Cas vs frekvemce - v case

$$y[n] = b_0x[n] + b_1x[n-1] + \ldots + b_Qx[n-Q] \quad -a_1y[n-1] - \ldots - a_Py[n-P]$$

$$y[n] = \sum_{k=0}^{Q} b_kx[n-k] - \sum_{k=1}^{P} a_ky[n-k]$$

$$y[n] = h[n] \star x[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

ve frekvenci

$$\omega = 2\pi \frac{f}{F_s}$$

$$X(e^{j\omega}) = DTFT\left\{x[n]\right\} = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$Y(e^{j\omega}) = DTFT\left\{y[n]\right\} = \sum_{n=-\infty}^{\infty} y[n]e^{-j\omega n}$$

prenos a cinitel prenosu

$$\omega_1 = 2\pi \frac{f_1}{F_s}$$

$$H(e^{j\omega_1}) \qquad |H(e^{j\omega_1})| \qquad \arg H(e^{j\omega_1})$$

pro vsechny frekvence - frekv char

$$H(e^{j\omega}) \qquad |H(e^{j\omega})| \qquad \arg H(e^{j\omega})$$

obrazem konvoluce je nasobeni

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$

stupid derivation - nejde ...

$$H(e^{j\omega}) = DTFT\left\{h[n]\right\} = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$$

z-transformace

$$y[n] = b_0x[n] + b_1x[n-1] + \ldots + b_Qx[n-Q] - a_1y[n-1] - \ldots - a_Py[n-P]$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$x[n] \xrightarrow{\mathcal{Z}} X(z)$$

operace s z

$$x[n] \xrightarrow{\mathcal{Z}} X(z)$$

$$ax[n] \xrightarrow{\mathcal{Z}} aX(z)$$

$$a_1x_1[n] + a_2x_2[n] \xrightarrow{\mathcal{Z}} a_1X_1(z) + a_2X_2(z)$$

$$x[n-k] \xrightarrow{\mathcal{Z}} X(z)z^{-k}$$

$$\sum_{n=-\infty}^{\infty} x[n-k]z^{-n} = \sum_{n=-\infty}^{\infty} x[n]z^{-(n+k)} = \sum_{n=-\infty}^{\infty} x[n]z^{-n}z^{-k} = z^{-k}\sum_{n=-\infty}^{\infty} x[n]z^{-n} = z^{-k}X(z)$$

$$x[n-1] \xrightarrow{\mathcal{Z}} X(z)z^{-1}$$

Vztah s DTFT

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \qquad X(z) = x[n]z^{-n}$$

$$z \longrightarrow e^{j\omega}$$

$$X(e^{j\omega}) = X(z)|_{z=e^{j\omega}},$$

z-transf dif rovnice

$$y[n] = b_0x[n] + b_1x[n-1] + \ldots + b_Qx[n-Q] \quad -a_1y[n-1] - \ldots - a_Py[n-P]$$

$$Y(z) = b_0X(z) + b_1X(z)z^{-1} + \ldots + b_QX(z)z^{-Q} \quad -a_1Y(z)z^{-1} - \ldots - a_pY(z)z^{-P}$$

prenos fce

$$H(z) = \frac{Y(z)}{X(z)}$$

$$Y(z) + a_1Y(z)z^{-1} + \ldots + a_pY(z)z^{-P} = b_0X(z) + b_1X(z)z^{-1} + \ldots + b_QX(z)z^{-Q}$$

$$Y(z)(1 + a_1z^{-1} + \ldots + a_pz^{-P}) = X(z)(b_0 + b_1z^{-1} + \ldots + b_Qz^{-Q})$$

$$\frac{Y(z)}{X(z)} = \frac{b_0 + b_1z^{-1} + \ldots + b_Qz^{-Q}}{1 + a_1z^{-1} + \ldots + a_pz^{-P}}$$

¿¿¿¿

$$H(z) = \frac{b_0 + b_1z^{-1} + \ldots + b_Qz^{-Q}}{1 + a_1z^{-1} + \ldots + a_pz^{-P}} = \frac{\sum_{k=0}^{Q} b_kz^{-k}}{1 + \sum_{k=1}^{P} a_kz^{-k}}$$

na DFT ...

