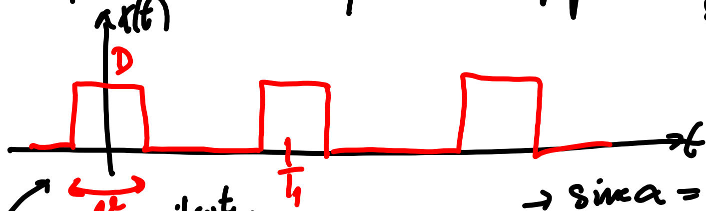
Fourier series – spec. anal. of periodic cont. time signals.



$c_k = \frac{1}{T_1}\int_{T_1} x(t)\, e^{-jk\omega_1 t}\, dt = \dots =$

$\frac{D\vartheta}{T_1}\,\mathrm{sinc}\left(\frac{\vartheta}{2}k\omega_1\right)$

→ $\mathrm{sinc}\, a = \frac{\sin a}{a}$

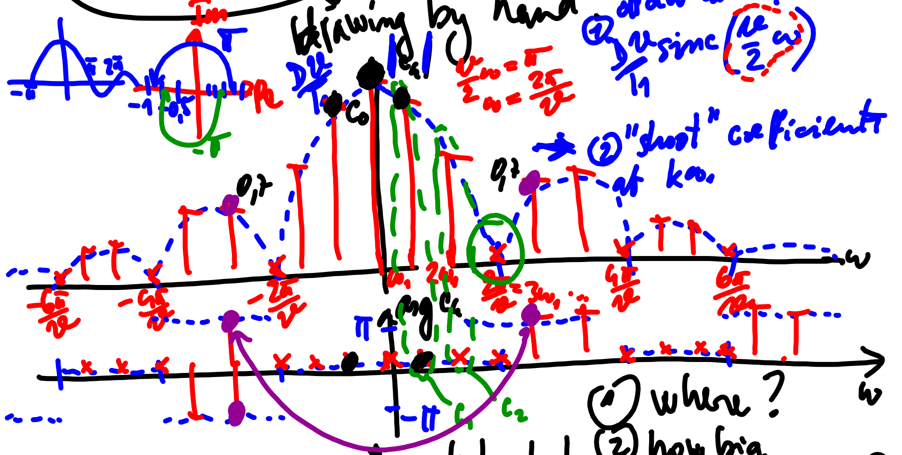
→ $\int_{-b}^{b} e^{\pm jxy}\, dy = 2b\,\mathrm{sinc}(bx)$

$\omega_1 = \frac{2\pi}{T_1}$ fundamental freq. [rad/s]

drawing by hand!

(1) draw cont. function $\frac{D\vartheta}{T_1}\mathrm{sinc}\left(\frac{\vartheta}{2}\omega\right)$

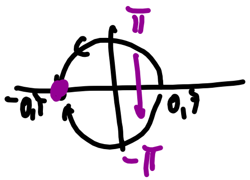
(2) "shoot" coefficients at $k\omega_1$



(1) where? (2) how big (3) how shifted?

