Fourier series (F.S.)

input: periodic signal with continuous time

output: series of coefficients c_k T_1 [s] $f_1 = \frac{1}{T_1}$ [Hz]

where? multiples of fund. frequency $k\omega_1$ $\omega_1 = \frac{2\pi}{T_1}$ [rad/s]

how much? $|c_k|$

how shifted? $\arg c_k$ $c_{-k} = c_k^*$

F.S. of shifted signal. $|c_k|$ the same as the original

$\arg c_k$ follow a linear pattern

"uphill" for advance

"downhill" for delay

$\boxed{\text{output} = \text{maybe a constant} \ \ \text{sum. operator} \ \ \text{input} \ \ e^{\mp j \cdot \text{time} \cdot \text{frequency}}}$

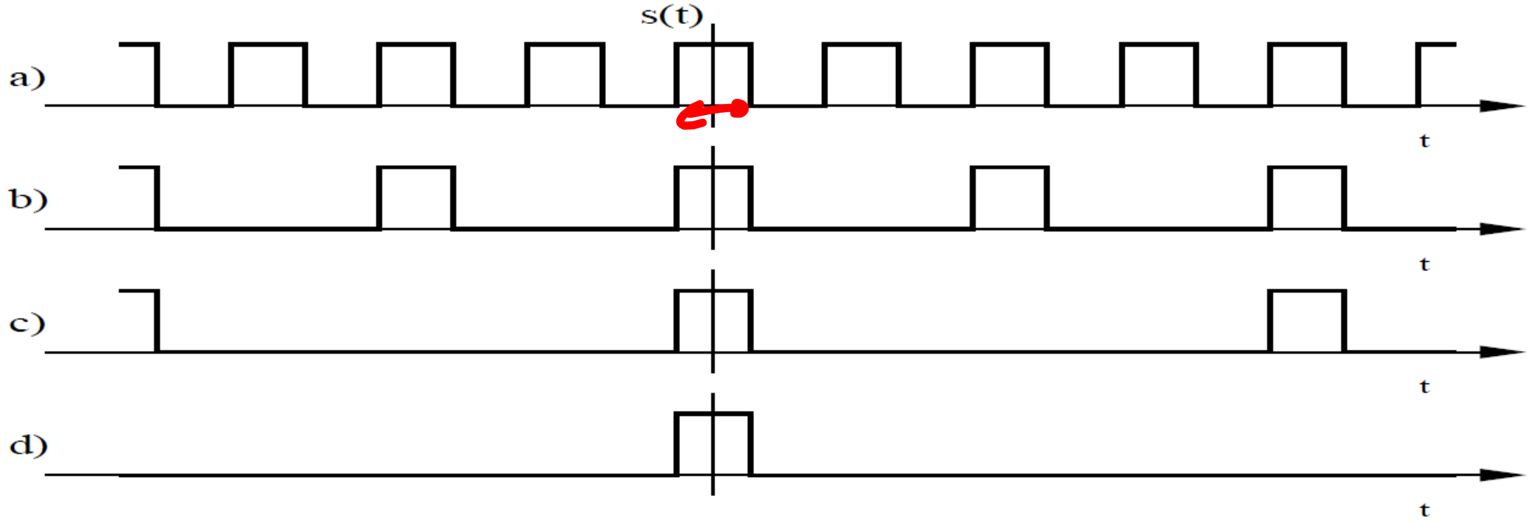
time → frequency

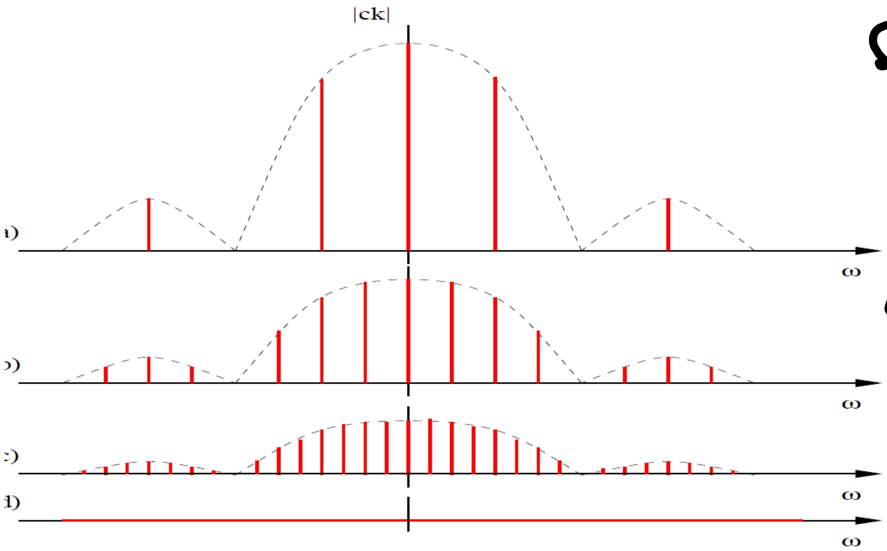
