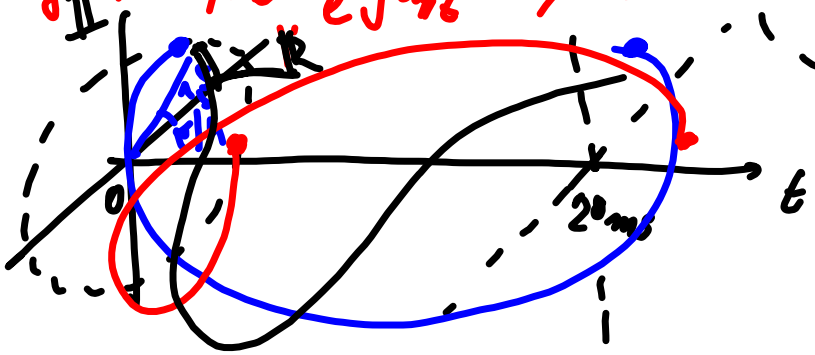
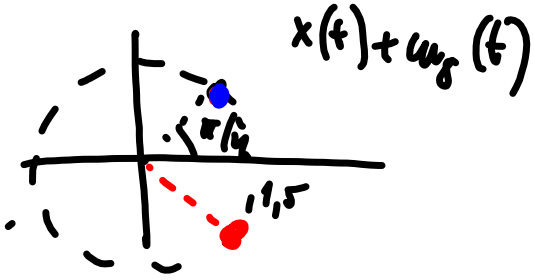
$x(t) = c_1 e^{j\omega_1 t}$ c_1 - complex coefficient

$x(t) = 1{,}5\, e^{j\frac{\pi}{4}} e^{j100\pi t}$

$u_g(t) = 1{,}5\, e^{j\frac{\pi}{4}} e^{-j\omega_1 t}$ $t=0$





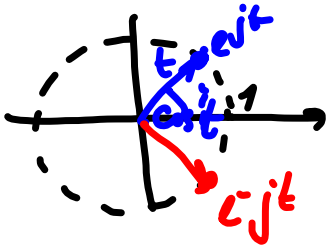
$\omega_1 = 100\pi$

$f_1 = 50\ Hz$

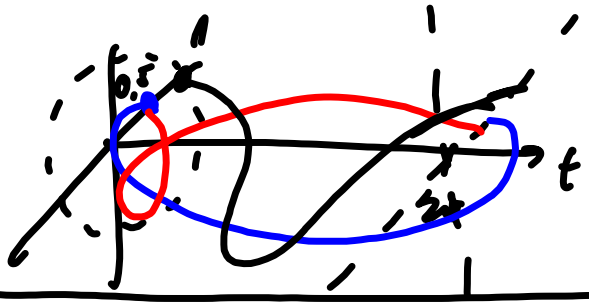
$T_1 = \frac{1}{f_1} = \frac{1}{50} = 20\ ms$

Expressing cosine as 2 complex exponentials.

$$\cos t = \frac{e^{jt} + e^{-jt}}{2}$$



$e^{jt} + e^{-jt} = 2\cos t$



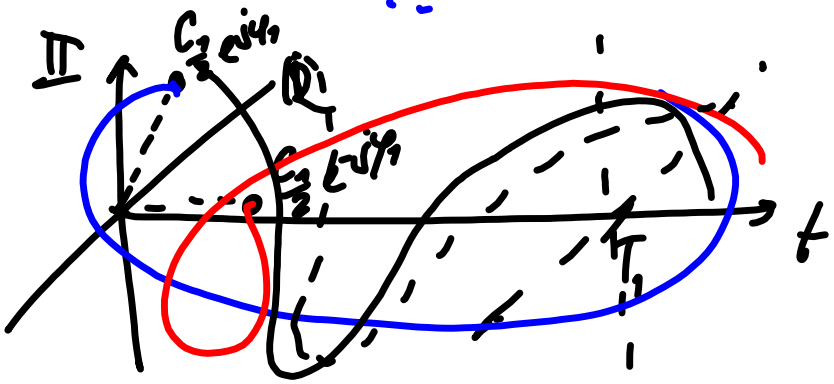
$2^{a+b} = 2^a \cdot 2^b$

$e^{a+b} = e^a e^b$ ...

$$x(t) = C_1 \cos(\omega_1 t + \varphi_1) = \frac{C_1}{2} e^{j(\omega_1 t + \varphi_1)} + \frac{C_1}{2} e^{-j(\omega_1 t + \varphi_1)} =$$

$$= \frac{C_1}{2} e^{j\varphi_1} e^{j\omega_1 t} + \frac{C_1}{2} e^{-j\varphi_1} e^{-j\omega_1 t}$$

$\varphi_1 = -\frac{\pi}{4}$

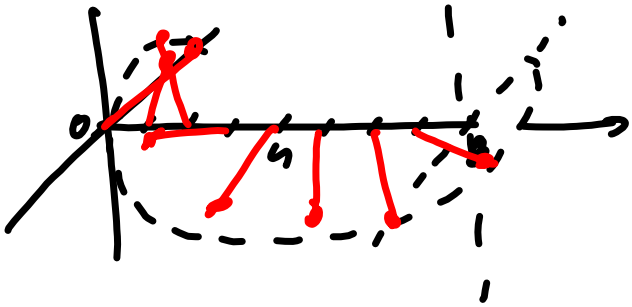


$f_1 = \frac{1}{T_1}$

$\omega_1 = 2\pi f_1$

$T_1 = \frac{2\pi}{\omega_1}$

Discrete complex. exponential



$$x[n] = e^{j 2\pi \frac{k}{N} n}$$

$\frac{k}{N}$ normalized f.