$c_k = c_{-k}^*$ $|c_k| = |c_{-k}|$

$\arg c_k = -\arg c_{-k}$

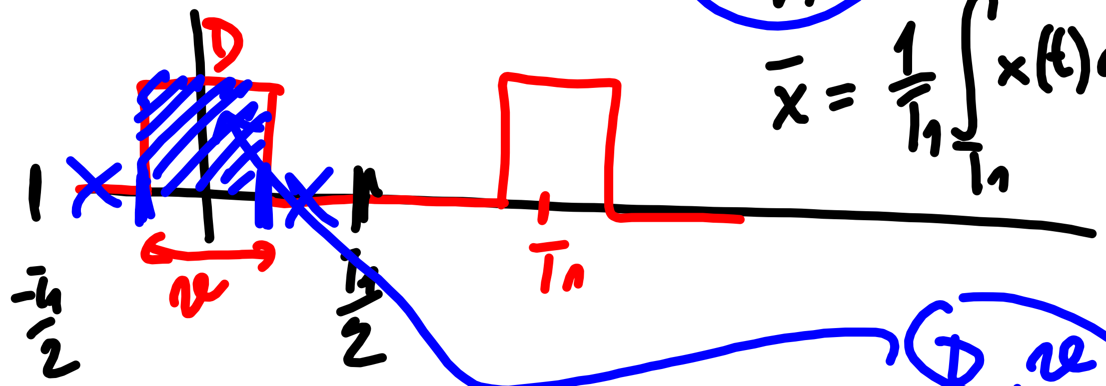


-0,7 and -0,7 are! $\Re(x) = \Re(y)$ $\Im(x) = \Im(y)$

$|x| = |y|$

$\arg x = -\arg y$ ✓

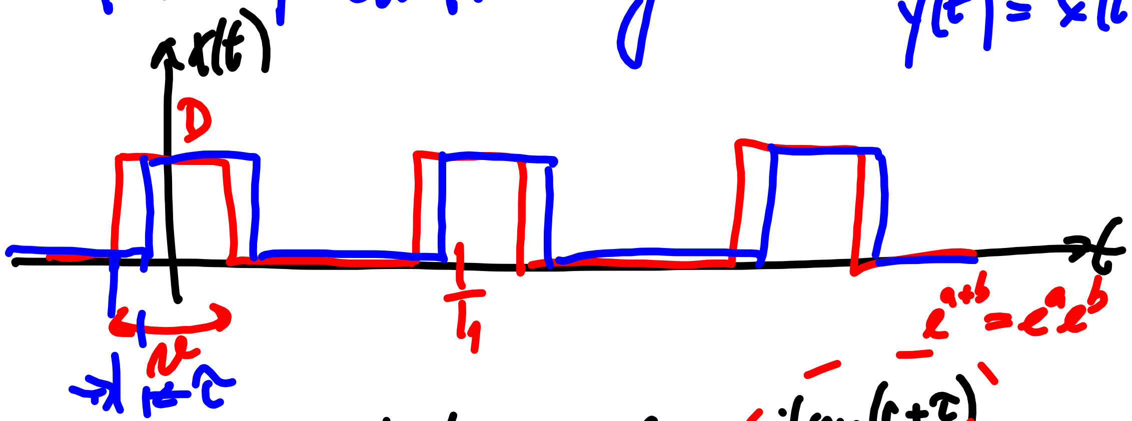
$$c_0 = \text{D.C. value} = \frac{D\tau}{T_1}$$



$$\bar{x} = \frac{1}{T_1}\int_{T_1} x(t)\,dt = \frac{1}{T_1}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} D\,dt = \frac{D\cdot\tau}{T_1}$$

F.S. of shifted signals.

$$y(t) = x(t-\tau)$$



$$e^{a+b} = e^a e^b$$

$$c_{yk} = \frac{1}{T_1}\int_{T_1} x(t-\tau)\,e^{-jk\omega_1 t}\,dt = \frac{1}{T_1}\int_{T_1} x(r)\,e^{-jk\omega_1 (r+\tau)}\,dr = \frac{1}{T_1}\int x(r)\,e^{-jk\omega_1 r}\,e^{-jk\omega_1 \tau}\,dr$$

$r = t-\tau$

$t = r+\tau$

$dt = dr$

$$= e^{-jk\omega_1 \tau}\underbrace{\frac{1}{T_1}\int x(r)\,e^{-jk\omega_1 r}\,dr}_{c_k} \leftarrow \text{def. F.S.!}$$

what?

$$= e^{-jk\omega_1 \tau}\, c_k \leftarrow \text{original coeffs.}$$

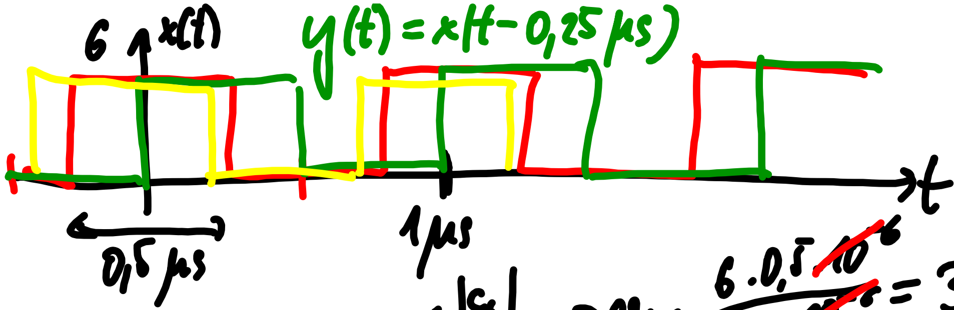
"correction" for the shift.

$$|c_{yk}| = \underbrace{|e^{-jk\omega_1 \tau}|}_{1} \cdot |c_k| = |c_k|$$

"how much" did not change!

$$\arg c_{yk} = \arg e^{-jk\omega_1 \tau} + \arg c_k = \arg c_k - k\omega_1 \tau$$