freq. → time

analysis:	synthesis:
$c_k = \frac{1}{T_1}\int_{T_1} x(t) e^{-jtk\omega_1} dt$	$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{+jtk\omega_1}$

FOURIER TRANSFORM (F.T.)

I want to analyze non-periodic signals.





Straight use of F.S. does ~~not~~ not work :(

How to hack F.S. to work with non-periodic signals?

$$T_1 \to \infty$$

$$\omega_1 = \frac{2\pi}{T_1} \to d\omega$$ infinitely small increment in ang. frequency

$$\frac{1}{T_1} = \frac{d\omega}{2\pi}$$

F.S.

$$c_k = \frac{1}{T_1}\int_{T_1} x(t)\, e^{-jk\omega_1 t}\, dt$$

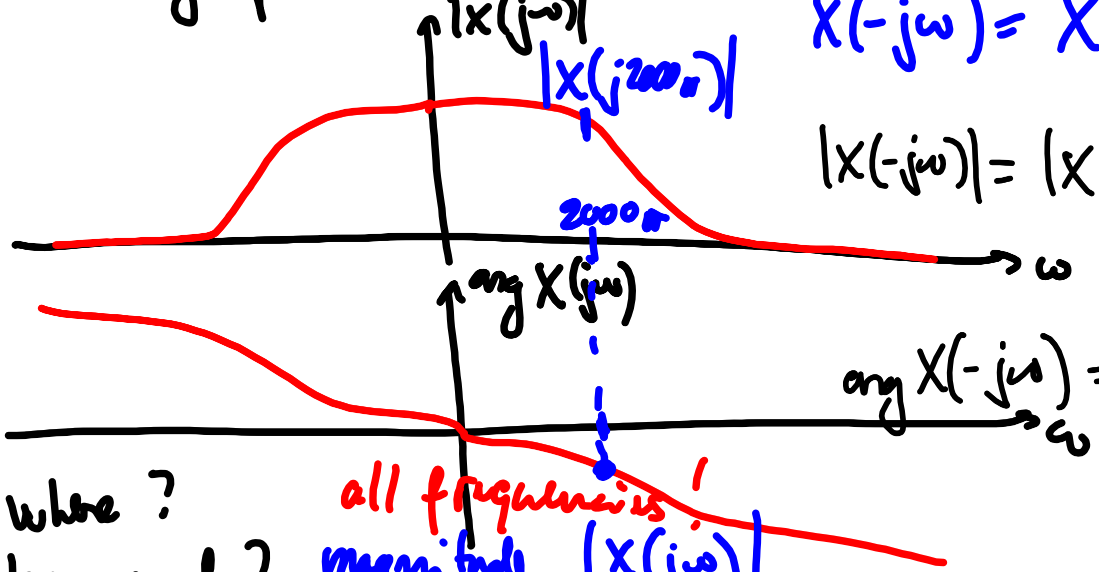
infinitely small in coefficient of. Fs ...

