DFT:

N vzorků signálu $\longrightarrow$ N vzorků spektra (komplexní)

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j k \frac{2\pi}{N} n} \qquad x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k]\, e^{+j k \frac{2\pi}{N} n}$$

k	0	k	$\frac{N}{2}$	$N-1$	N	
		$f = \frac{k}{N}$	$\frac{1}{2}$		1	norm. frekv. [ ]
		$f = \frac{k}{N} F_s$	$F_s/2$		F_s	běžná frekv. [Hz]
		$\omega = k \frac{2\pi}{N}$	π		2π	norm. kruhová frekvence [rad]
		$\omega = k \frac{2\pi F_s}{N}$	πF_s		$2\pi F_s$	běžná kruhová frekvence [rad/s]

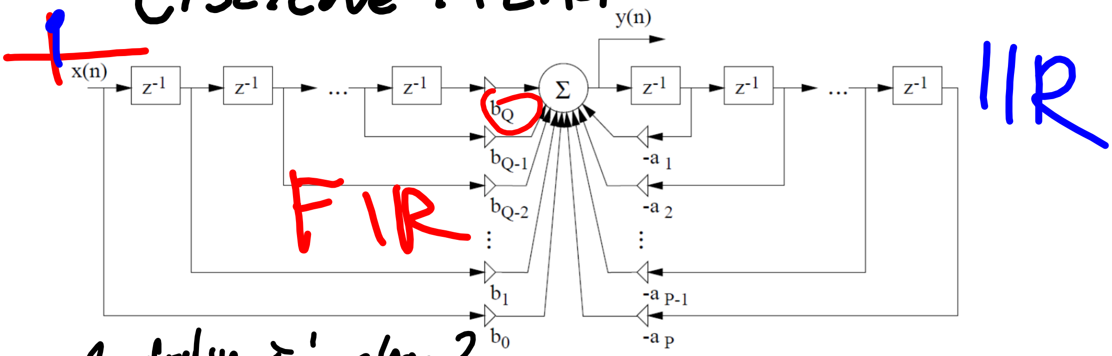
pro $x[n] \in \mathbb{R} \Rightarrow$ symetrie ~~$X[k] = X^*[-k]$~~

$$X[k] = X^*[N-k]$$

kruhové zpoždění $x[n]$ o m vzorků: $X[k] \cdot e^{-j k \frac{2\pi}{N} m}$

kruhová konvoluce $x_1[n] \circledast x_2[n]$: $X_1[k] \cdot X_2[k]$

ČÍSLICOVÉ FILTRY



1, frekvenční char?
2, stabilita

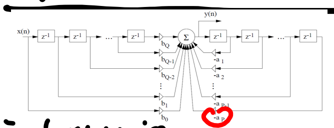
impulsní odezva: $h[n] \xrightarrow{DTFT} H(e^{j\omega}) = \sum_{n=0}^{Q\ (\infty)} h[n]\, e^{-j\omega n}$

ω - nutno vzorkovat ve frekvenci DTFT → DFT

N pro DFT: prodloužit Q o nuly: zero padding (bourrage des zéros)

IIR: nutno "oříznout" impulsní odezvu na vhodné N ⇒ nepřesnost.

Frekv. char. a stabilita přesně



Diferenční rovnice

$$y[n] = \sum_{k=0}^{Q} b_k x[n-k] - \sum_{k=1}^{P} a_k y[n-k]$$

$$y[n] = x[n] - 0{,}5\, y[n-1]$$

$b_0 = 1$ $\quad -a_1 = -0{,}5$ $\quad a_1 = 0{,}5$

z-transformace

$$X(z) = \sum x[n] z^{-n} \qquad X(e^{j\omega}) = \sum x[n] e^{-j\omega n}$$

DTFT

slovníček:

$x[n] \longrightarrow X(z)$

$a\,x[n] \longrightarrow a\,X(z)$

$x[n-k] \longrightarrow X(z) z^{-k}$

$$Y(z) = X(z) - 0{,}5\, Y(z) z^{-1}$$

$$Y(z)[1 + 0{,}5 z^{-1}] = X(z)$$

Přenos. fce:

$$\frac{Y(z)}{X(z)} = H(z) = \frac{1}{1 + 0{,}5 z^{-1}}$$

$$Y(z) = b_0 X(z) + b_1 X(z) z^{-1} + \dots b_Q X(z) z^{-Q} - a_1 Y(z) z^{-1} - \dots - a_P Y(z) z^{-P}$$

$$Y(z) = \sum_{k=0}^{Q} b_k X(z) z^{-k} - \sum_{k=1}^{P} a_k Y(z) z^{-k}$$

Frekv. char.:

$$H(e^{j\omega}) = \frac{1}{1 + 0{,}5 e^{-j\omega}}$$

Přenosová funkce $\quad H(z) = \dfrac{Y(z)}{X(z)}$

$$Y(z)[1 + a_1 z^{-1} + \dots + a_P z^{-P}] = X(z)[b_0 + b_1 z^{-1} + \dots + b_Q z^{-Q}]$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{b_0 + b_1 z^{-1} + \dots + b_Q z^{-Q}}{1 + a_1 z^{-1} + \dots + a_P z^{-P}} = \frac{\sum_{k=0}^{Q} b_k z^{-k}}{1 + \sum_{k=1}^{P} a_k z^{-k}}$$

Frekvenční charakteristika

$$H(e^{j\omega}) = H(z)\Big|_{z = e^{j\omega}} = \frac{\sum_{k=0}^{Q} b_k e^{-j\omega k}}{1 + \sum_{k=1}^{P} a_k e^{-j\omega k}}$$

Rozklad přenos. fce.