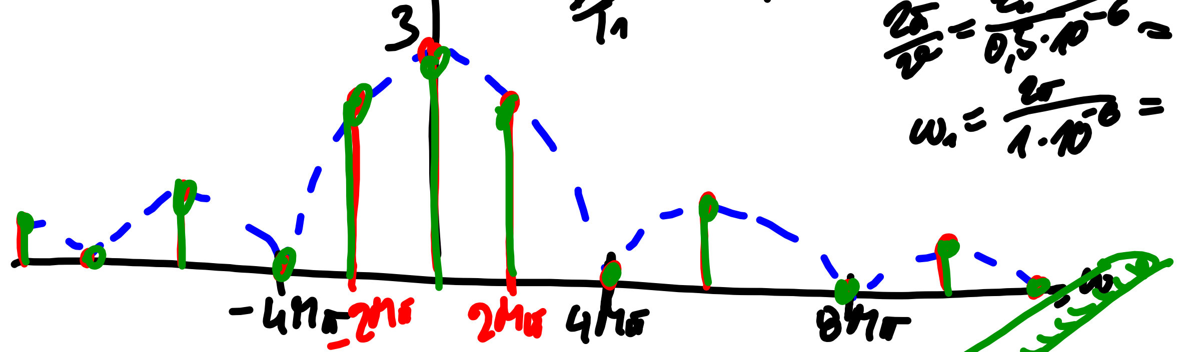
"correction" of arguments.



$D = 6$

$\tau = 0{,}5 \cdot 10^{-6}$

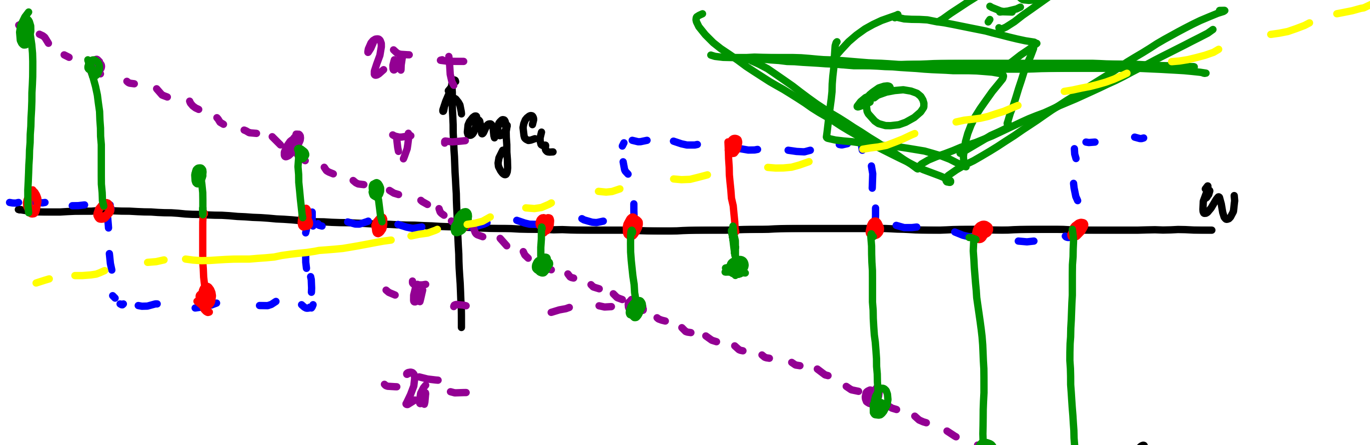
$T_1 = 1 \cdot 10^{-6}$



$$\frac{D\tau}{T_1} = \frac{6 \cdot 0{,}5 \cdot \cancel{10^{-6}}}{1 \cdot \cancel{10^{-6}}} = 3$$

$$\frac{2\pi}{\tau} = \frac{2\pi}{0{,}5 \cdot 10^{-6}} = 4\pi \cdot 10^{6} = 4M\pi$$

$$\omega_1 = \frac{2\pi}{1 \cdot 10^{-6}} = 2M\pi$$



$$\text{ang}\, c_k = \text{ang}\, c_k - k\omega_1\tau = \text{ang}\, c_k - k\, 2\pi \cdot \cancel{10^{6}} \cdot 0{,}25 \cdot \cancel{10^{-6}} = \text{ang}\, c_k - k\frac{\pi}{2}$$

Summary for F.S.

