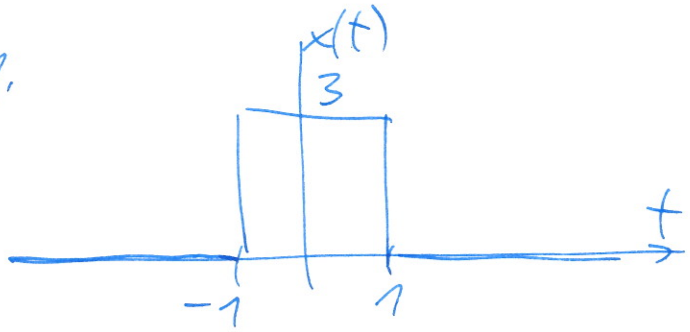
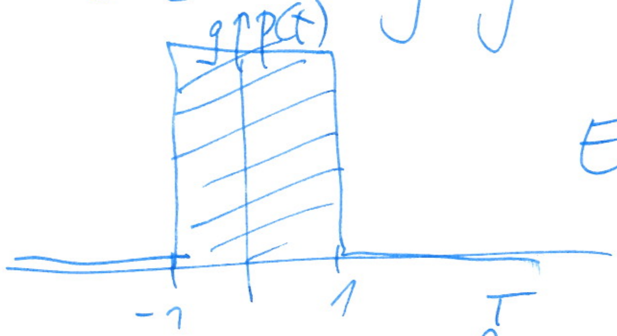
ISS Numerical exercise #3

1.



2. We will need instantaneous power (okamžitý výkon) $p(t) = x^2(t)$

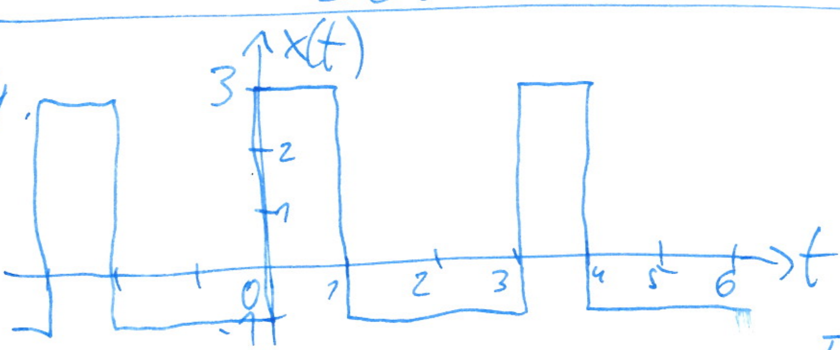


$$E = \int_{-\infty}^{\infty} p(t)\,dt = 2 \cdot 9 = 18$$

3. $$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} p(t)\,dt = \frac{1}{2 \cdot \infty} \underbrace{\int_{-\infty}^{\infty} p(t)\,dt}_{18} =$$

$$= \frac{18}{2 \cdot \infty} = \emptyset.$$

4.

