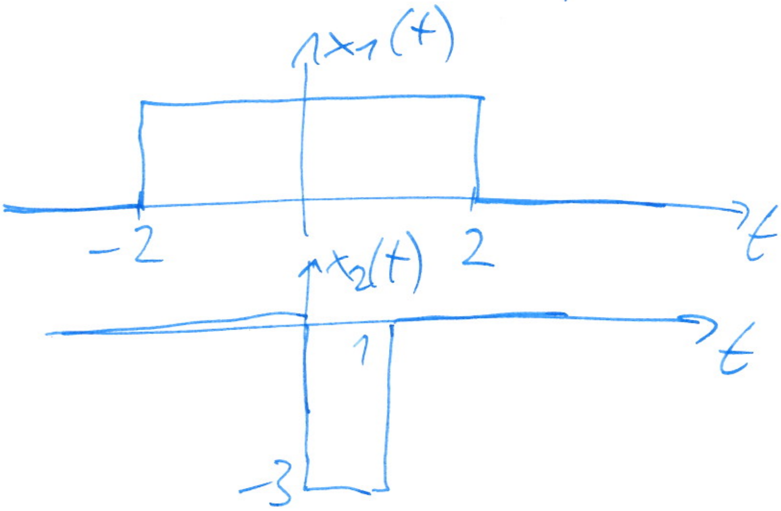
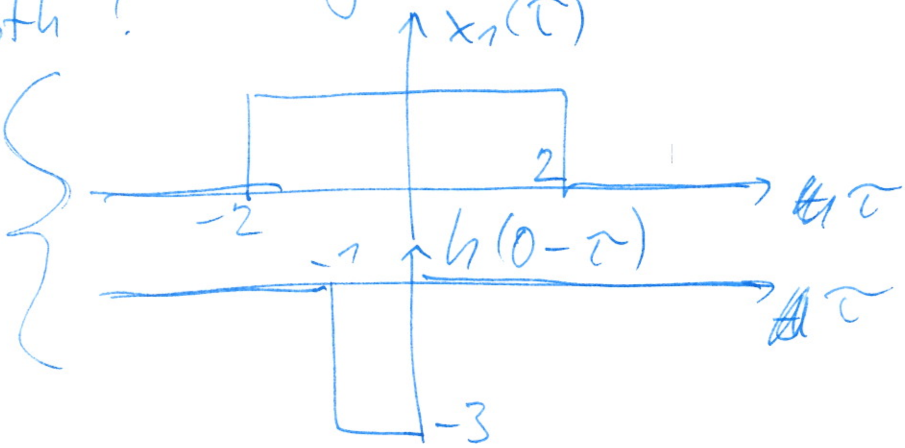
1

ISS Exercise #4

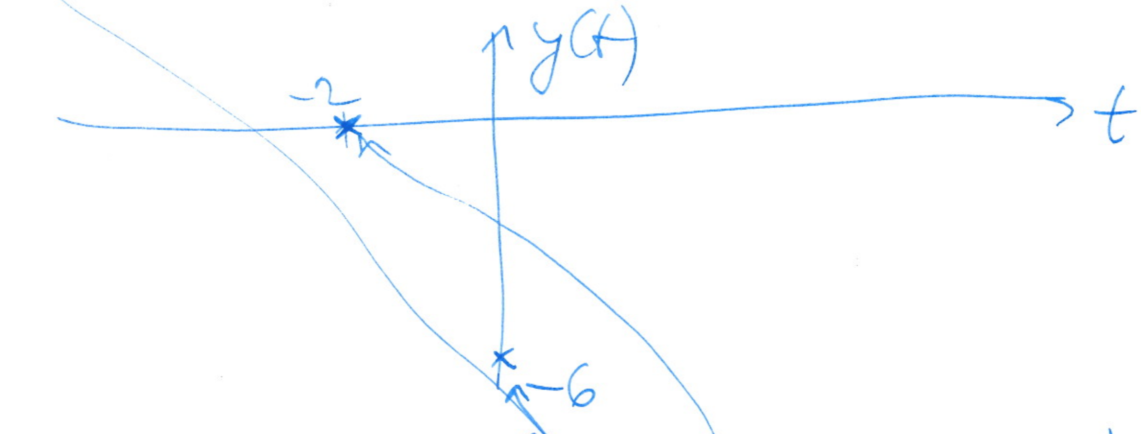
1



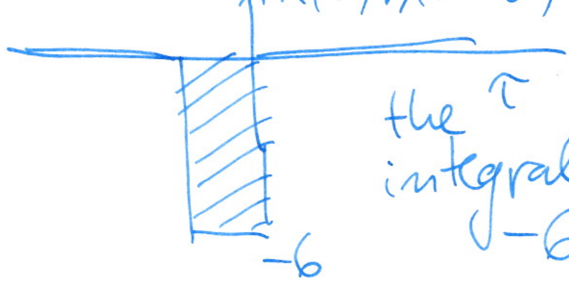
2 Selecting the 1st variant but divide students in group and let them pick both!