- cont. time signal periodic
- series of coefficients $k\omega_1$ multiples of fund. freq.

magnitudes "how much"

phases "how shifts"

where?

synt : freq → time

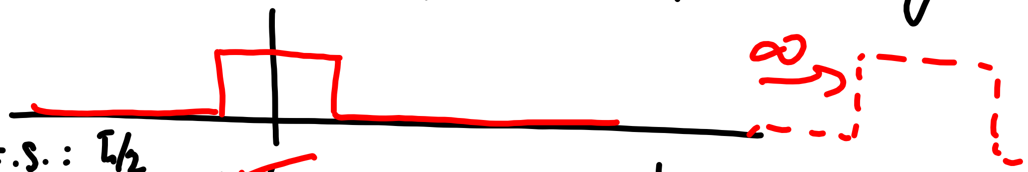
anal : time → freq.

Fourier game: output = sometimes constant summation input $e^{\pm j\,\text{freq.}\,\text{time}}$

$$c_k = \frac{1}{T_1}\int_{T_1} x(t)\,e^{-jk\omega_1 t}\,dt$$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k\,e^{+jk\omega_1 t}$$

Fourier transform → non-periodic signals



F.S.: $T_1/2$

$$c_k = \frac{1}{T_1}\int_{-T_1/2}^{T_1/2} x(t)\, e^{-jk\omega_1 t}\, dt$$

ang. freq.

$T_1 \to \infty$!

$\omega_1 = \frac{2\pi}{T_1}$ $d\omega = \frac{2\pi}{T_1}$ infinitely small step in frequency

dc infinitely small increment of coefficient.

$$dc = \frac{d\omega}{2\pi}\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$\frac{1}{T_1} = \frac{d\omega}{2\pi}$

$$2\pi \frac{dc}{d\omega} = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

density

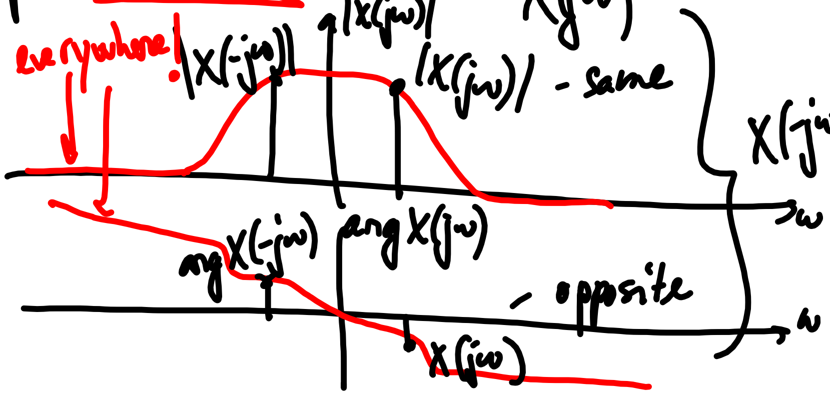
$$\boxed{X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$

ρ volume V mass m

$\rho = \frac{m}{V}$

$\rho(x,y,z) = \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$

Complex spectral function F.T. $X(j\omega)$



$X(-j\omega) = X^*(j\omega)$

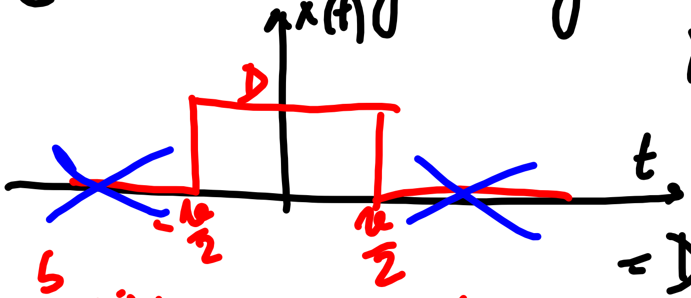
$$\boxed{X(j\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt}$$

spectral function F.T.

$$\boxed{x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{+j\omega t}\,d\omega}$$

inverse F.T. (IFT)

Ex #1: rectangular signal



$$X(j\omega)=D\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-j\omega t}\,dt =$$

