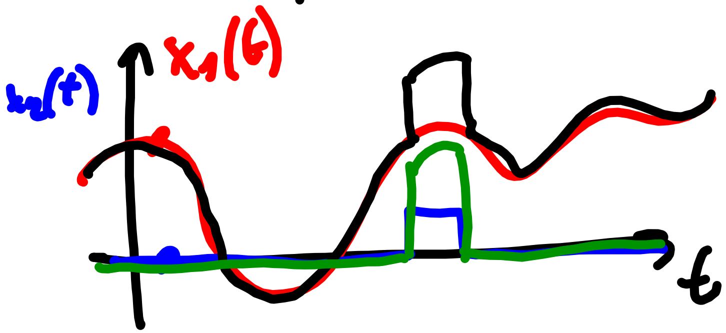
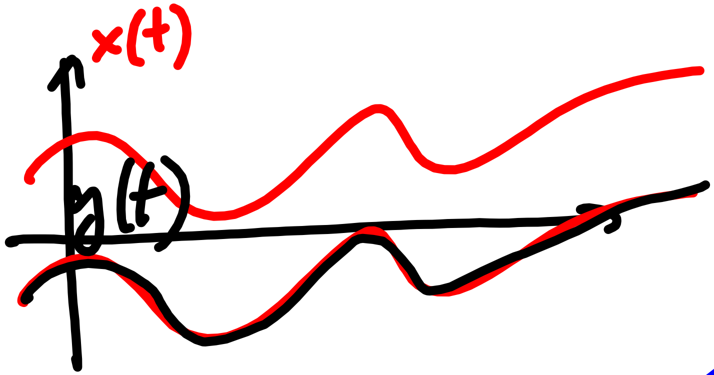
Spojitý čas

- posuny v čase
- energie a výkony
- periodický
- spek. analýza per. signálů - Fourierova řada