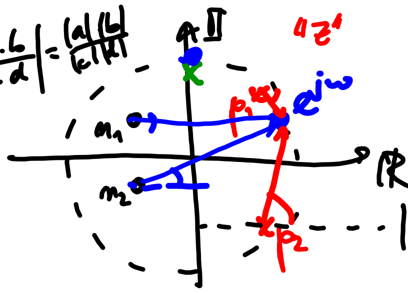
kořeny čitatele — NULOVÉ BODY

$$H(z) = \frac{b_0 z^{-Q}(z^Q + b_1 z^{Q-1} + \dots b_Q)}{z^{-P}(z^P + a_1 z^{P-1} + \dots a_P)} = b_0 z^{P-Q} \frac{(z - n_1)(z - n_2)\dots(z - n_Q)}{(z - p_1)(z - p_2)\dots(z - p_P)}$$

(NULY)

kořeny jmenovatele PÓLY

$$\left|\frac{a \cdot b}{c\, d}\right| = \frac{|a|\,|b|}{|c|\,|d|}$$

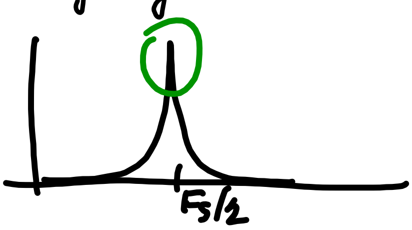


$$H(e^{j\omega}) = b_0 e^{j\omega(P-Q)} \frac{(e^{j\omega} - n_1)(e^{j\omega} - n_2)\dots}{(e^{j\omega} - p_1)(e^{j\omega} - p_2)\dots}$$

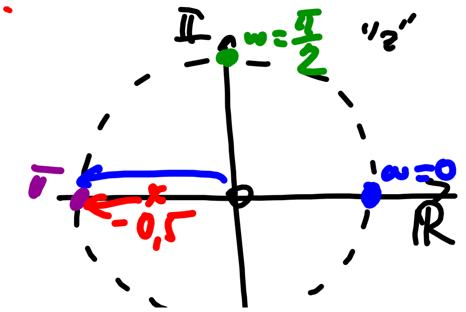
$$|H(e^{j\omega})| = b_0 \frac{\text{součin délek modrých vektorů}}{\text{součin délek červených vektorů}}$$

$$\arg H(e^{j\omega}) = \text{součet úhlů modrých vektorů} - \text{součet úhlů červených vektorů} + \omega(P - Q)$$

$$\arg \frac{a\,b}{c\,d} = \arg a + \arg b - \arg c - \arg d$$



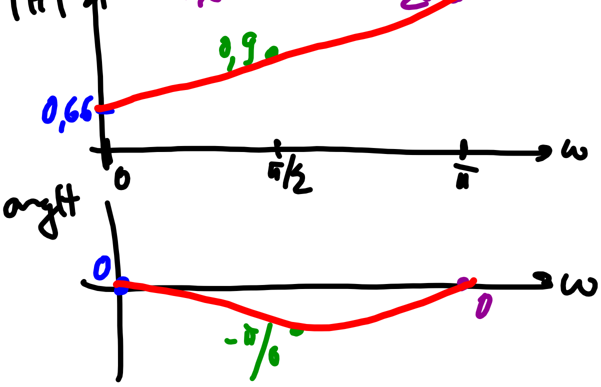
$$H(z) = \frac{1}{1+0{,}5z^{-1}} = \frac{1}{z^{-1}(z+0{,}5)} = \frac{z-0}{(z-(-0{,}5))}$$

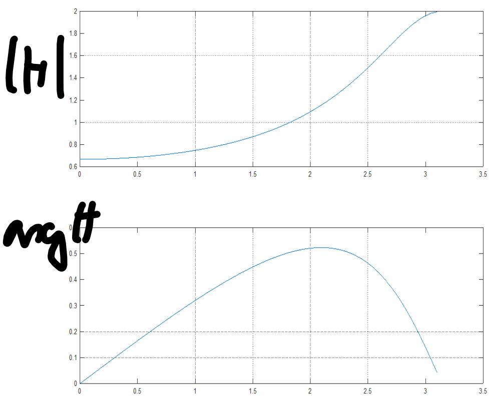


$|H| = \frac{1}{1{,}5} = 0{,}66 \quad \text{ang}\,H = 0-0 = 0$

$|H| = \frac{1}{\sqrt{1^2+0{,}5^2}} = 0{,}9 \quad \text{ang}\,H = \frac{\pi}{2} - \frac{\pi}{3} \cong \frac{\pi}{6}$

$|H| = \frac{1}{0{,}5} = 2 \quad \text{ang}\,H = \pi - \pi = 0$





Stabilita

$|p_k| < 1$

zplane (B, A)