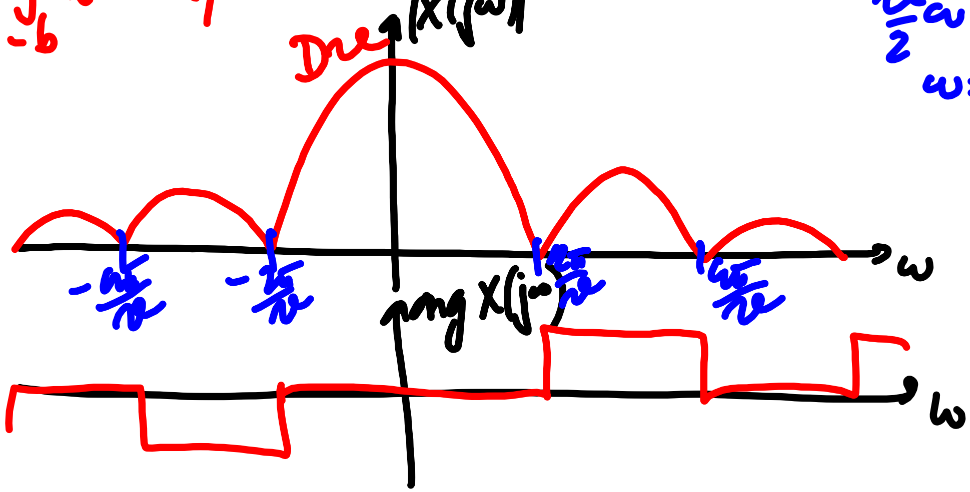
S.T.

$$= D\,2\cdot\frac{\tau}{2}\,\mathrm{sinc}\left(\frac{\tau}{2}\omega\right)=D\tau\,\mathrm{sinc}\left(\frac{\tau}{2}\omega\right)$$