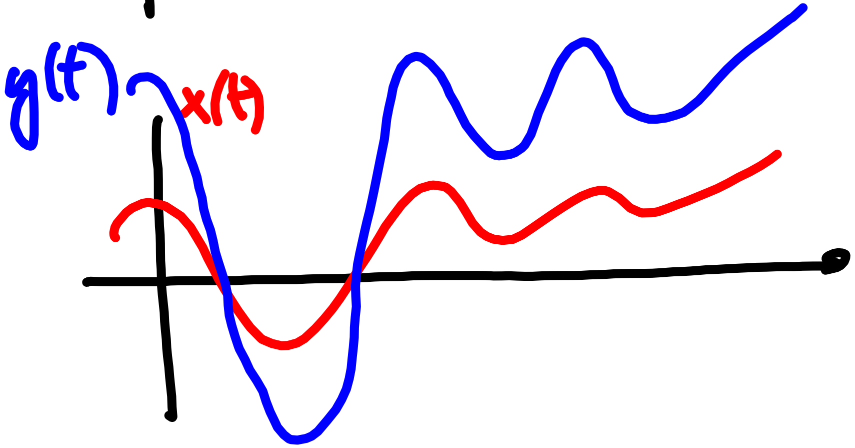


$x(t) = x_1(t) + x_2(t)$

$y(t) = x_1(t)\,x_2(t)$



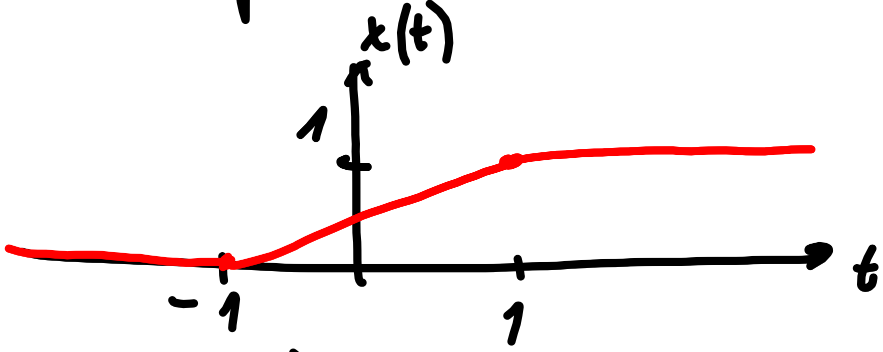
$$y(t) = x(t) + C$$



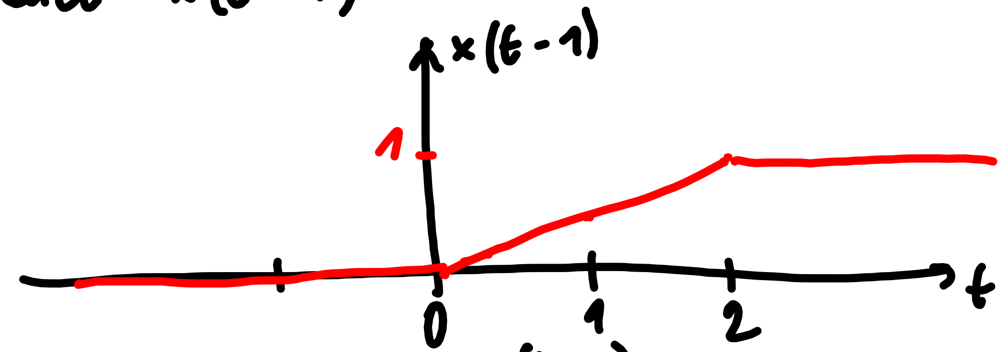
$$y(t) = 3x(t)$$

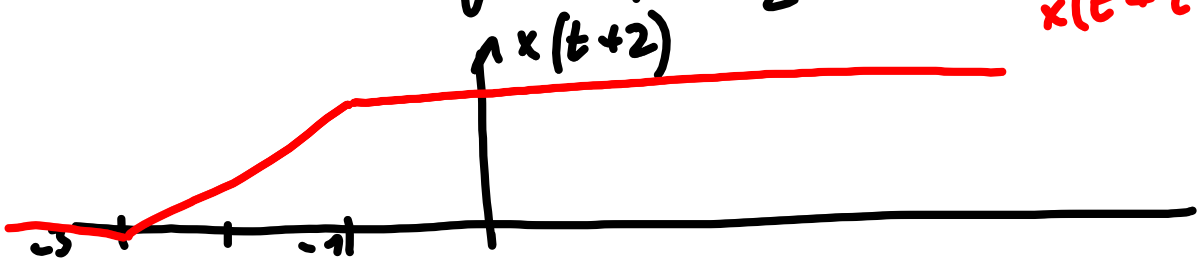
Modifikace času - posuny v čase

$$x(t)=\begin{cases}0 & \text{pro } t<-1\\ 0{,}5t+0{,}5 & \text{pro } t\in\langle -1,1\rangle\\ 1 & \text{pro } t>1\end{cases}$$