3

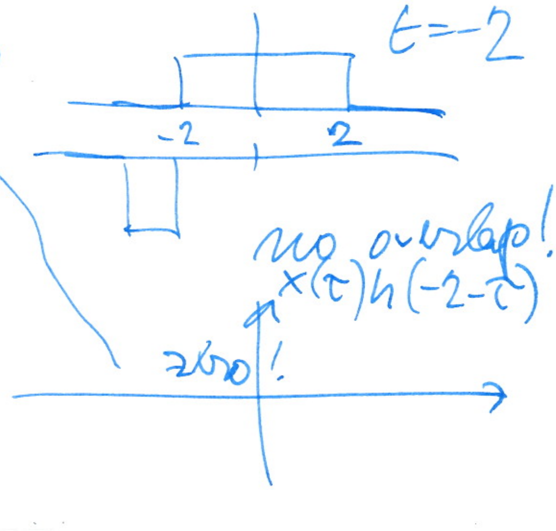


the product of signals



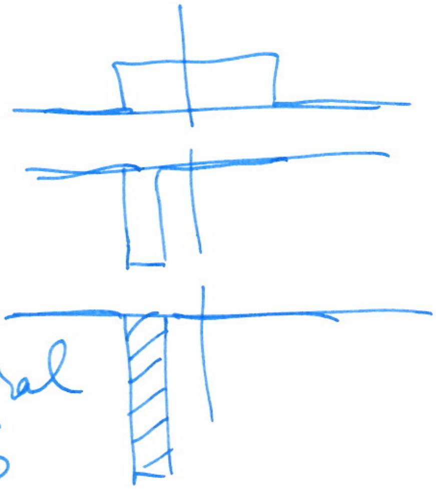
the integral is -6

4



5

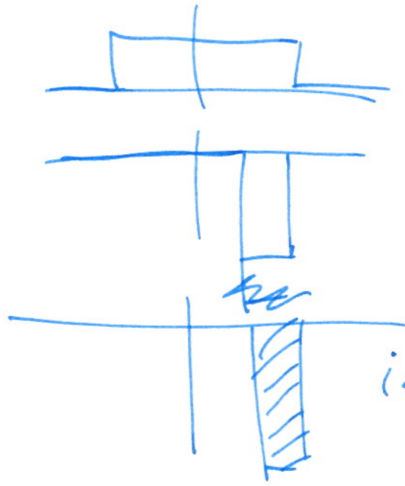
t = -1



integral = −6

6

t = 2

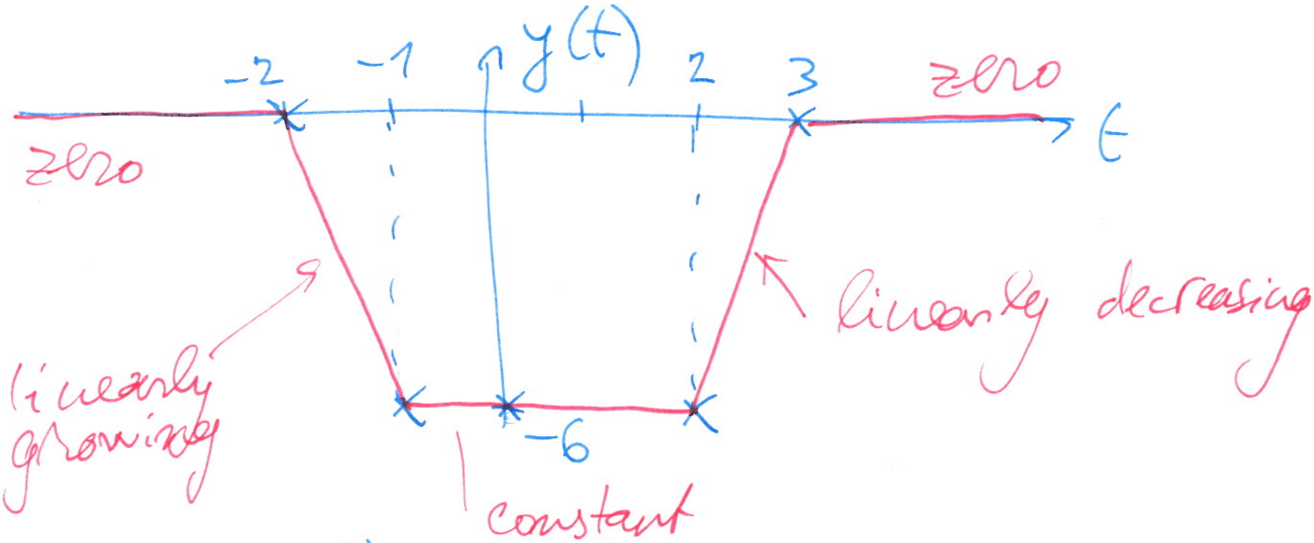


integral = −6

7 t = 3

no overlap ⇒ zero.

