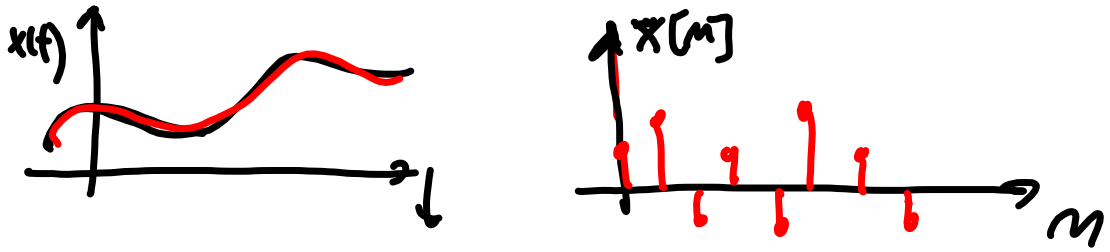
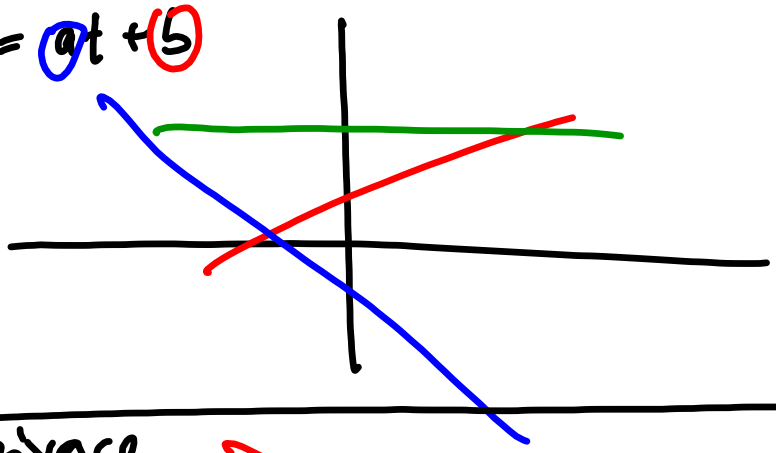


$$y(t) = x_1(t) + x_2(t) \quad \checkmark$$
$$= x_1(t) - x_2(t) \quad \checkmark$$
$$= x_1(t) \cdot x_2(t) \quad \checkmark$$

$$y[n] = x_1[n] + x_2[n] \quad \checkmark$$
$$x_1[n] - x_2[n] \quad \checkmark$$
$$x_1[n] \cdot x_2[n] \quad \checkmark$$

$x(t) = at + 5$



Derivace

$x(t) = t^2 + 2t + 3$

chci derivaci v bodě $t_1 = 0$

analyticky: $\frac{dx(t)}{dt} = 2t + 2$ $\qquad \left.\frac{dx(t)}{dt}\right|_{t_1=0} = 2$

numericky: $\left.\frac{dx(t)}{dt}\right|_{t_1} \approx \frac{x(t_2) - x(t_1)}{t_2 - t_1}$

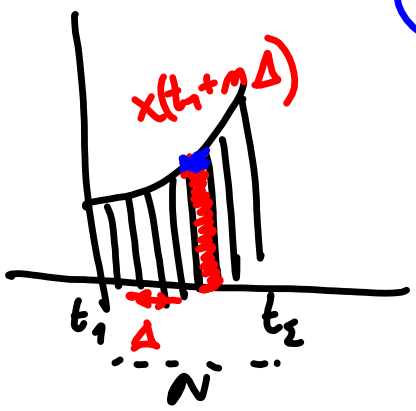
Integrály

$$\int_{t_1}^{t_2} x(t)\,dt = \left[x_p(t)\right]_{t_1}^{t_2} = x_p(t_2) - x_p(t_1)$$

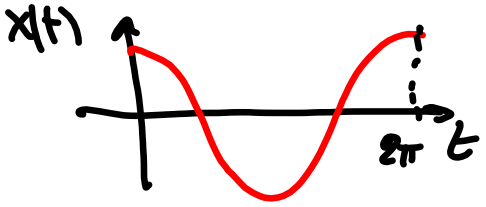
$t_1 = 0 \quad t_2 = 1$

1. Analyticky: $\int_0^1 x(t)\,dt = \left[\frac{t^3}{3} + t^2 + 3t\right]_0^1 = \frac{1}{3} + 1 + 1 = 4{,}33$

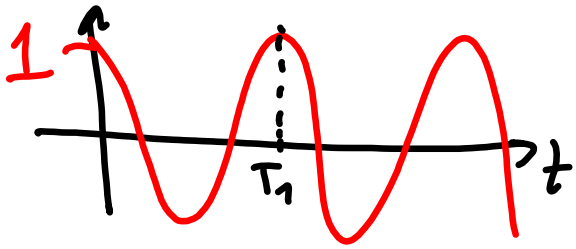
2. Numericky: $\int_{t_1}^{t_2} x(t)\,dt \approx \sum_{n=0}^{N-1} x(t_1 + n\Delta)\Delta$