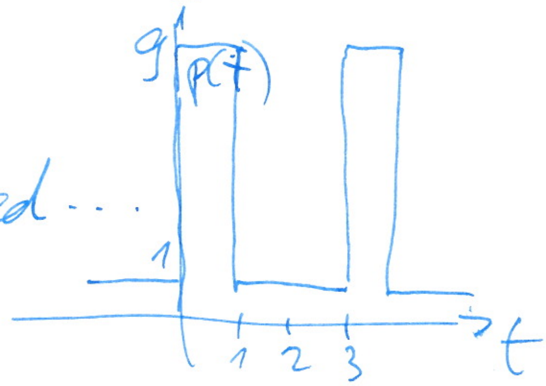


5. Mean value $$\overline{x} = \frac{1}{T_1} \int_0^{T_1} x(t)\,dt = \frac{1}{3}(3 \cdot 1 + 2 \cdot (-1)) =$$

$$= \underline{\underline{\frac{1}{3}}}$$

6. Again, inst. power is needed...

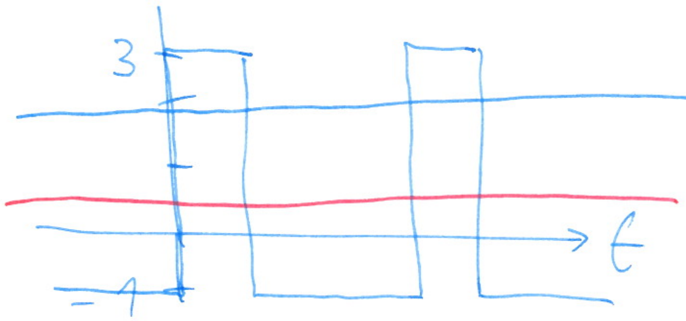
$$E = \int_0^{T_1} p(t)\,dt = 9 + 2 = \underline{\underline{11}}$$



7. $P = \frac{E}{T} = \frac{1}{T}\int_0^{T_1} p(t)\,dt = \frac{1}{3}\cdot 11 = \frac{11}{3} = 3{,}66$

8. $C_{ef} = \sqrt{P} = \sqrt{3{,}66} = 1{,}91$

9.



$1{,}91 = C_{ef}$

$\frac{1}{3} = \bar{x}$ Different because:

- for $\bar{x}$, the negative part "pulls it down"
- for C_{ef}, the negative part "contributes".

10. For cosine, $P = \frac{C_1^2}{2}$, where C_1 is magnitude.

$x(t) = \cancel{2} + \underbrace{4\cdot\cos(2000\pi t)}$

$P = \frac{4^2}{2} = \frac{16}{2} = 8$

11. $\omega_1 = 2\pi\cdot f_1 = 2000\pi$ rad/s

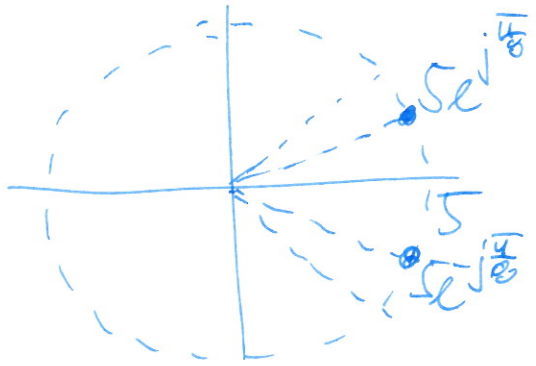
$x(t) = 10\cos\left(2000\pi t + \frac{\pi}{8}\right) = \frac{10}{2}e^{j(2000\pi t + \frac{\pi}{8})} + \frac{10}{2}e^{-j(2000\pi t + \frac{\pi}{8})} =$

$= \left[\text{using } e^{a+b} = e^a\cdot e^b\right] =$

$= \underbrace{5e^{j\frac{\pi}{8}}}_{c_1}\cdot e^{j2000\pi t} + \underbrace{5\cdot e^{-j\frac{\pi}{8}}}_{c_{-1}} e^{-j2000\pi t}$