chci: $x(t-1)$



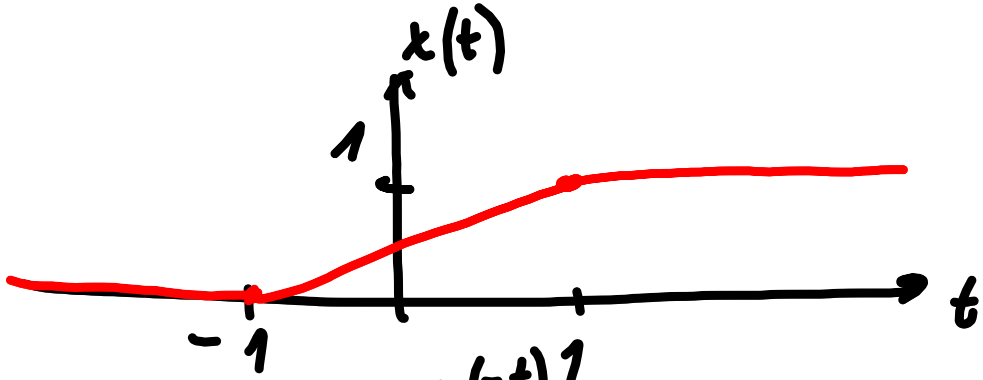


kladné číslo

$x(t-\tau)$ zpoždění delay

$x(t+\tau)$ předběhnutí advance

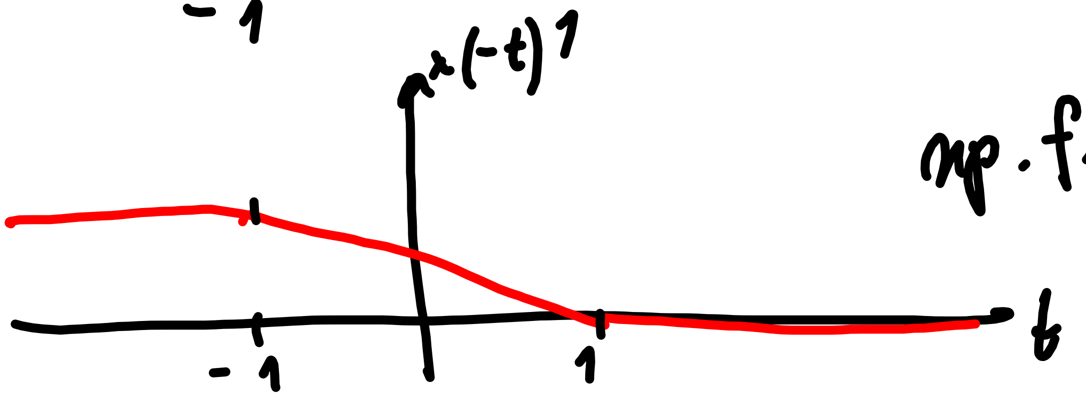
OTOČENÍ ČASOVÉ OSY



time reverse

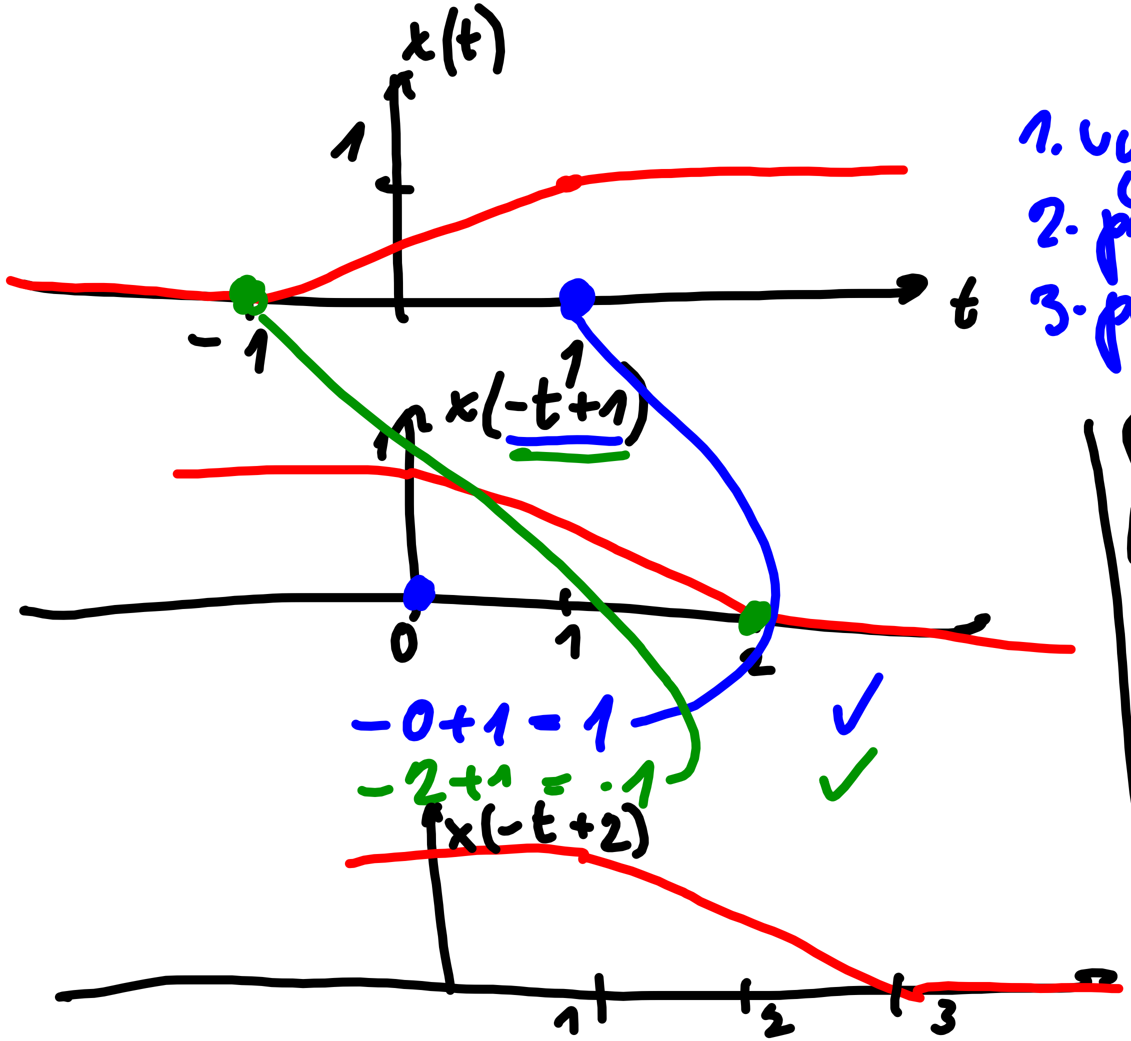
reversing the time

flipping the time



np.fliplr

OTOČENÍ ČASU S POSUNUTÍM