3. Prasácky! $3 \times 1 + \frac{3 \times 1}{2} = 4{,}5$



$x(t) = \cos t$

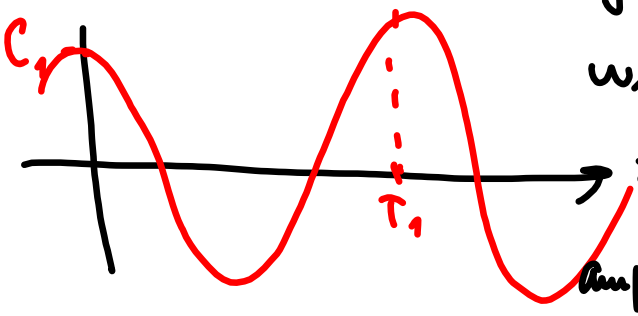


$x(t) = \cos\left(2\pi \frac{t}{T_1}\right)$

T_1 perioda [s]

$f_1 = \frac{1}{T_1}$ frekvence $s^{-1} = Hz$

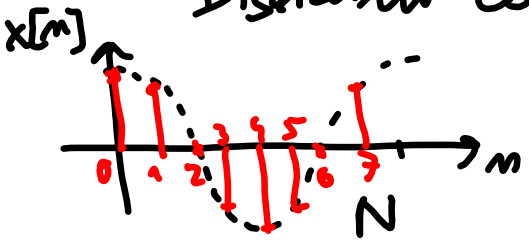
$\omega_1 = 2\pi f_1 = \frac{2\pi}{T_1}$ kruh. frekv. $rad\,s^{-1}$



$x(t) = C_1 \cos(\omega_1 t + \varphi_1)$

amplituda; $rad\,s^{-1}$; poč. fáze rad

Diskrétní cos



$x[n] = \cos n$ ~~cos n~~

$x[n] = \cos\left(2\pi \frac{n}{N}\right)$

N - perioda ve vzorcích []

$\frac{1}{N}$ základní normované frekv. []

$\omega_1 = \frac{2\pi}{N}$ normovaná kruhová frekv. rad

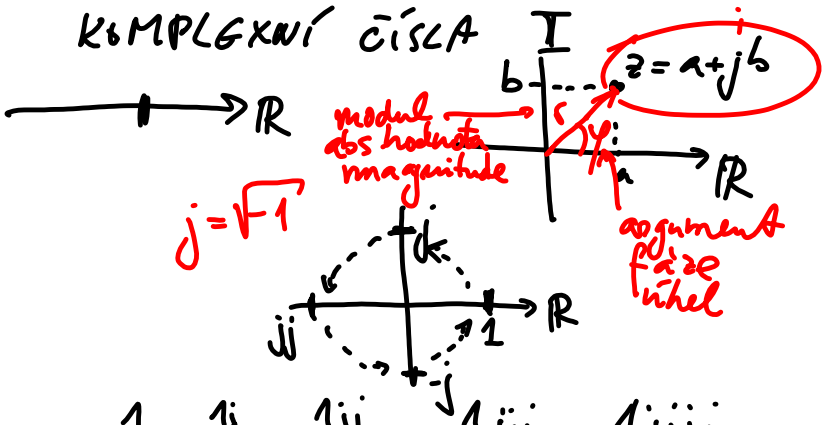
normování ≈ dělení
odnormování ≈ násobení } F_s

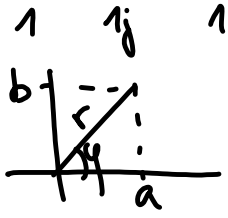
$x[n] = C_1 \cos(\omega_1 n + \varphi_1)$

amplituda; rad; poč. fáze rad.

$x[n] = 10 \cos\left(\frac{2\pi}{32} 5 n\right)$

KOMPLEXNÍ ČÍSLA





$a = r\cos\varphi$
$b = r\sin\varphi$
$z = r\cos\varphi + jr\sin\varphi$

$r = \sqrt{a^2+b^2}$
$\varphi = \arctan\frac{b}{a}$ arg. angle (z)

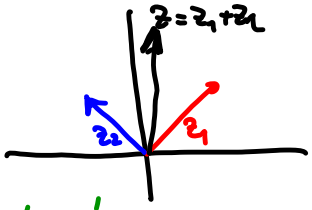
Exp. tvar kompl. čísla

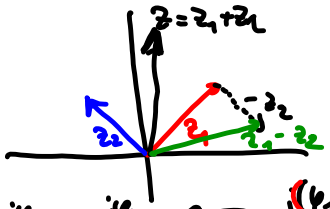
$z = re^{j\varphi}$ $z = r\exp(j\varphi)$ $z = r\angle\varphi$

operace

⊕ ⊖ ⊗

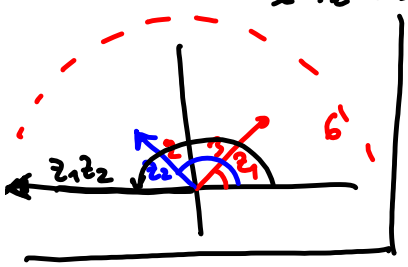
$z_1 = a_1 + jb_1$
$z_2 = a_2 + jb_2$
$z = a_1 + a_2 + j(b_1+b_2)$ $z = a_1 - a_2 + j(b_1-b_2)$