12. $c_1 = 5e^{j\frac{\pi}{8}}$ $\qquad c_{-1} = 5 \cdot e^{-j\frac{\pi}{8}}$

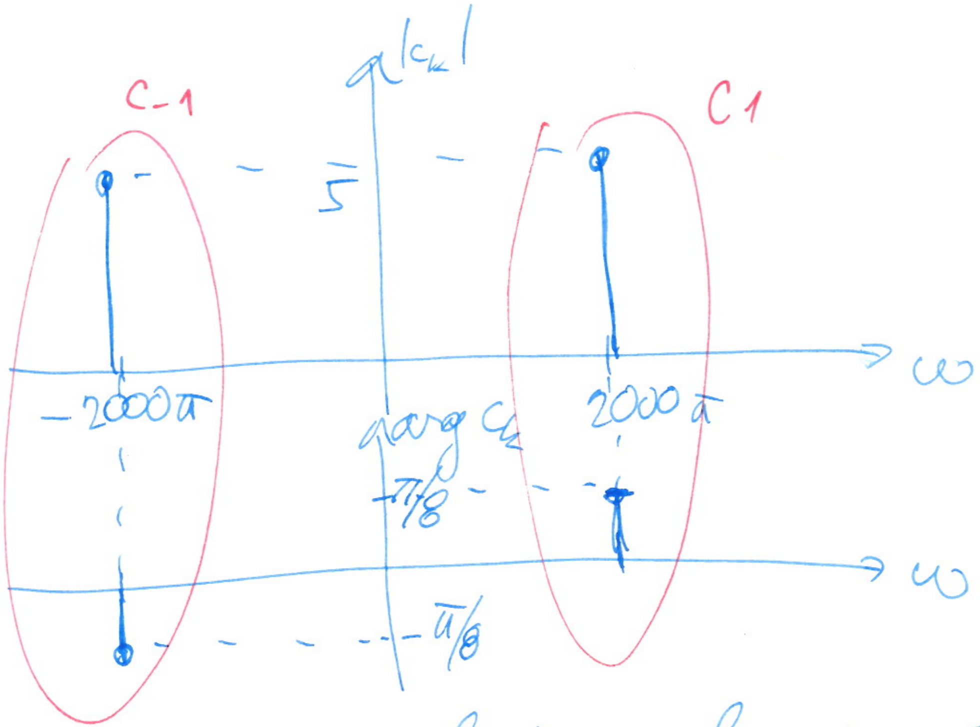
You can also draw them to complex plane



Komplex conjugation:

- magnitude the same - YES
- angles opposite $\pi/8$ v $-\pi/8$ - YES.

13.



14. If the signal is real, $c_k = c_{-k}^*$ must hold, so completing: $c_{-1} = 4e^{-j\frac{\pi}{4}}$, $c_2 = 2e^{j\frac{\pi}{2}}$

Making a cosine out of a pair of coefficients:

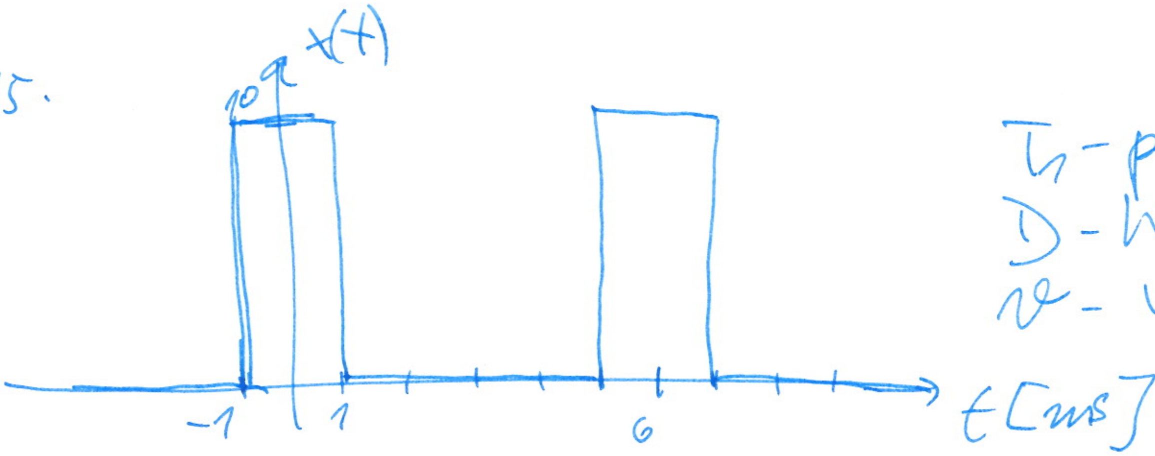
$$C_k = 2|c_k| = 2|c_{-k}| \qquad \varphi_k = \arg c_k = -\arg c_{-k}$$

$$\omega_1 = \frac{2\pi}{T_1} = 2000\pi \text{ rad/s}$$

$$x(t) = 8\cos\left(2000\pi t + \frac{\pi}{4}\right) + 4\cos\left(4000\pi t + \frac{\pi}{2}\right)$$

4.

15.