$$\int_{-b}^{b} e^{\pm jxy}\,dy = 2b\,\mathrm{sinc}\,bx$$

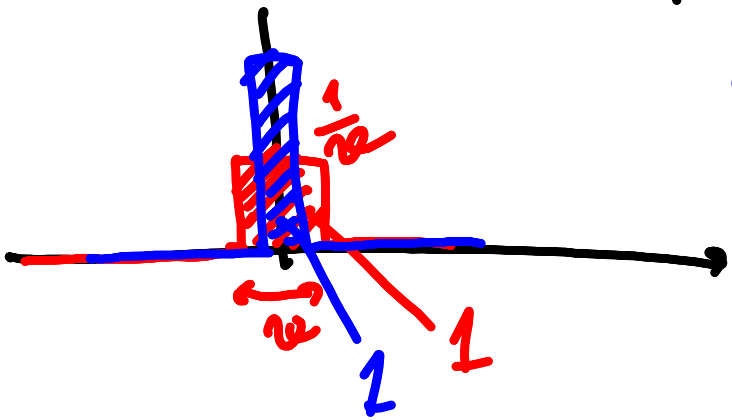
$$\frac{\tau}{2}\omega=\pi$$

$$\omega=\frac{2\pi}{\tau}$$

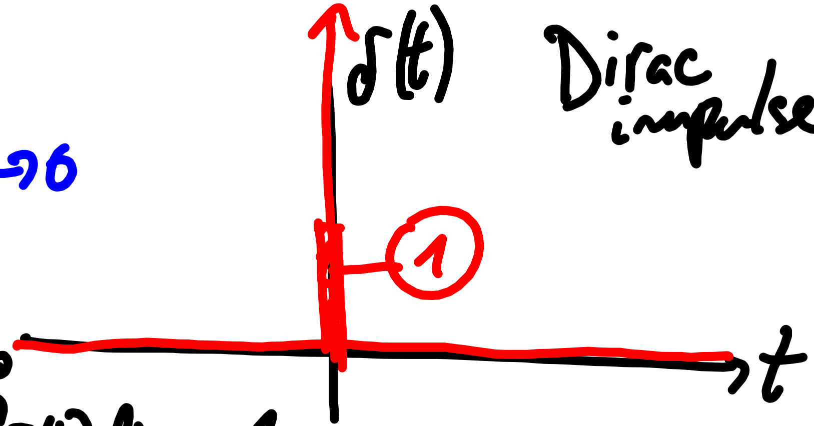


spectral function

F.T. of Dirac pulse



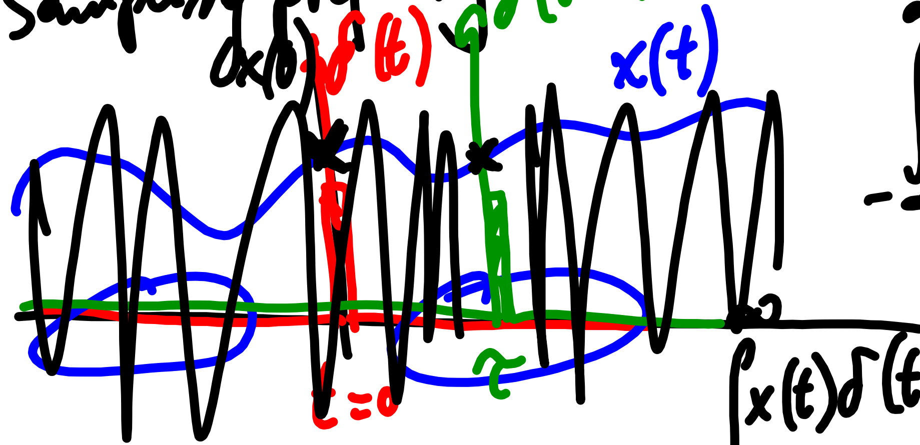
$w \to 0$



Dirac impulse

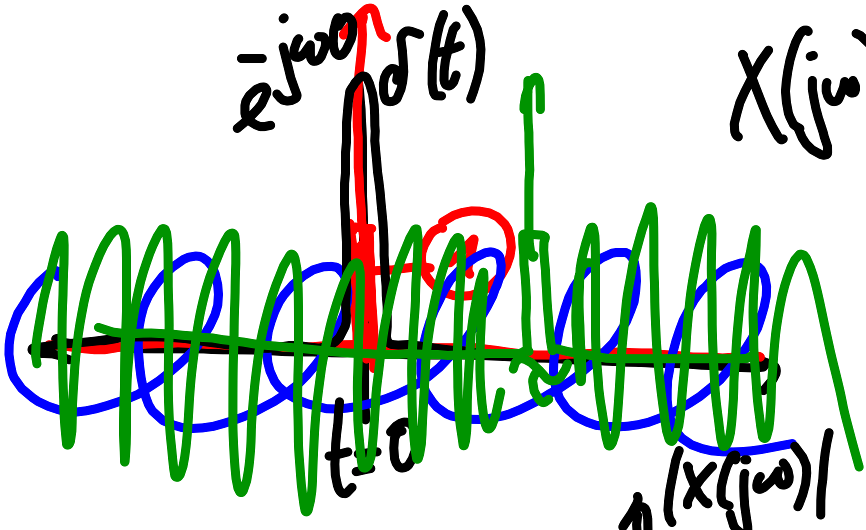
$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

"Sampling property"



$$\int_{-\infty}^{\infty} x(t)\delta(t)\,dt = \int_{-\infty}^{\infty} x(0)\delta(t)\,dt = x(0)$$

$$\int_{-\infty}^{\infty} x(t)\delta(t-\tau)\,dt = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,dt = x(t-\tau)$$



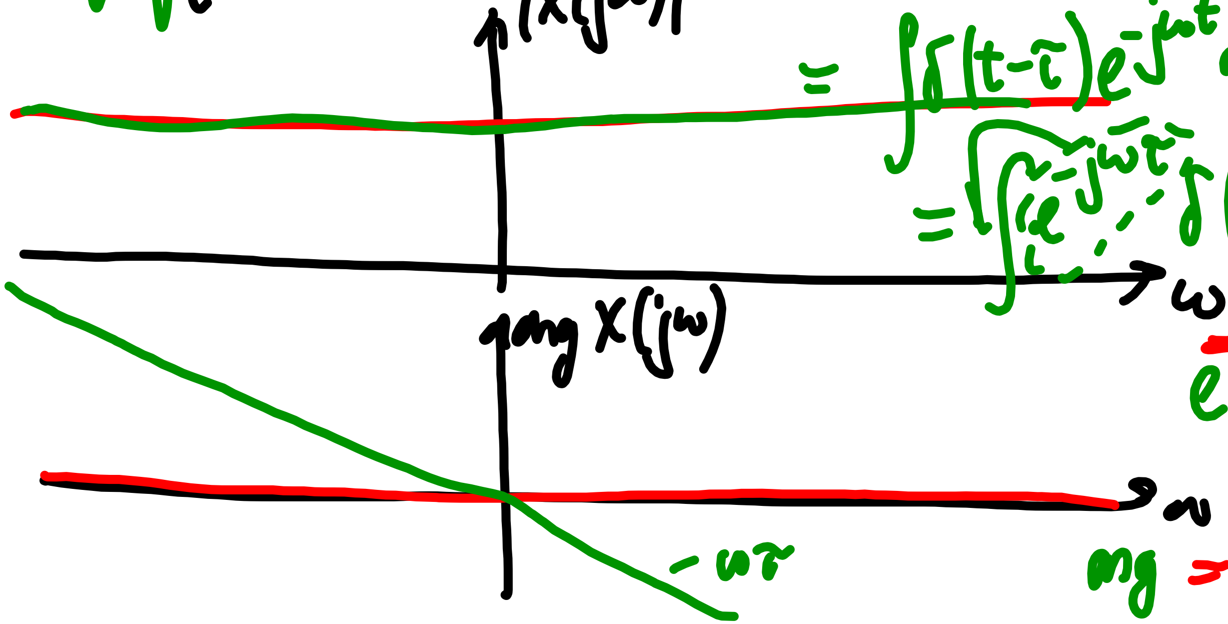
$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt =$$