$$dc = \frac{d\omega}{2\pi}\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \qquad 2\pi\frac{dc}{d\omega} = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\boxed{X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$ Fourier transform.

spectral function

Plotting spectral function



$X(-j\omega) = X^*(j\omega)$

$|X(-j\omega)| = |X(j\omega)|$

$\angle X(-j\omega) = -\angle X(j\omega)$

where? all frequencies!

how much? magnitude $|X(j\omega)|$

how shifted? angle of $X(j\omega)$

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output = maybe a constant c · some op. · input $e^{\pm j \,\text{time frequency}}$

time → f

f → time

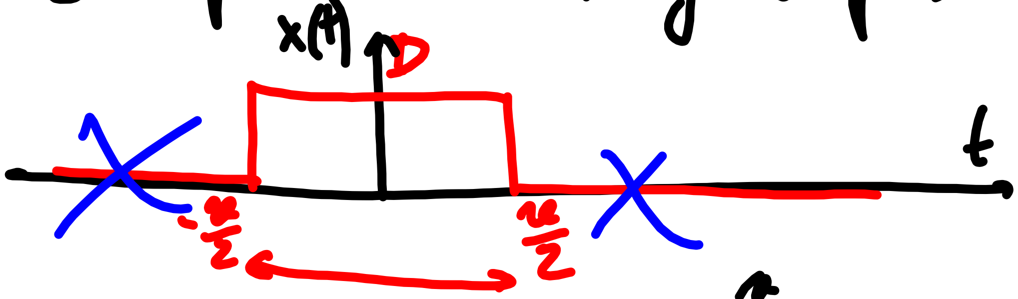
analysis

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-jt\omega}\, dt$$

synthesis Inverse F.T.

$$\boxed{x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{jt\omega}\, d\omega}$$

Example #1: rectangular pulse!



$$\int_{-b}^{b} e^{\mp jxy}\,dy = 2b\,\mathrm{sinc}(bx)$$

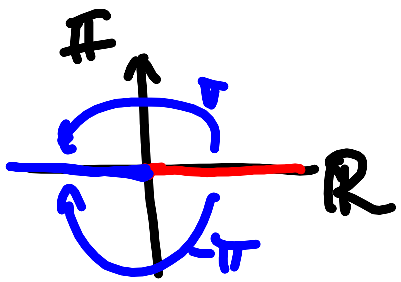
$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt = D\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-j\omega t}\,dt = D\,\cancel{2}\cdot\frac{\tau}{\cancel{2}}\,\mathrm{sinc}\left(\frac{\tau}{2}\omega\right) =$$

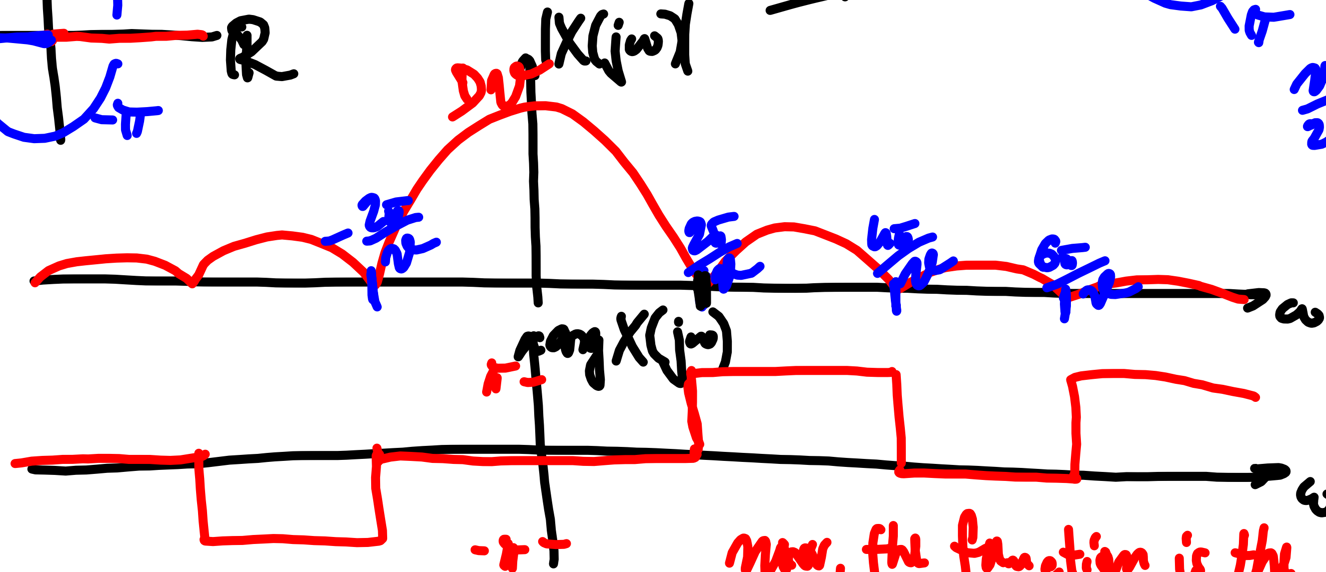
$$= D\tau\,\mathrm{sinc}\left(\frac{\tau}{2}\omega\right)$$