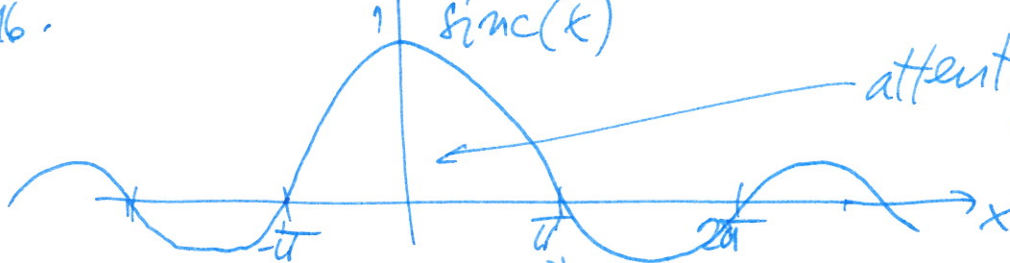
T_1 - period $6\cdot 10^{-3}$

D - height 10

ϑ - width $2\cdot 10^{-3}$

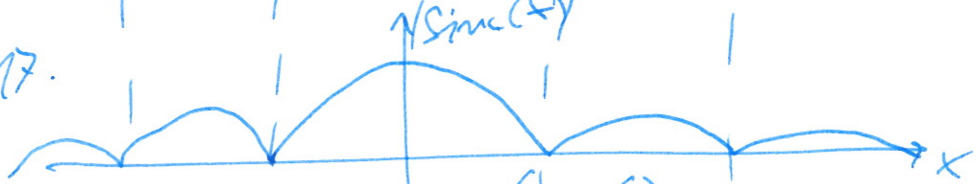
$\frac{\vartheta}{T_1}$ = "duty cycle"

16.



attention! Twice the width of the other "lobes" !!!

17.



can be any odd multiple of π, picking $+\pi$/$-\pi$ for having it "beautiful".

18. Only **blue** part first !!!

It is $\frac{D\vartheta}{T_1}\cdot \text{sinc}\left(\frac{\vartheta}{2}\omega\right)$

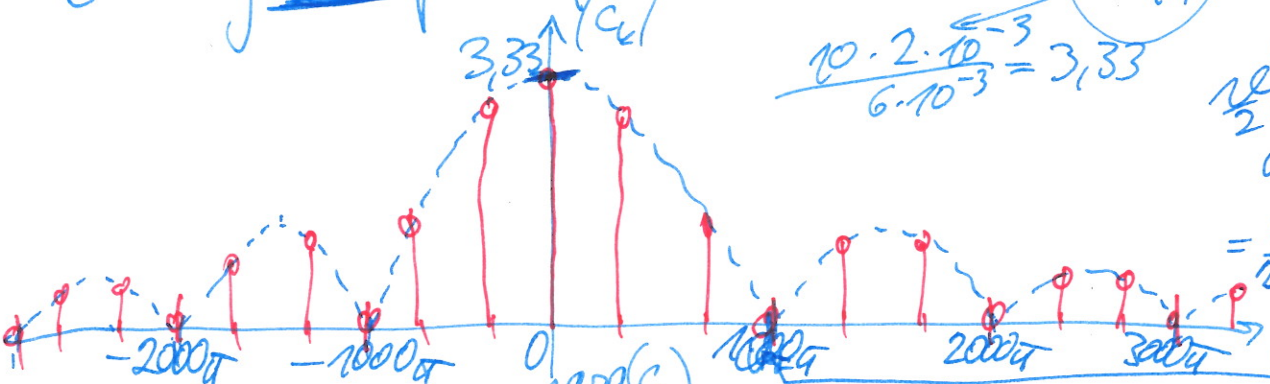
$\frac{10\cdot 2\cdot 10^{-3}}{6\cdot 10^{-3}} = 3{,}33$