8



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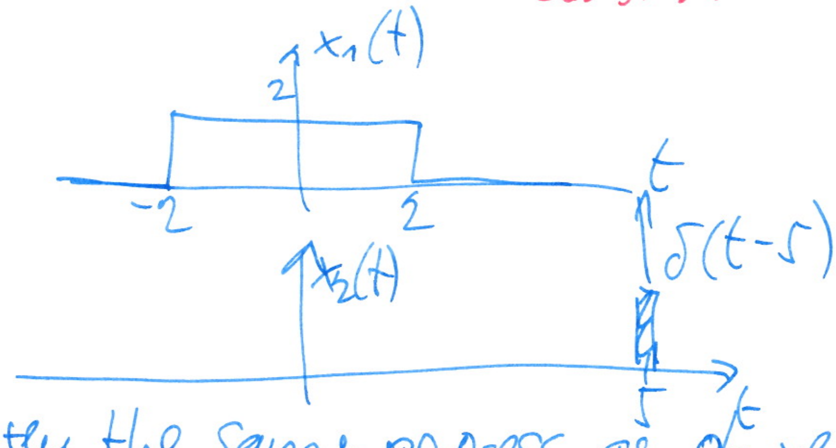
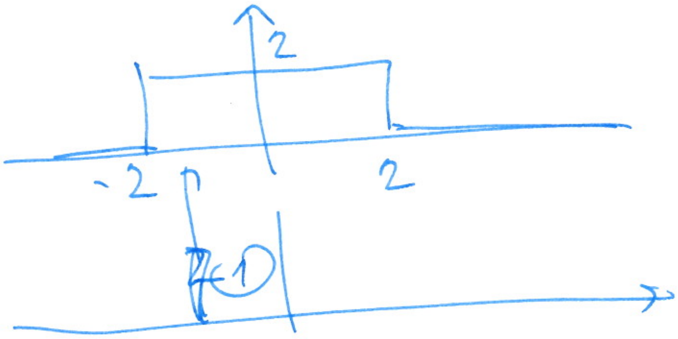


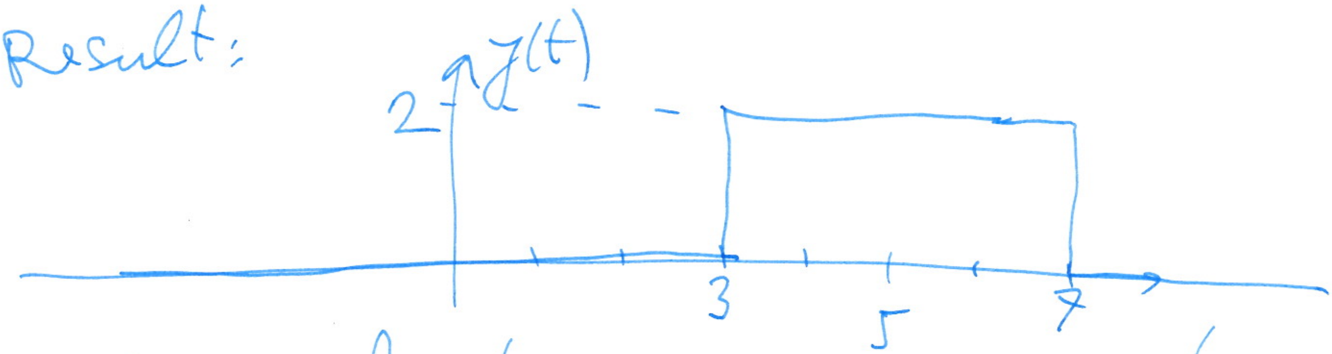
exactly the same process as above … flipping one of the signals and shifting it. Anytime the Dirac hits the rectangle, we have:



product:

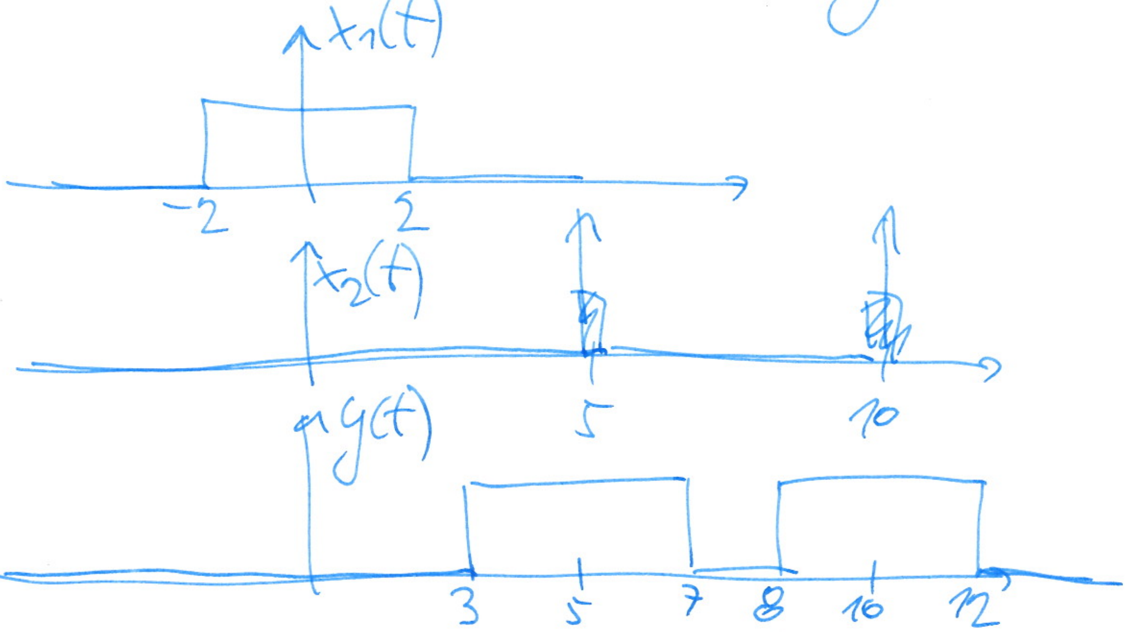
the integral of this is 2.

Result:

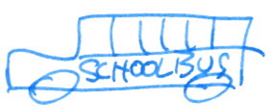


The Dirac functions as a <u>copy-machine</u>.

(10)



Again a <u>copy-machine</u>!

(11) Input $x_1 =$ 

output $y_1 = 40$

Input $x_2 =$ 

output $y_2 = 1000$

(12) Linearity:

$ax_1 + bx_2$ leads to $ay_1 + by_2$.

For us: a school buses and b trucks with carps: input $ax_1 + bx_2$

the result will be $\underbrace{a \cdot 40 \text{ (kids)} + b\,1000 \text{ (carps)}}_{\text{living beings}}$

The same linear combination of original outputs:

$$ay_1 + by_2 = a \cdot 40 + b \cdot 1000$$

the same
⇒ linear!

(13) the result for ~~$ay_1 + by_1$~~ $ax_1 + bx_2$ will be: $\cancel{a \cdot 40} + b \cdot 1000 = b \cdot 1000$

eaten by piranhas!

The same lin. combination of original outputs

$$ay_1 + by_2 = a \cdot 40 + b \cdot 1000$$

not same
non-linear!

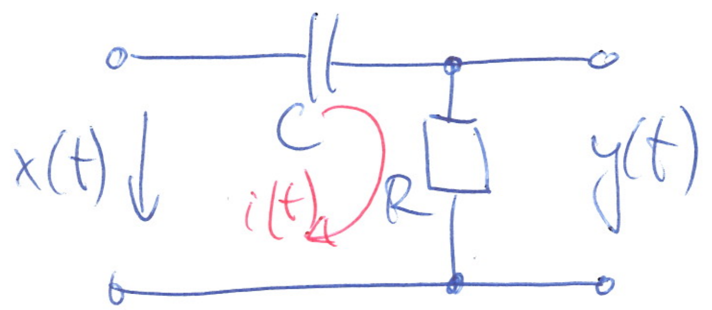
14 $\omega_1 = 160\pi$ rad/s, needs to have $H(j\omega_1)$ for this frequency. $80\,Hz \approx 160\pi$, we have it $H(j160\pi) = 10 \cdot e^{-j0{,}5\pi}$

The cosine has magnitude multiplied by 10 and phase delayed by $0{,}5\pi$:

$$y(t) = 10 \cdot 45 \cos(160\pi t + 0{,}4\pi - 0{,}5\pi) =$$
$$= \underline{\underline{450 \cos(160\pi t - 0{,}1\pi)}}$$

~~Lab #3 Continuous-time systems~~

We will study the behavior of the following circuit:



1 Find differential equation describing the circuit!

15 We'll use current $i(t)$ that can be determined in two ways:

$$\left.\begin{aligned} i(t) &= \frac{y(t)}{R} \\ i(t) &= C\,\frac{d(x(t)-y(t))}{dt} \end{aligned}\right\} \text{this is the same current!}$$

16 $$y(t) = RC\,\frac{dx(t)}{dt} - RC\,\frac{dy(t)}{dt}$$

17 Do its Laplace transform and find the transfer function of the circuit:

"Vocabulary" of LT:

$$x(t) \longrightarrow X(s)$$

$$a\,x(t) \longrightarrow a\,X(s)$$

That's all we need!

$$\frac{dx(t)}{dt} \longrightarrow s\,X(s)$$

$$Y(s) = RCs\,X(s) - RCs\,Y(s)$$

... let's try to group everything depending on $Y(s)$ on the ~~righ~~ left, $X(s)$ on the right ...