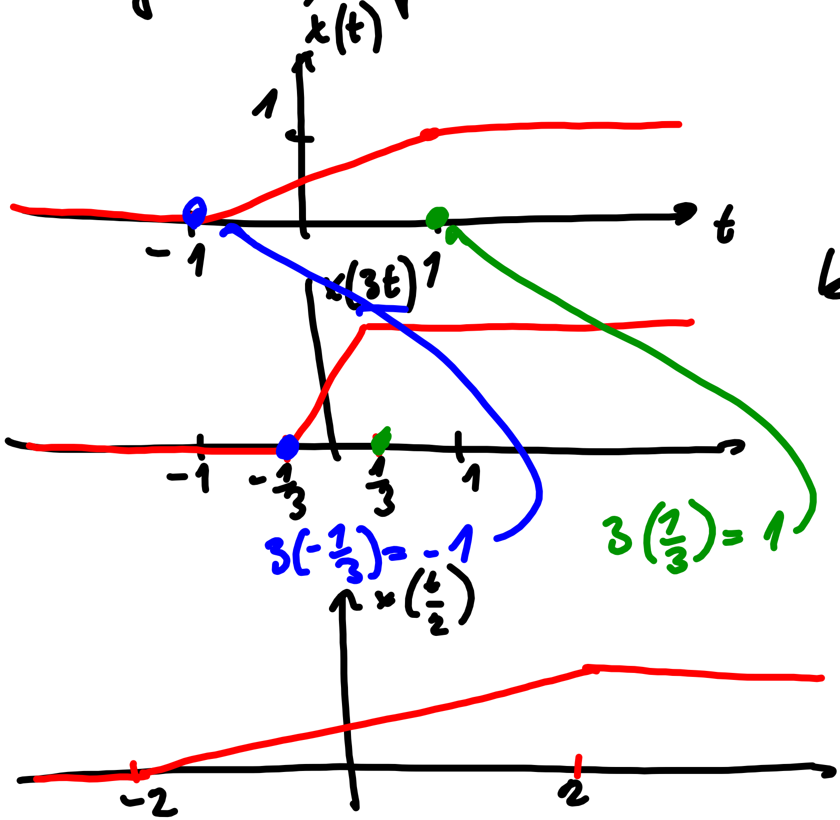
1. vyber důležitý čas
2. proveď čas. modifikaci
3. podívej se do originálu

Remember!

konvolnce

$$x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

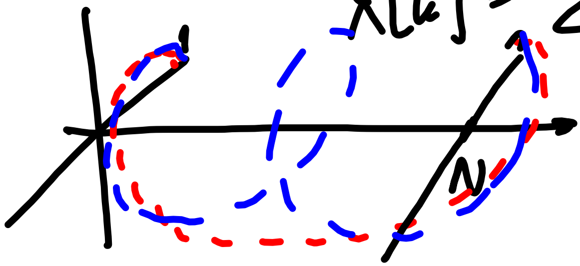
Zrychlení / zpomalení času



kontrakce času
contraction
speed-up ...

dilatace času
dilation ?
slowing-down ?