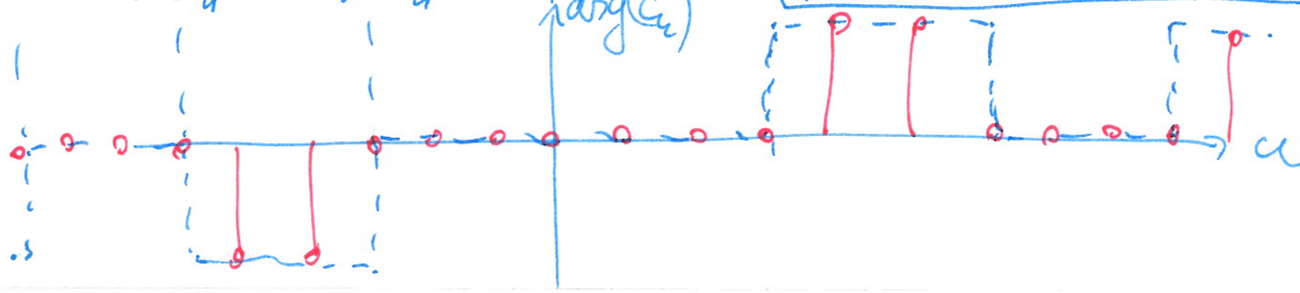
must be π

$\frac{\vartheta}{2}\omega = \pi$

$\omega = \frac{2\pi}{\vartheta} =$

$= \frac{2\pi}{2\cdot 10^{-3}} = 1000\pi$ rad/s





5

19. $\omega_1 = \frac{2\pi}{T_1} = \frac{2\pi}{6 \cdot 10^{-3}} = 333{,}3\pi$ rad/s

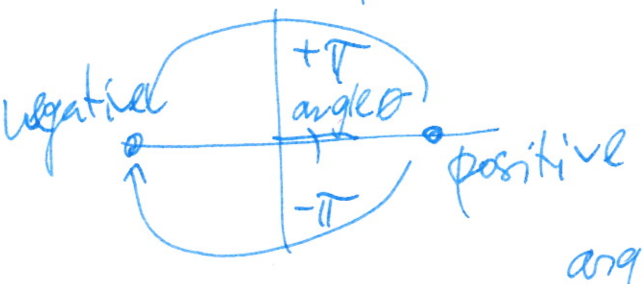
Now drawing the red coefficients at the multiples ~~of~~ $333{,}3\pi$ ...

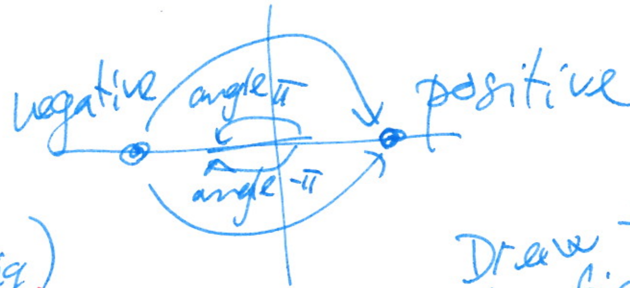
... actually, for $c_3, c_6, c_9, \ldots, c_{-3}, c_{-6}, c_{-9}, \ldots$ the phase can be zero, or $\pm\pi$ or anything else, but we're reasonable and set it to zero.

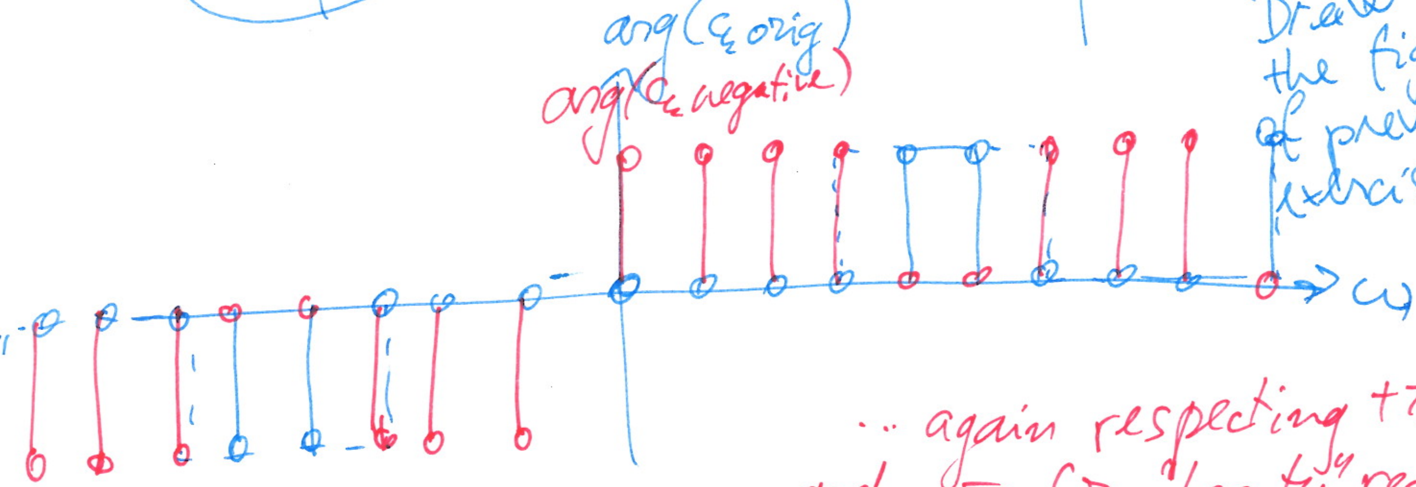
20. If the signal changes the sign, it's enough to change sign of all coefficients c_k.