$$Y(s)\left[1 + RCs\right] = X(s)\,RCs$$

Now, find $H(s)$ that is defined as $\frac{Y(s)}{X(s)}$

18

$$H(s) = \frac{Y(s)}{X(s)} = \frac{RCs}{1 + RCs}$$

19 What are the coefficients b_k, a_k?

$$H(s) = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k}$$

$b_0 = 0$ $b_1 = RC$ $M = 1$

$a_0 = 1$ $a_1 = RC$ $N = 1$

20 Convert it to the form including zeros and poles

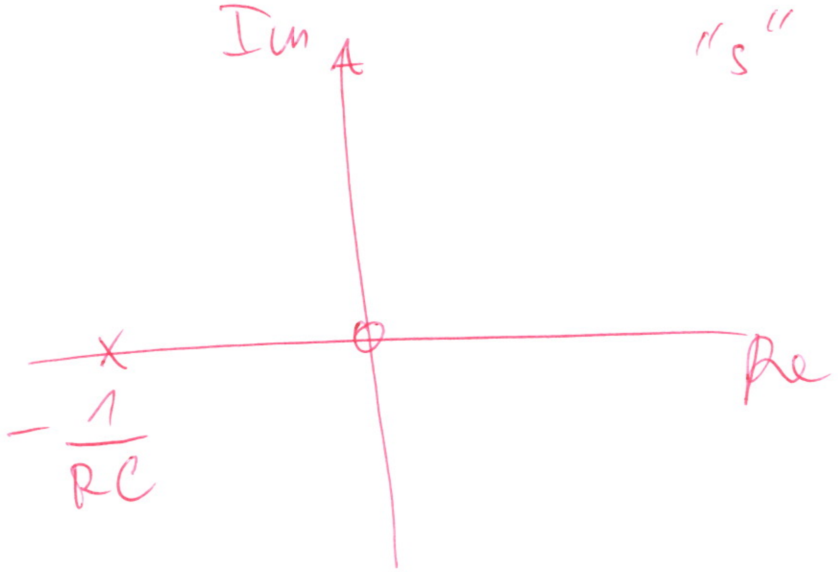
$$H(s) = \frac{b_M}{a_N} \cdot \frac{\prod_{k=1}^{M} (s - n_k)}{\prod_{k=1}^{N} (s - p_k)} = \frac{RC\,(s - 0)}{RC\left(s - \left(-\frac{1}{RC}\right)\right)} =$$

$$= \frac{s - 0}{s - \left(-\frac{1}{RC}\right)}$$

0 is the only zero

$-\frac{1}{RC}$ is the only pole.

(21) Draw them into the "s" plane and (8) check stability of the circuit ...



Real part of $-\frac{1}{RC}$ smaller than zero

$\Rightarrow$ <u>stable</u> :-)

(22) Determine the frequency response, pay particular attention to

$\omega = 0{,}000001$

$\omega = \frac{1}{RC}$

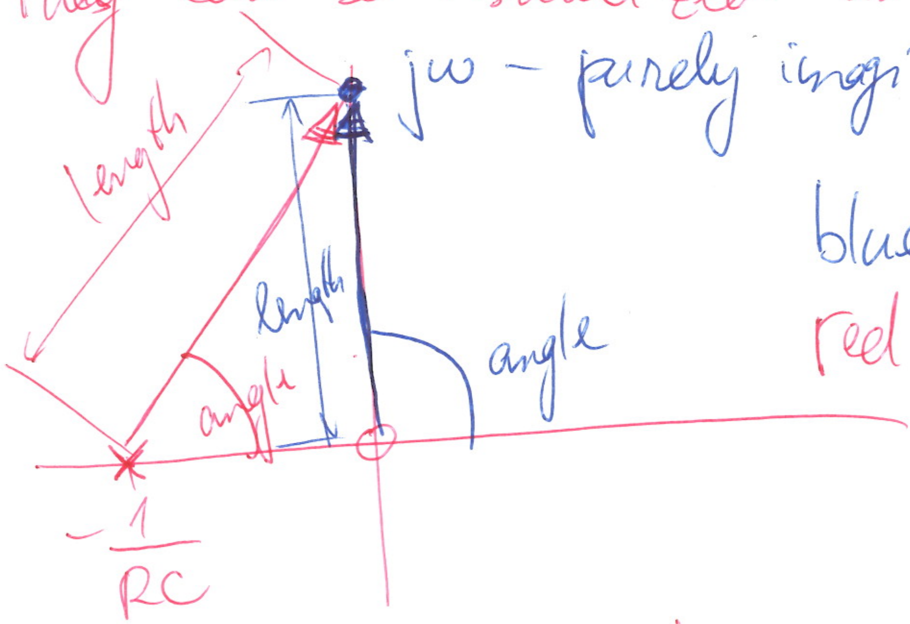
$\omega = \infty$

H(s) to H(jω) by simply re-(writing) all occurrences of 's' by 'jω'

$$H(j\omega) = \frac{b_M}{a_N} \frac{\prod_{k=1}^{M} (j\omega - h_k)}{\prod_{k=1}^{N} (j\omega - p_k)} =$$

$$= \frac{j\omega - 0}{j\omega - \left(-\frac{1}{RC}\right)}$$

Numerator and denominator are complex numbers. They can be visualized as vectors!