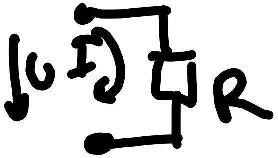
$$X[k] = \sum x[n]\, e^{-j 2\pi k \frac{1}{N} n}$$



$k=2$

TDNN
CNN mohou mít také dilataci

Energie a výkony

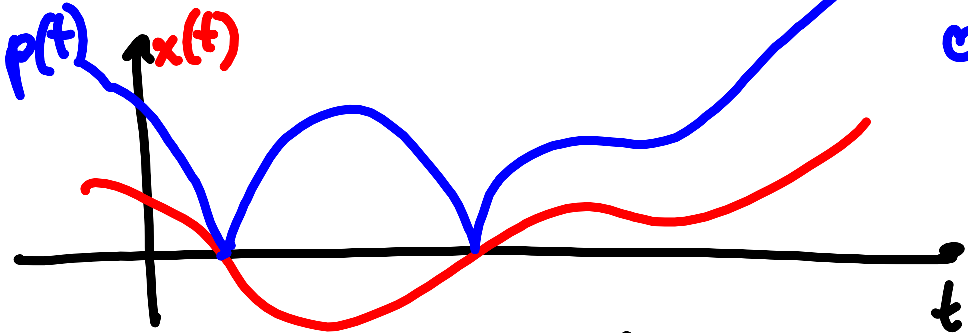


$P = UI = \frac{U^2}{R} = I^2 R$

$U = IR$

signál x $\quad P = x^2$

výkon.

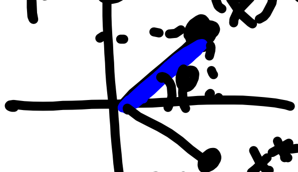


okamžitý výkon

$p(t) = x^2(t)$

instantaneous power

Ok pro $x(t) \in \mathbb{R}$



$x(t) \in \mathbb{C}$ $\quad p(t) = |x(t)|^2$

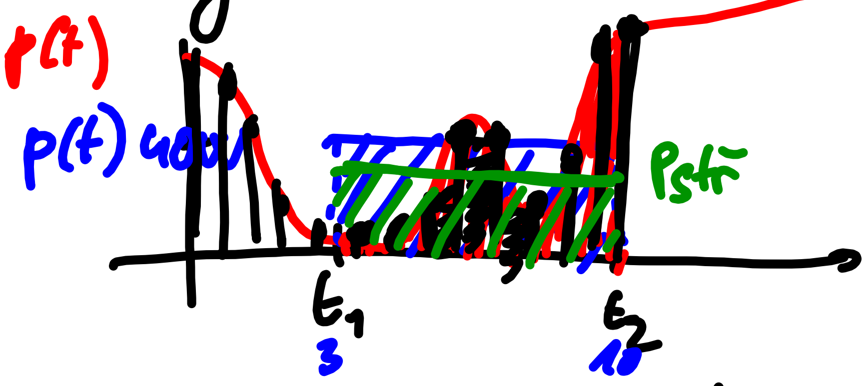
x v poli

```
p = np.pow(np.abs(x), 2.0)
p = np.pow(np.real(x), 2.0) + np.pow(np.imag(x), 2.0)
p = x * np.conj(x)
```