π

$$\frac{\tau}{2}\omega = \pi$$

$$\omega = \frac{2\pi}{\tau}$$

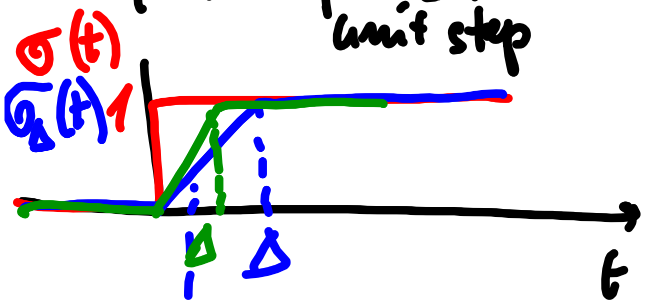




now, the function is the result (unlike F.S.)

F.T. of Dirac impulse

unit step

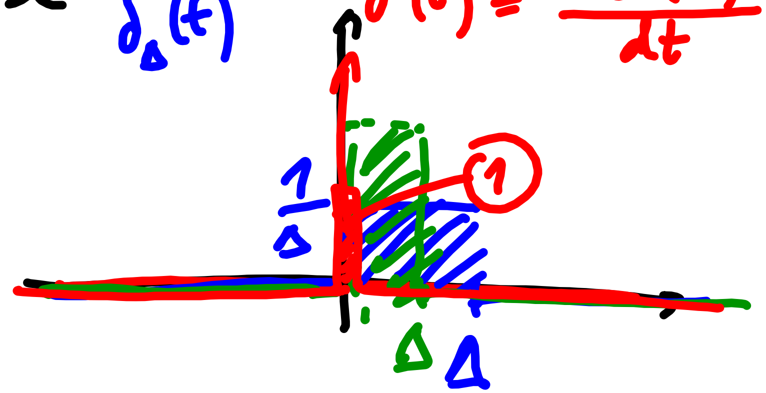


for $t=\Delta$ it must be 1

$\frac{1}{\Delta}t$

$\delta_\Delta(t)$

$$\delta(t) = \frac{d\sigma(t)}{dt}$$

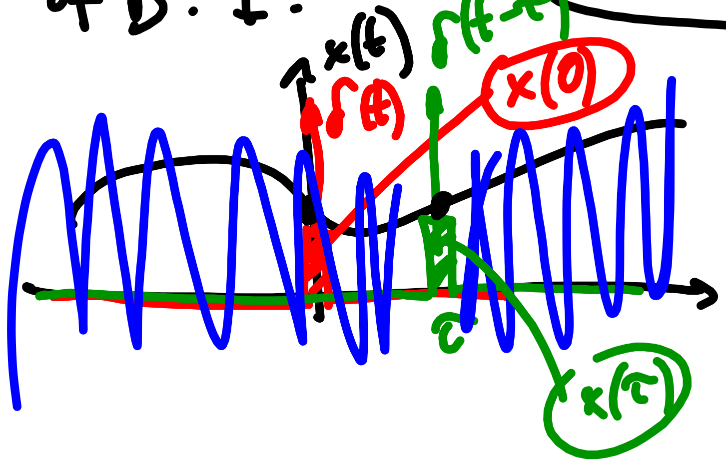


$$\int_{-\infty}^{\infty} \delta_\Delta(t)\,dt = 1$$