magnitudes $\longrightarrow$ stay the same

Angles $\longrightarrow$ change them from 0 to $\pm\pi$, and from $\pm\pi$ to zero. Explanation:







Draw to the figure of previous exercise...

... again respecting $+\pi$ and $-\pi$ for "beauty" reasons.

21. Delay $\tau = 0{,}5 \cdot 10^{-3}$ s.

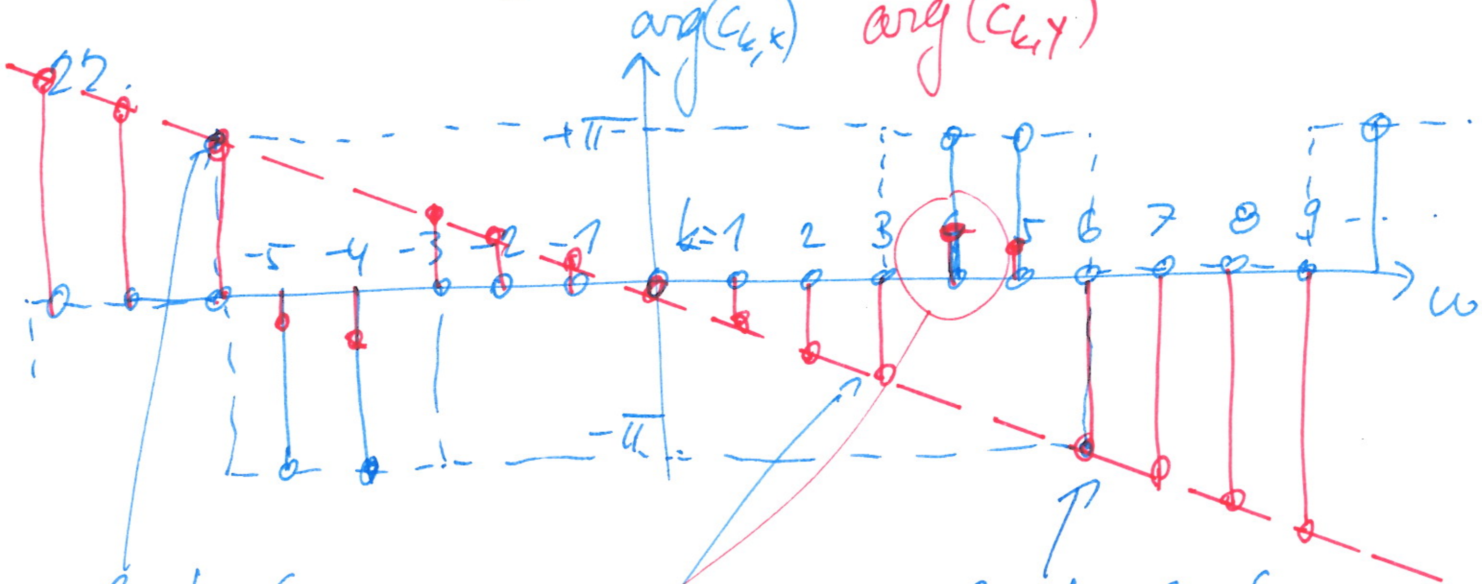
$c_{k,y} = c_{k,x} \cdot e^{-jk\omega_1\tau} = c_{k,x} \cdot e^{-jk \frac{1000}{3}\pi \cdot 0{,}5 \cdot 10^{-3}} =$