$j\omega$ – purely imaginary, imag. axis!

blue one: $j\omega - 0$

red one: $j\omega - \left(-\frac{1}{RC}\right)$

$H(j\omega)$ is given by division of these complex numbers:

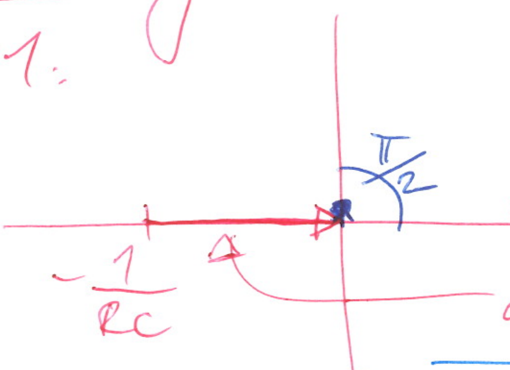
module is <u>division</u> of modules

argument is <u>subtraction</u> of arguments

Module ≈ length of vector

Arg ≈ angle of vector

$\omega = 0{,}000001$:

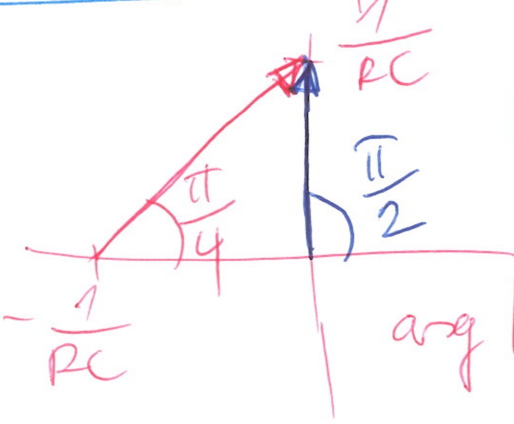


$$|H(j0)| = \frac{\text{almost } 0}{\frac{1}{RC}} = 0$$

$$\arg H(j0) = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

23

$\omega = \frac{1}{RC}$

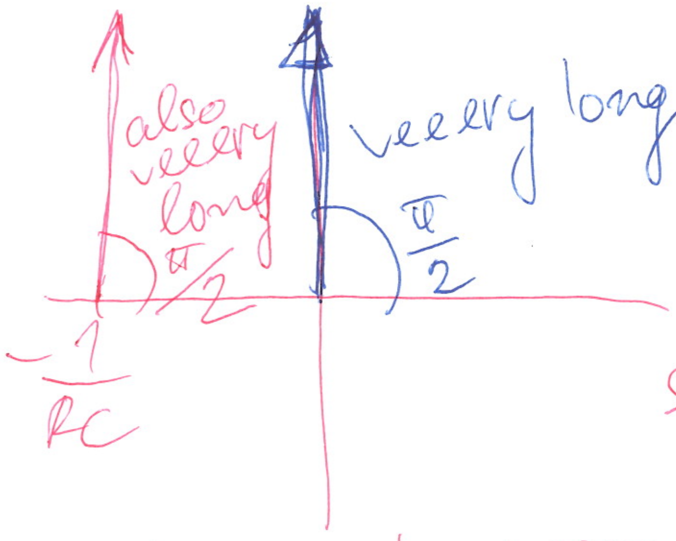


$$\left|H\left(j\frac{1}{RC}\right)\right| = \frac{\omega}{\sqrt{\omega^2+\omega^2}} = \frac{1}{\sqrt{2}}$$

$$\arg H\left(j\frac{1}{RC}\right) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