$$\int_{-\infty}^{\infty} \delta_\Delta(t)\,dt = 1$$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1\ !$$

Sampling property of D.I.

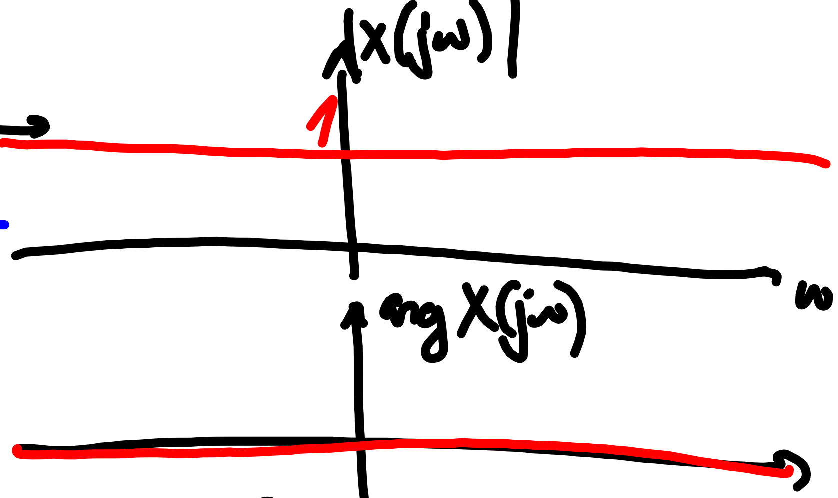


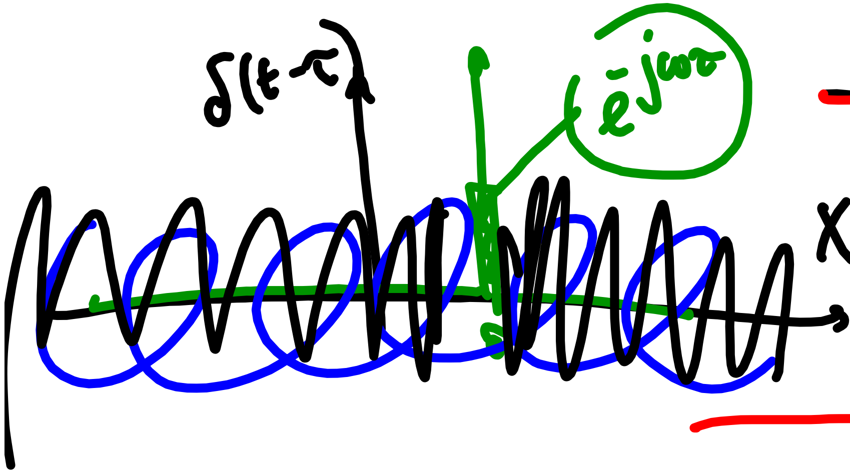
$$\int_{-\infty}^{\infty} x(t)\delta(t)\,dt = x(0)$$

$$\int_{-\infty}^{\infty} x(t)\delta(t-\tau)\,dt = x(\tau)$$

F.T. of a Dirac

$$X(j\omega) = \int_{-\infty}^{\infty} \underline{\delta(t) e^{-j\omega t}} dt = 1$$





$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t-\tau) e^{-j\omega t} dt = e^{-j\omega\tau}$$