$$= \int \underbrace{e^{-j\omega 0}}_{1} \delta(t)\, dt = 1$$

$$= \int \delta(t-\tau)\, e^{-j\omega t}\, dt =$$

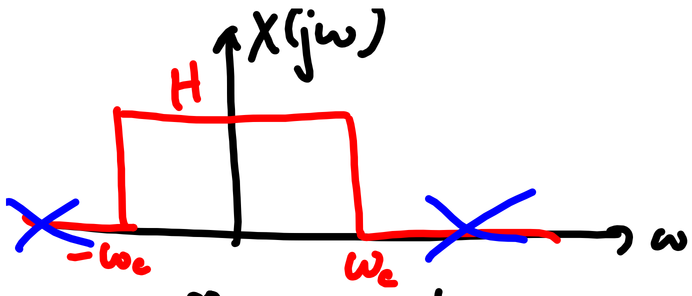
$$= \int e^{-j\omega\tau} \delta(t-\tau)\, dt =$$

$$e^{-j\omega\tau}$$



$$|e^{-j\omega\tau}| = 1$$

$$\text{ang} = -\omega\tau$$



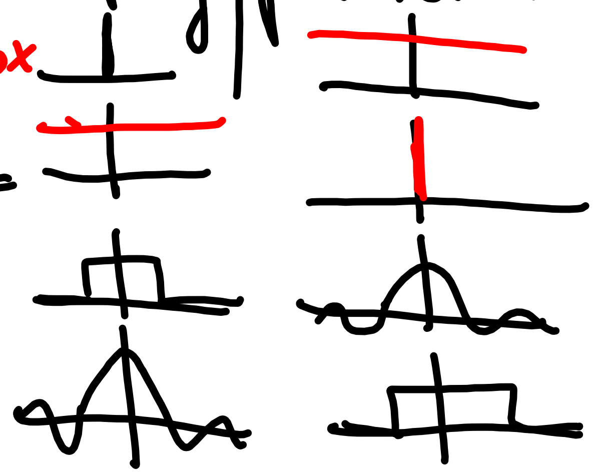
$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{+j\omega t}\,d\omega=$$

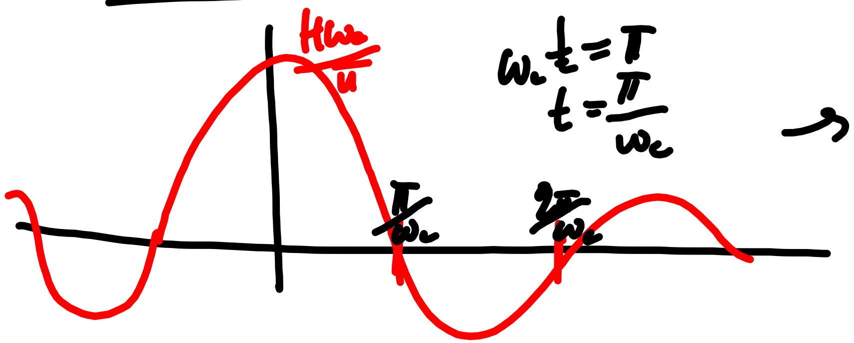
$$\int_{-b}^{b}e^{+jxy}\,dy=2b\,\mathrm{sinc}\,bx$$

$$=\frac{1}{2\pi}H\int_{-\omega_c}^{\omega_c}e^{+j\omega t}\,d\omega=\frac{H}{2\pi}2\omega_c\,\mathrm{sinc}(\omega_c t)=$$

$$=\frac{H\omega_c}{\pi}\,\mathrm{sinc}(\omega_c t)$$

Time $\Leftrightarrow$	Freq.
multiplic.	convolution
convolution	multipl.
periodicity	sampling (discretization)
sampling	periodization





$$\omega_c t=\pi$$
$$t=\frac{\pi}{\omega_c}$$

→ interpolation of signals using sinc function!