24

$\omega = \infty$

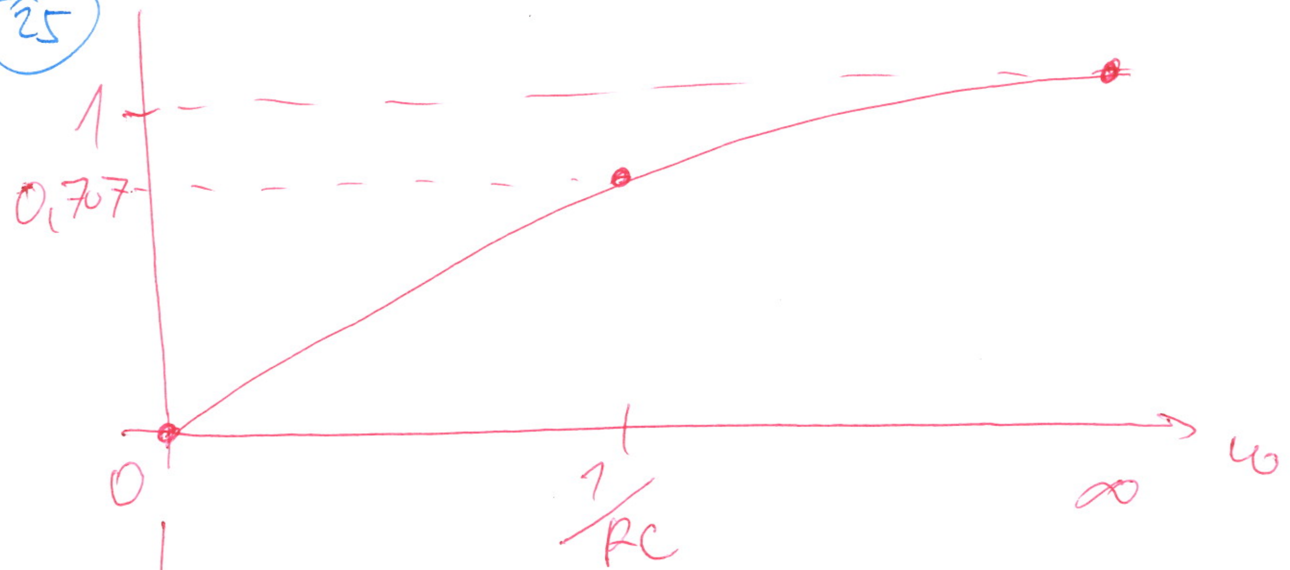


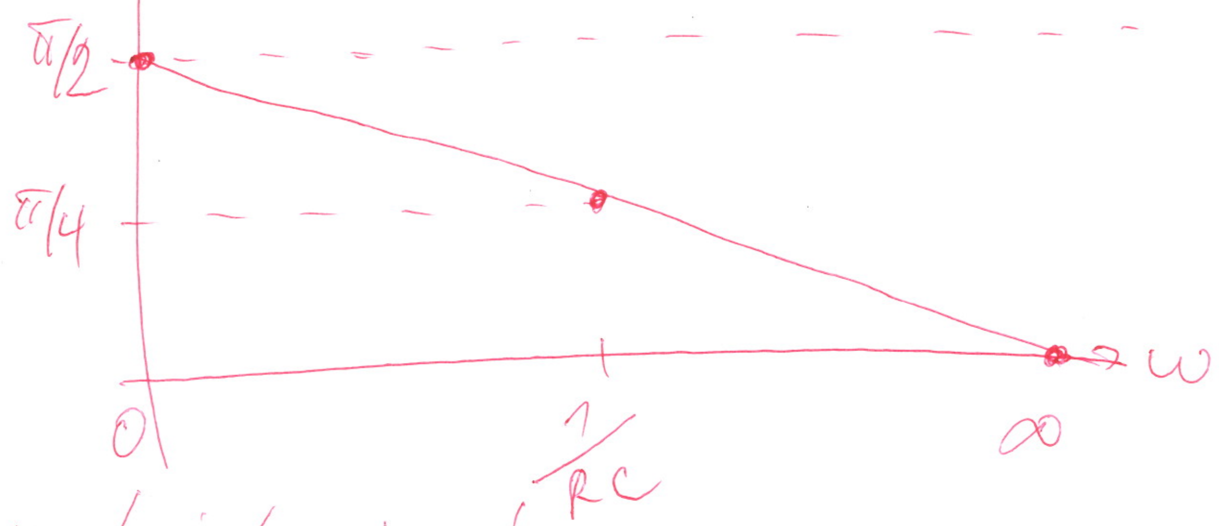
Their lengths are almost the same.

$$|H(j\infty)| = \frac{\text{veeery long}}{\text{veeery long}} = 1$$

$$\arg H(j\infty) = \frac{\pi}{2} - \frac{\pi}{2} = 0$$

25





$\Rightarrow$ high-pass!

$\Rightarrow$ look at the plot for $R = 1\,k\Omega$, $C = 1000\,pF$

[redacted]

26 Check stability of system described by transfer function

$$H(s) = \frac{1}{s^2 - 2s + 2}$$

We want to express it as

$$H(s) = \frac{b_M}{a_N} \frac{\prod_{k=1}^{M} (s - n_k)}{\prod_{k=1}^{N} (s - p_k)}$$

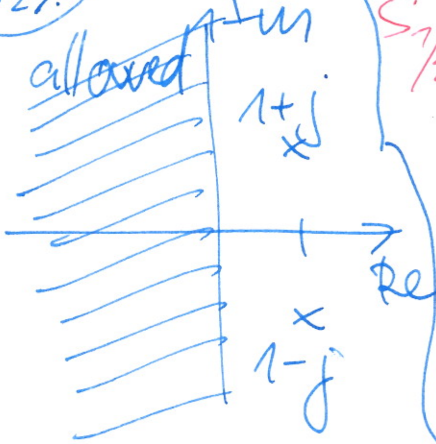
$b_M = a_N = 1$, ok, no need.

numerator – no zero, ok.