$= \underline{c_{k,x} \cdot e^{-jk\frac{\pi}{6}}}$

This multiplication does not change the magnitude, but it changes the phase

$$\arg(c_{k,y}) = \arg(c_{k,x}) - k\frac{\pi}{6}$$

22.



for $k=-6$, $-(-6)\frac{\pi}{6} = +\pi$

for $k=6$, $-\frac{6}{6}\pi = -\pi$

auxiliary line, generating values $-k\frac{\pi}{6}$ but need to add the original coefficients!

for example: original one: π

$-\frac{4}{6}\pi = -\frac{2}{3}\pi$ new one: $\pi - \frac{2}{3}\pi = \frac{1}{3}\pi$

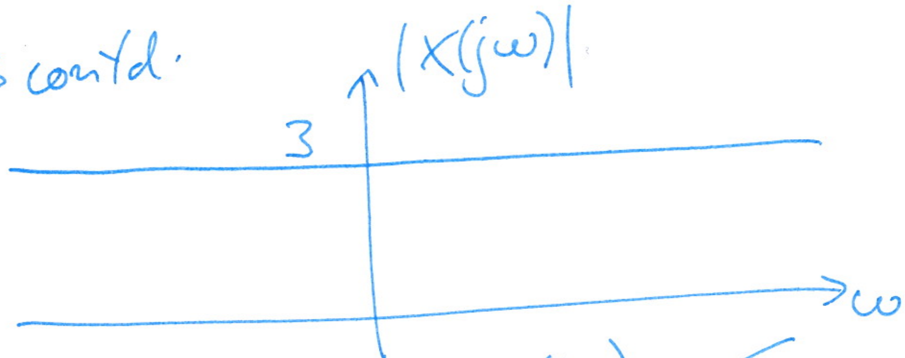
---

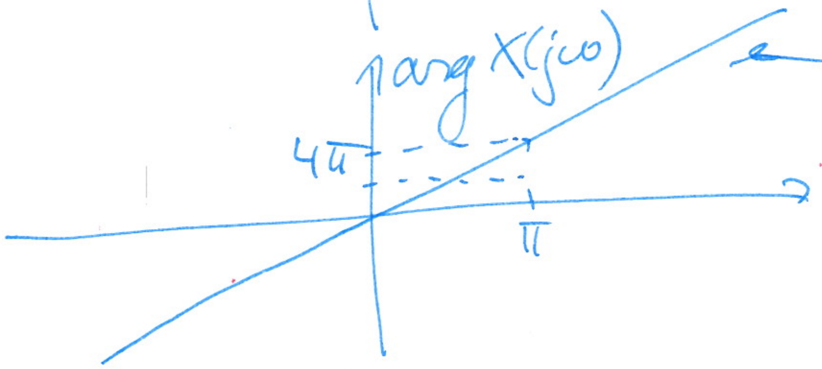
23. Dirac impulse

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\underbrace{e^{-j\omega t}}_{} dt = 3e^{-j\omega(-4)} = \underline{\underline{3e^{j4\omega}}}$$

Dirac is sampling it!

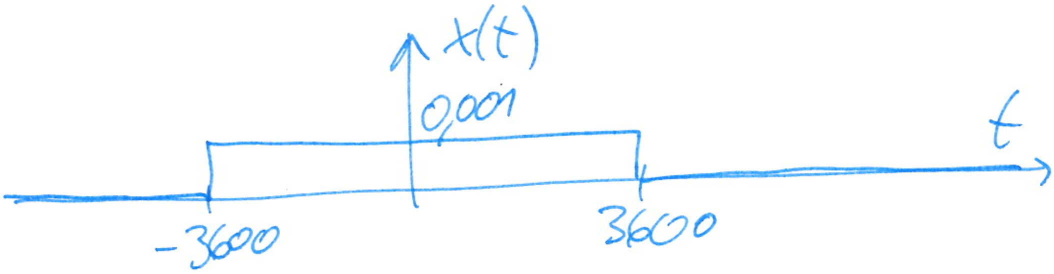
23 cont'd.





the slope of this line is 4.