$x \cdot x^* = |x| e^{j\varphi} \cdot |x| e^{-j\varphi} = |x|^2$

Matlab `p = x .* conj(x)`

Energie



$E = \underline{P \cdot T} = 40 \cdot 7 = 280\ J$

$E = \int_{t_1}^{t_2} p(t)\,dt = \sum$ vzorky . výkonu vzorkovací periodu

Střední výkon signálu v intervalu $\langle t_1, t_2 \rangle$

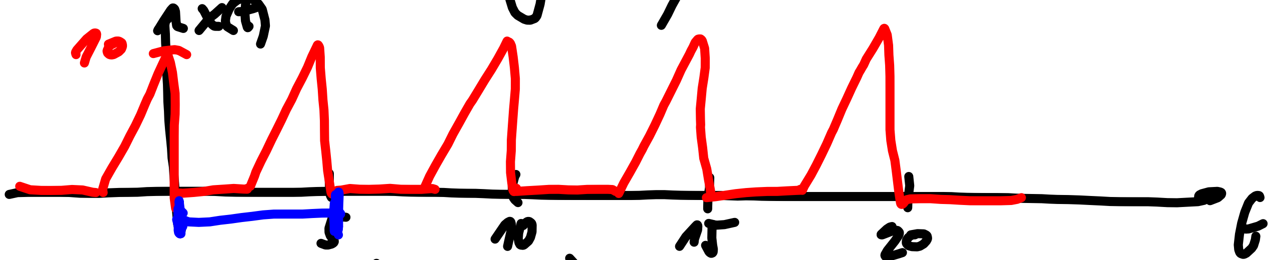
$$P_{stř} = \frac{E_{t_1,t_2}}{t_2 - t_1}$$

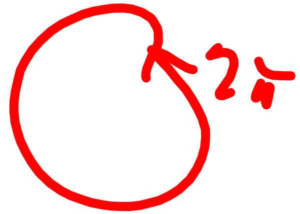
Efektivní hodnota ?

$X_{ef}^2 = P_{stř}$ $X_{ef} = \sqrt{P_{stř}}$

stejnosměrný signál, který na intervalu t_1 do t_2 dá stejnou energii jako pův. signál.

periodické signály





$x(t) = x(t+10)$
$x(t-15)$

$T_1 = 5\,s$ základní perioda

$x(t) = x(t - T_1)$
$x(t + T_1)$

$f_1 = \frac{1}{T_1}$ základní frekvence [Hz]

$\omega_1 = 2\pi f_1$ kruhová frekvence [rad/s]

Energie / výkony per. signálů – vždy přes 1 periodu

Kam nastavit 1 periodu? je to jedno!

$$E_{T_1} = \int_{T_1} x^2(t)\,dt = \int_0^{T_1} x^2(t)\,dt = \int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} x^2(t)\,dt = \int_{999T_1}^{1000T_1} x^2(t)\,dt$$

střední výkon

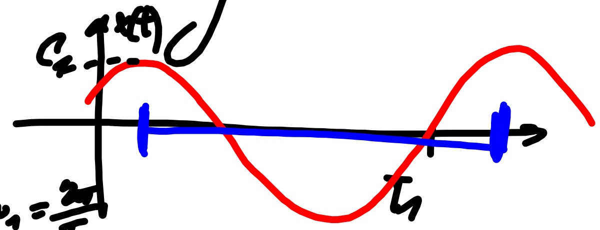
$$P_{stř} = \frac{1}{T_1}\int_{T_1} x^2(t)\,dt$$

Střední výkon cosinusovky

$$x(t) = C_1 \cos(\omega_1 t + \cancel{\varphi_1})$$

amplitu   rad/s   rad

$$f_1 = \frac{1}{T_1} \qquad \omega_1 = \frac{2\pi}{T_1}$$



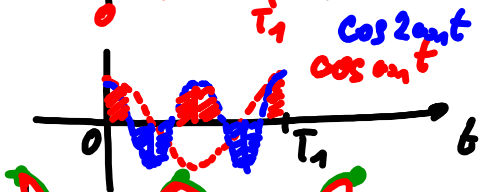
$$\underline{x(t) = C_1 \cos(\omega_1 t)}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$P_{stř} = \frac{E_{T_1}}{T_1} = \frac{1}{T_1}\int_{T_1} x^2(t)\,dt = \frac{1}{T_1}\int \left[C_1 \cos(\omega_1 t)\right]^2 dt =$$

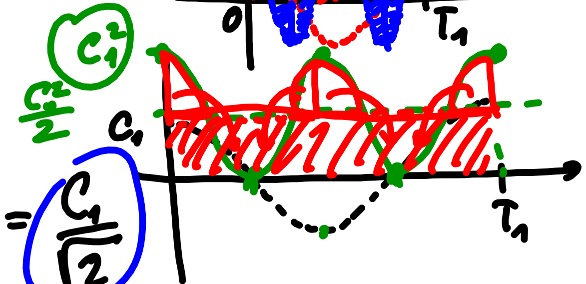
$$= \frac{C_1^2}{T_1}\int \cos^2(\omega_1 t)\,dt = \frac{C_1^2}{T_1}\int \frac{1+\cos(2\omega_1 t)}{2}\,dt =$$

$$= \frac{C_1^2}{2T_1}\left[\underbrace{\int_{T_1} 1\,dt}_{T_1} + \underbrace{\cancel{\int_{T_1} \cos 2\omega_1 t\,dt}}_{0}\right] =$$