$2\pi \frac{k}{N}$ normalized angular f.

---

general discrete c. e.

$$x[n] = c_k e^{j 2\pi \frac{k}{N} n}$$

k ... number of turns (periods) over N samples

$c_k = |c_k| e^{j \angle c_k}$

$|c_k|$ ... "thickness"

$\angle c_k$ ... "pre-rotation"

$$x_2[n] = 5 e^{-j\frac{\pi}{3}} e^{j 2\pi \frac{1}{32} n}$$

$$x_3[n] = 5 e^{+j\frac{\pi}{3}} e^{j 2\pi \frac{1}{32} n}$$

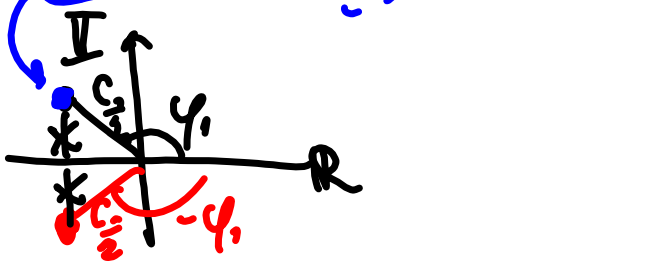
Crafting a discrete cos from 2 c.e.

$x[n] = C_1 \cos(\omega_1 n + \varphi_1)$

C_1 magnitude [anything]
ω_1 normalized ang. freq [rad]
φ_1 initial phase [rad]

$\cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$

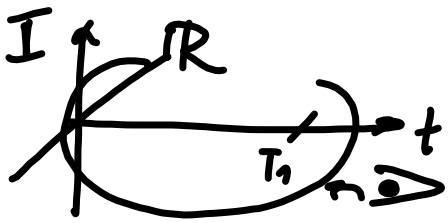
$x[n] = \frac{C_1}{2} e^{j\varphi_1} e^{j\omega_1 n} + \frac{C_1}{2} e^{-j\varphi_1} e^{-j\omega_1 n}$



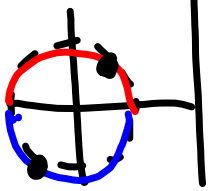
Summing c.e. over 1 period

Continuous time

$x(t) = e^{j\omega_1 t}$, $\omega_1 = \frac{2\pi}{T_1}$

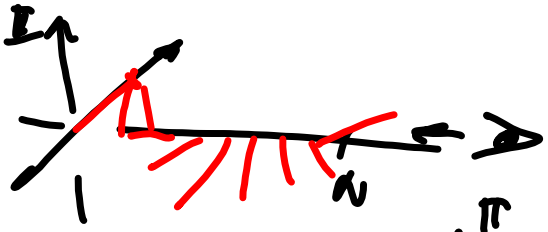


$$\int_0^{T_1} x(t)\,dt = 0$$

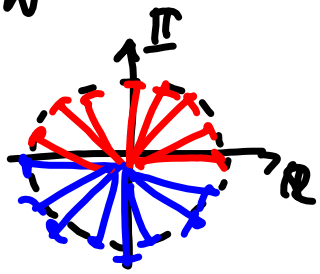


Discrete time

$x[n] = e^{j\Omega_1 n}$, $\Omega_1 = \frac{2\pi}{N}$

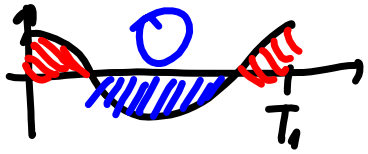


$$\sum_{n=0}^{N-1} x[n] = 0$$

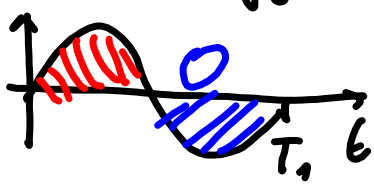


$$\int_0^{T_1} \left(\cos 2\pi \frac{1}{T_1} t + j \sin 2\pi \frac{1}{T_1} t\right) dt = \int \cos \ldots dt + j \int \sin \ldots dt$$

cos



sin



$f_1 = 440\,Hz \qquad T_1 = \frac{1}{440} \qquad F_S = 8000\,Hz$

$\frac{8000}{440} \qquad 8000 \cdot \frac{1}{440} =$