24.

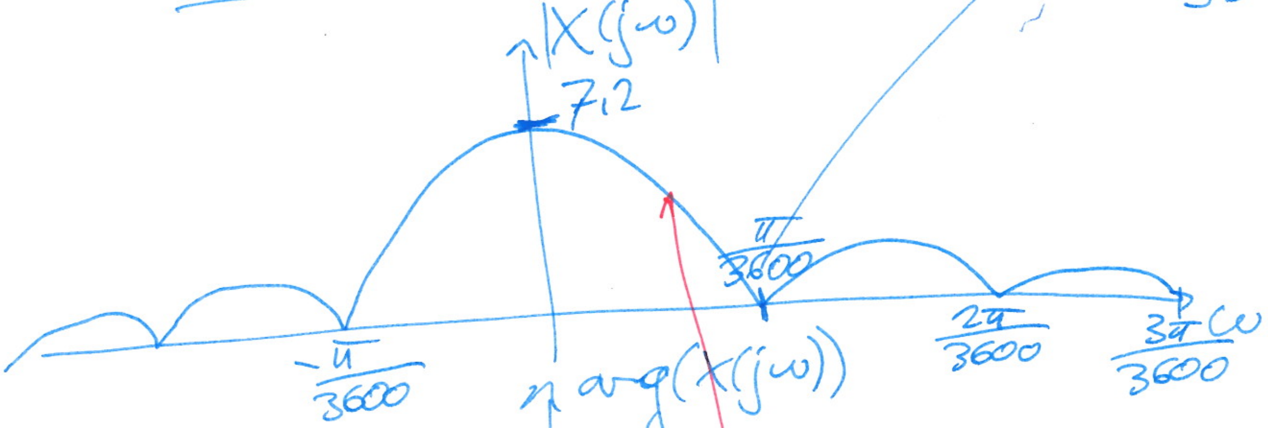


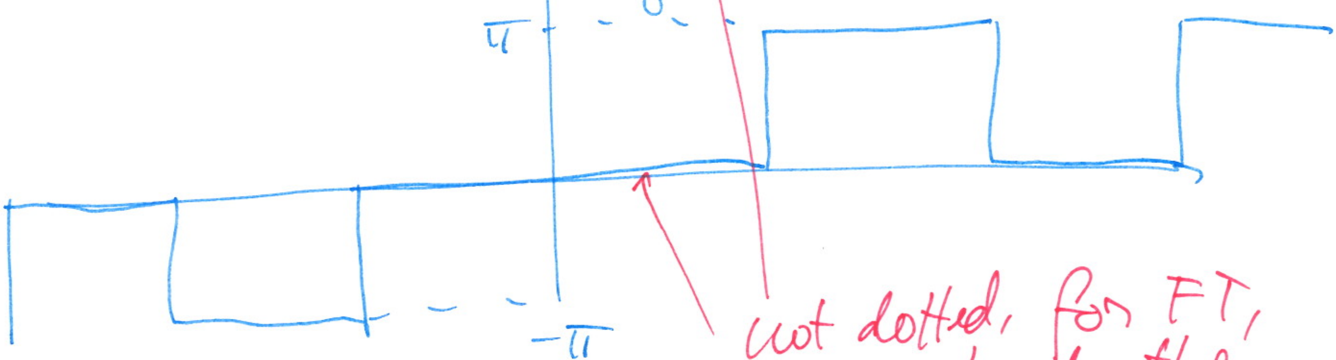
$$X(j\omega) = D\tau \,\mathrm{sinc}\left(\frac{\tau}{2}\omega\right) = 0{,}001 \cdot 7200 \,\mathrm{sinc}(3600\,\omega) =$$

$$= \underline{\underline{7{,}2 \,\mathrm{sinc}(3600\,\omega)}}$$

$3600\,\omega = \pi$

$\omega = \dfrac{\pi}{3600}$





not dotted, for FT, this is already the result.

H

25. $y(t) = x(t + 3600)$

$Y(j\omega) = X(j\omega) \cdot e^{j\omega 3600}$

magnitudes do not change

Angles do:

$\arg Y(j\omega) = \arg X(j\omega) + 3600 \cdot \omega.$