Denominator needs some work...

$$s^2 - 2s + 2 = 0$$

27.



$$s_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{4 - 4.1.2}}{2} = \frac{2 \pm \sqrt{-4}}{2} =$$

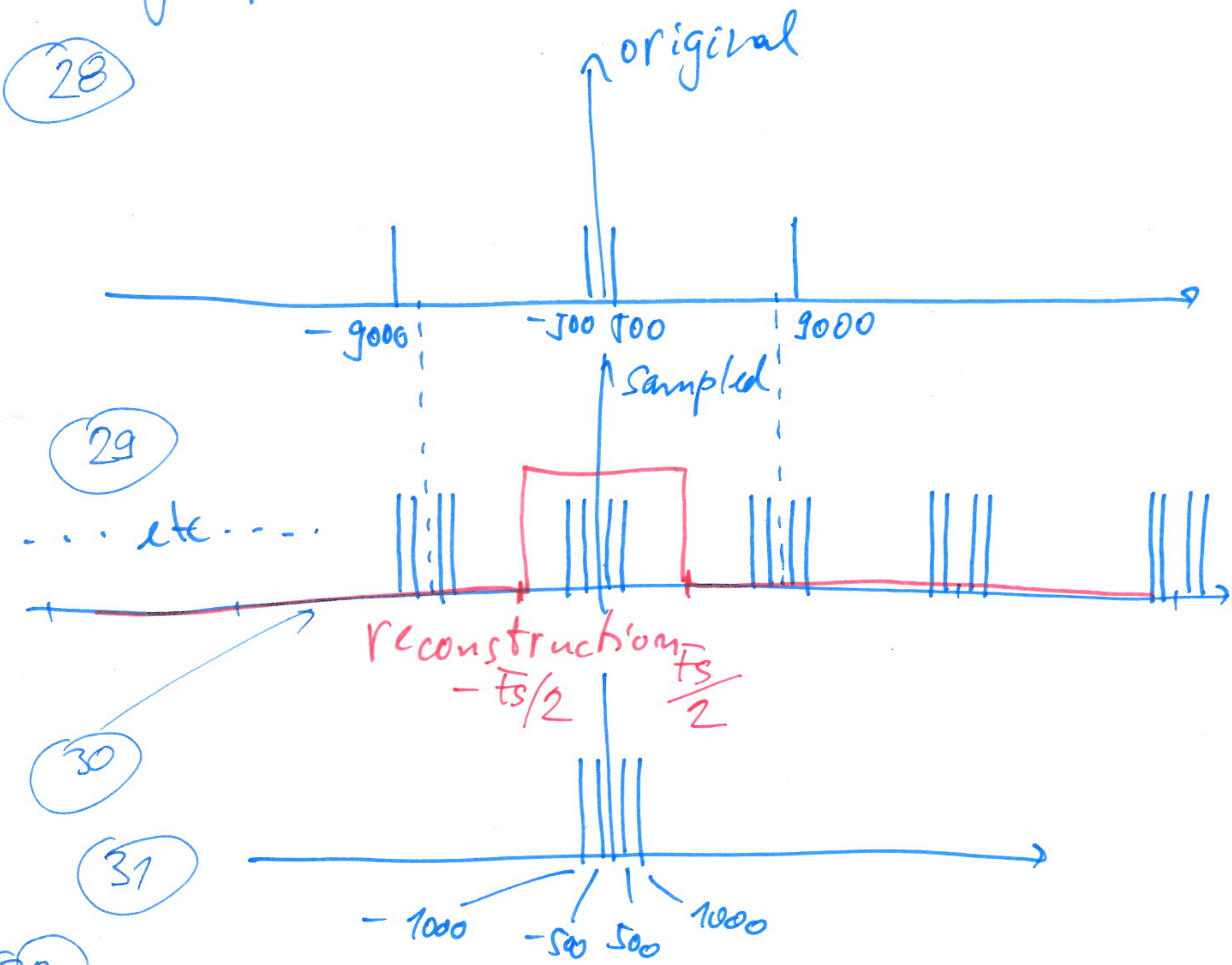
$$= 1 \pm j$$

Real component > 0

$\Rightarrow$ not stable

Continuous time signal is a mix of two cosines: at 500 Hz and 9000 Hz.
It undergoes perfect sampling and perfect reconstruction at $F_s = 8000$ Hz.
What will be the output?

Sampling means periodization of spectrum, reconstruction means filtering from $-\frac{F_s}{2}$ to $\frac{F_s}{2}$:



32
This is a spectrum of a mix of two cosines: at 500 Hz and at 1 kHz. Different from original, FAILED!