$$H(e^{j\omega}) = H(z)|_{z=e^{j\omega}} = \frac{\sum_{k=0}^{Q} b_ke^{-j\omega k}}{1 + \sum_{k=1}^{P} a_ke^{-j\omega k}}$$

polynomy

$$B(z) = \sum_{k=0}^{Q} b_kz^{-k}$$

$$A(z) = 1 + \sum_{k=1}^{P} a_kz^{-k}$$

rozklad polynomu

$$x^2 + bx + c = 0 \quad \longrightarrow \quad x_1, x_2$$

$$x^2 + bx + c = (x - x_1)(x - x_2)$$

$$x^2 + 2x + 1 = (x+1)(x+1) = (x - (-1))(x - (-1)) \quad \longrightarrow \quad x_1 = -1, \quad x_2 = -1$$

$$x^2 - 2x + 2$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm 2j}{2}$$

$$x_1 = 1 + j, \quad x_2 = 1 - j$$

rozklad prenos fce

$$H(z) = \frac{b_0 + b_1z^{-1} + \ldots + b_Qz^{-Q}}{1 + a_1z^{-1} + \ldots + a_Pz^{-P}} = \frac{z^{-Q}(b_0z^Q + b_1z^{Q-1} + \ldots + b_Q)}{z^{-P}(z^P + a_1z^{P-1} + \ldots + a_P)}$$

$$= b_0z^{P-Q}\frac{(z-n_1)(z-n_2)\ldots(z-n_Q)}{(z-p_1)(z-p_2)\ldots(z-p_P)}$$

$$H(z) = b_0z^{P-Q}\frac{\prod_{k=1}^{Q}(z-n_k)}{\prod_{k=1}^{P}(z-p_k)}$$

$$H(e^{j\omega}) = b_0e^{j\omega(P-Q)}\frac{(e^{j\omega}-n_1)(e^{j\omega}-n_2)\ldots(e^{j\omega}-n_Q)}{(e^{j\omega}-p_1)(e^{j\omega}-p_2)\ldots(e^{j\omega}-p_P)}$$

modul

$$|H(e^{j\omega})| = b_0\frac{|e^{j\omega}-n_1|\;|e^{j\omega}-n_2|\ldots|e^{j\omega}-n_Q|}{|e^{j\omega}-p_1|\;|e^{j\omega}-p_2|\ldots|e^{j\omega}-p_P|} = b_0\frac{\text{product of lengths of blue vectors}}{\text{product of lengths of red vectors}}$$

argument

$$\arg H(e^{j\omega}) = (P-Q)\omega \; \frac{\arg(e^{j\omega}-n_1)\;\arg(e^{j\omega}-n_2)\ldots\arg(e^{j\omega}-n_Q)}{\arg(e^{j\omega}-p_1)\;\arg(e^{j\omega}-p_2)\ldots\arg(e^{j\omega}-p_P)}$$

$$= (P-Q)\omega + \text{sum of angles of blue vectors} \;\; -\text{sum of angles of red vectors}$$

——————- nas filtr —————— $\omega = 0$

$$|H(e^{j\omega})| = 1\frac{|1-1|\;|1-(-j)|\;|1-j|}{|1-0.9e^{j\pi/4}|\;|1-0.9e^{-j\pi/4}|} = 0$$

$\omega = \pi/4$

$$|H(e^{j\omega})| = 1\frac{|e^{j\pi/4}-1|\;|e^{j\pi/4}-(-j)|\;|e^{j\pi/4}-j|}{|e^{j\pi/4}-0.9e^{j\pi/4}|\;|e^{j\pi/4}-0.9e^{-j\pi/4}|} = \frac{0.75\;0.75\;1.8}{0.1\;1.4} = 7.3$$

$\omega = \pi/2$

$$|H(e^{j\omega})| = 1\frac{|j-1|\;|j-(-j)|\;|j-j|}{|j-0.9e^{-j\pi/4}|\;|j-0.9e^{-j\pi/4}|} = 0$$

$\omega = \pi$

$$|H(e^{j\omega})| = 1\frac{|-1-1|\;|-1-(-j)|\;|-1-j|}{|-1-0.9e^{-j\pi/4}|\;|-1-0.9e^{-j\pi/4}|} = \frac{\sqrt{2}\;2\;\sqrt{2}}{1.75\;1.75} = 1.84$$

argukmenty delat nebudem ...

——————— stabilita

$$x[n] \in [-C, \quad +C] \quad \longrightarrow \quad x[n] \in [-D, \quad +D]$$

FIR

$$D = \sum_{k=0}^{Q} C|b_k|$$

IIR

$$|p_k| < 1$$

zobrazovani a design

$$H_{dB}(e^{j\omega}) = 20\log_{10} H(e^{j\omega}) = 10\log_{10}|H(e^{j\omega})|^2$$

$$W_p = \frac{f_{pass}}{F_s/2}$$

$$W_s = \frac{f_{stop}}{F_s/2}$$