F.T. / Fourier game:

$t \to f$
$f \to t$

output = maybe constant · summation op. · input · $e^{\mp j \, \text{freq.} \, \text{time}}$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt \qquad x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{+j\omega t} \, d\omega$$

Input: non-periodic signal w/ continuous time

Out: spectral function defined for $\forall$ frequencies

$$X(j\omega) = X^*(-j\omega)$$

spectrum

$$x(t) \to X(j\omega)$$

$$x(t-\tau) \to X(j\omega) e^{-j\omega\tau}$$

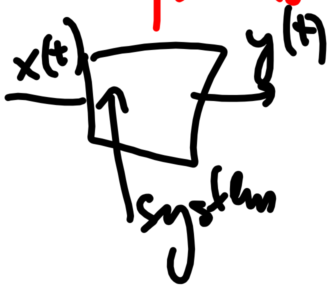
arg

1) where? → everywhere!

2) how much?

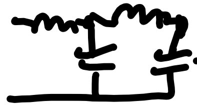
3) how shifted?

Systems - Filters



Memory / memoryless

$y(t) = A\,x(t)$ → no mem | $y[n] = A\,x[n]$



→ mem. | $y[n] = x[n] - x[n-1]$

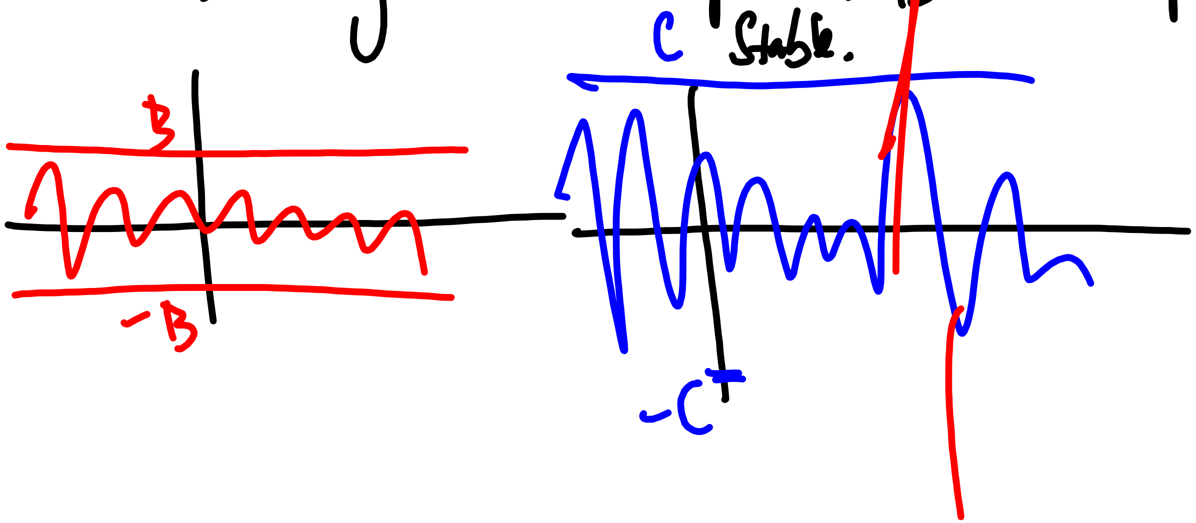
Causality

$$y(t) = f(x(t), x(\text{anything} \le t), y(\text{anything} < t))$$

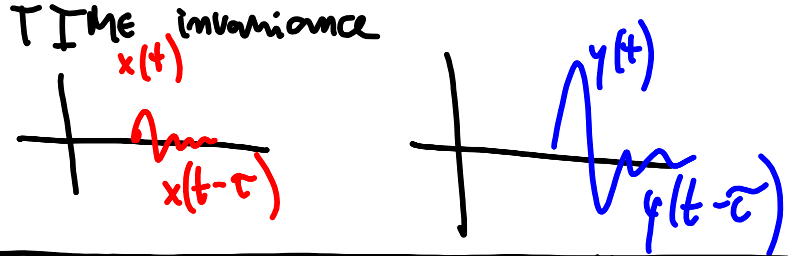
Stability

Bounded Input → Bounded Output

Stable.



TIME invariance



Linearity

$x_1(t) \longrightarrow y_1(t)$

$x_2(t) \longrightarrow y_2(t)$

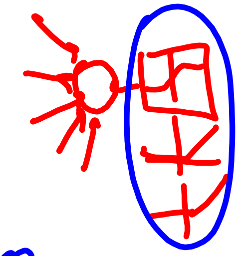
$x_1(t)$, $x_2(t)$ → $a x_1(t) + b x_2(t)$ → System → $a y_1(t) + b y_2(t)$ ⇒ linear.

~~LTI~~ - linear & time invariant.

some difficult signal → system → ???

Mach. learning

NN



cut it!

simple part 1 → syst. →

simple part 2 → syst. →

⋮

Σ

filter ~20 ms