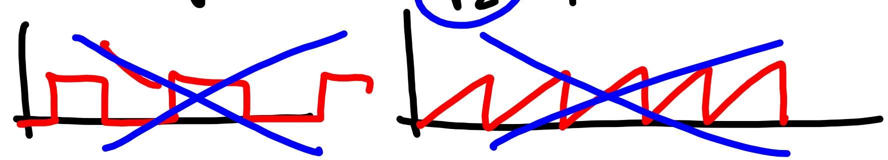


$$= \frac{C_1^2}{2\cancel{T_1}}\cdot \cancel{T_1} = \frac{C_1^2}{2}$$

$$= \frac{1}{\cancel{T_1}}\frac{C_1^2}{2}\cancel{T_1} = \frac{C_1^2}{2}$$



efektivní hodnota $\quad X_{ef} = \sqrt{P_{stř}} = \frac{C_1}{\sqrt{2}}$

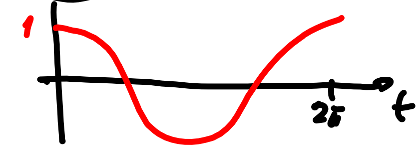


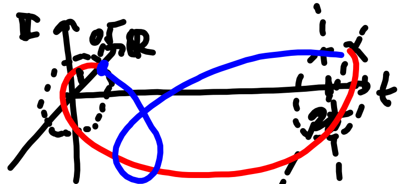
Rozklad cosinusovky na 2 komplexní exp.

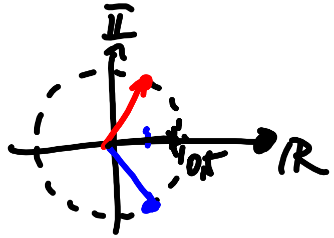
$x(t) = \cos t$

$$\cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$$

$$x(t) = \boxed{0{,}5\, e^{jt}} + \boxed{0{,}5\, e^{-jt}}$$







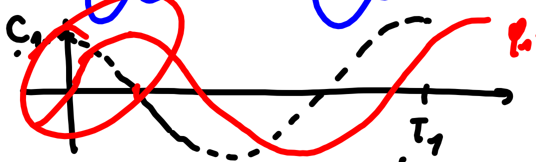
$$x(t) = C_1 \cos(\omega_1 t + \varphi_1) =$$

$$= \frac{C_1}{2} e^{j(\omega_1 t + \varphi_1)} + \frac{C_1}{2} e^{-j(\omega_1 t + \varphi_1)} =$$

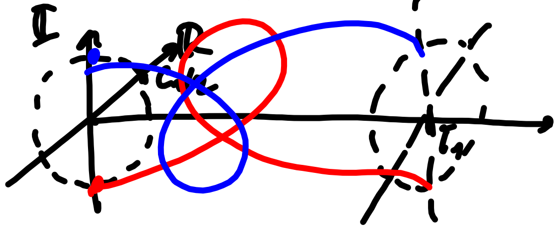
$$= \frac{C_1}{2} e^{j\varphi_1} e^{j\omega_1 t} + \frac{C_1}{2} e^{-j\varphi_1} e^{-j\omega_1 t}$$

$$\cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$$

$$e^{a+b} = e^a \cdot e^b$$



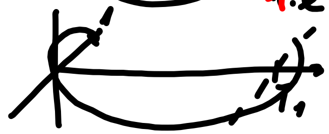
$\varphi_1 = -\frac{\pi}{2}$



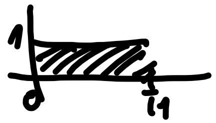
Střední výkon komp. exp.

$x(t) = e^{j\omega_1 t}$ $\qquad T_1 = \frac{2\pi}{\omega_1}$

$1 \cdot e^{j\omega_1 t}$



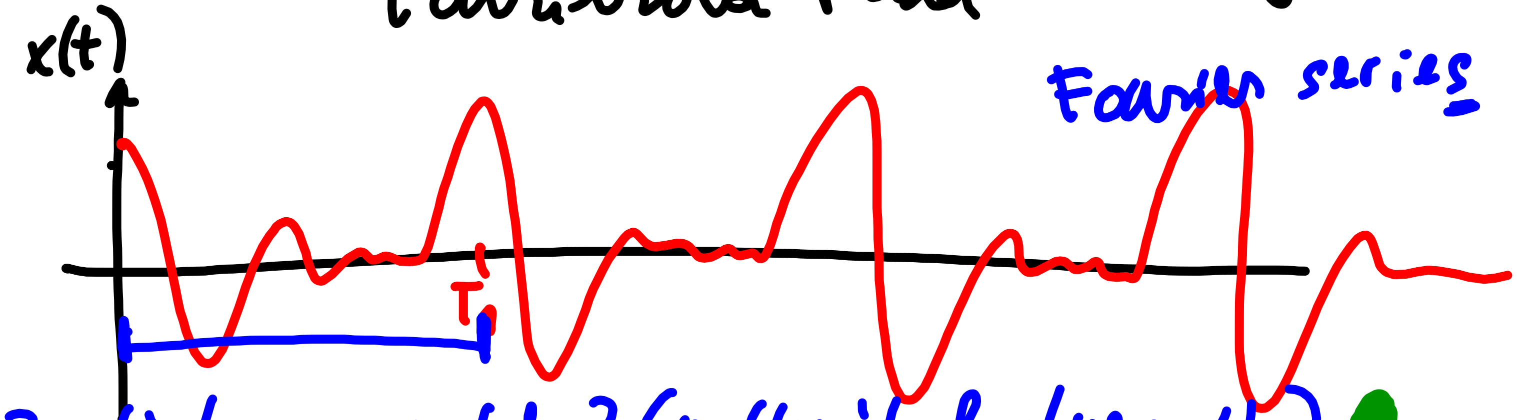
$$P_{stř} = \frac{1}{T_1} \int_{T_1} |x(t)|^2 \, dt = \frac{1}{T_1} \int_{T_1} 1 \, dt =$$



$$= \frac{T_1}{T_1} = \underline{1}$$

Frekvenční analýza periodických signálů
Fourierova řada

Fourier series



3 otázky:
1. kde? (na kterých frekvencích)
2. kolik?
3. jak je posunutý?

Harmonický vzlezené cosinusovky

$$x(t) = c_0 + C_1\cos(\omega_1 t + \varphi_1) + C_2\cos(2\omega_1 t + \varphi_2) + \dots + C_k\cos(k\omega_1 t + \varphi_k)$$

syntezační vzoreček.

$$x(t) = c_0 + C_1\cos(\omega_1 t + \varphi_1) + C_2\cos(2\omega_1 t + \varphi_2) + \dots + C_k\cos(k\omega_1 t + \varphi_k) =$$

$$x(t) = \dots c_{-k} e^{-jk\omega_1 t} + \dots + c_{-2} e^{-j2\omega_1 t} + c_{-1} e^{-j\omega_1 t} + c_0 + c_1 e^{j\omega_1 t} + c_2 e^{j2\omega_1 t} + \dots + c_k e^{jk\omega_1 t} + \dots$$

c_1 a c_{-1} musí být komplex. sdružené

c_k a c_{-k} — // —

$|c_k|$ kolik ?

ang c_k jak posunuté

Syntezační vzoreček kompaktně

Syntezační vzorce Fourierovy řady

$$\boxed{x(t) = c_0 + \sum_{k=1}^{\infty} C_k \cos(k\omega_1 t + \varphi_k)}$$

$\mathbb{R}$ FŘ

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_1 t}}$$

$\mathbb{C}$ FŘ

Jak odhadnout koeficienty FŘ? Analýza?

$$x(t) = \sum_{k=-\infty}^{\infty} c_k \underline{e^{jk\omega_1 t}}$$

neznámý signál

analyzační signály

c_k - podobnost $x(t)$ s k.e.
korelace $x(t)$ s k.e.
promítnout $x(t)$ do k.e.

$$\boxed{c_k = \frac{1}{T_1}\int_{T_1} x(t)\, e^{-jk\omega_1 t}}$$

analyzační vzorec F.Ř.