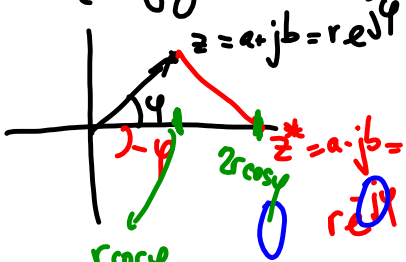


Násobení: $z = z_1 z_2 = r_1 e^{j\varphi_1} \cdot r_2 e^{j\varphi_2} = (r_1 r_2) e^{j(\varphi_1+\varphi_2)}$

$e^a \cdot e^b = e^{a+b}$

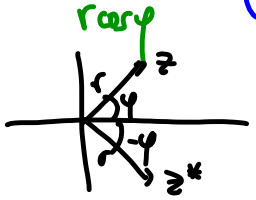


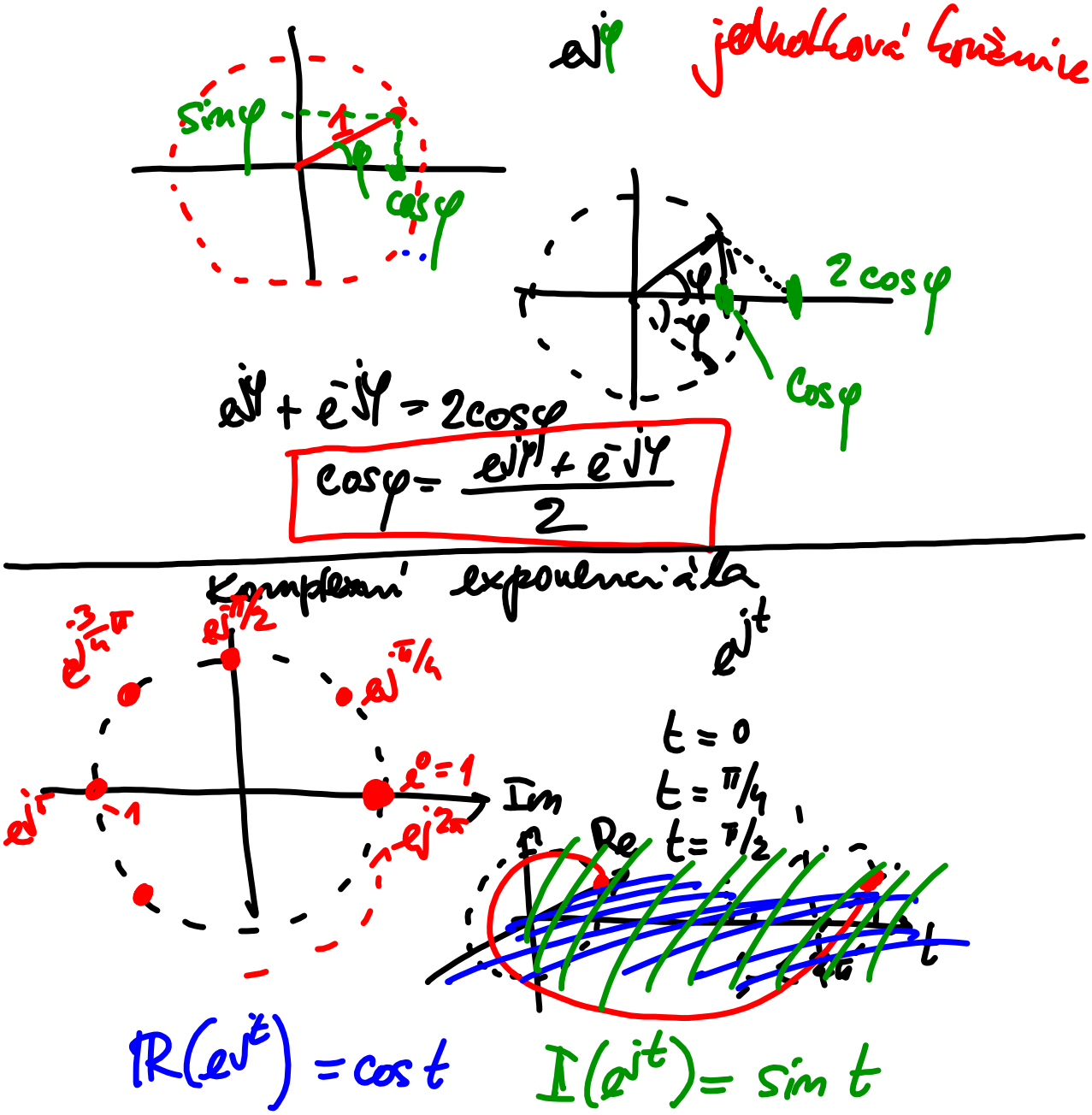
Komplexní sdružení (conjugation)

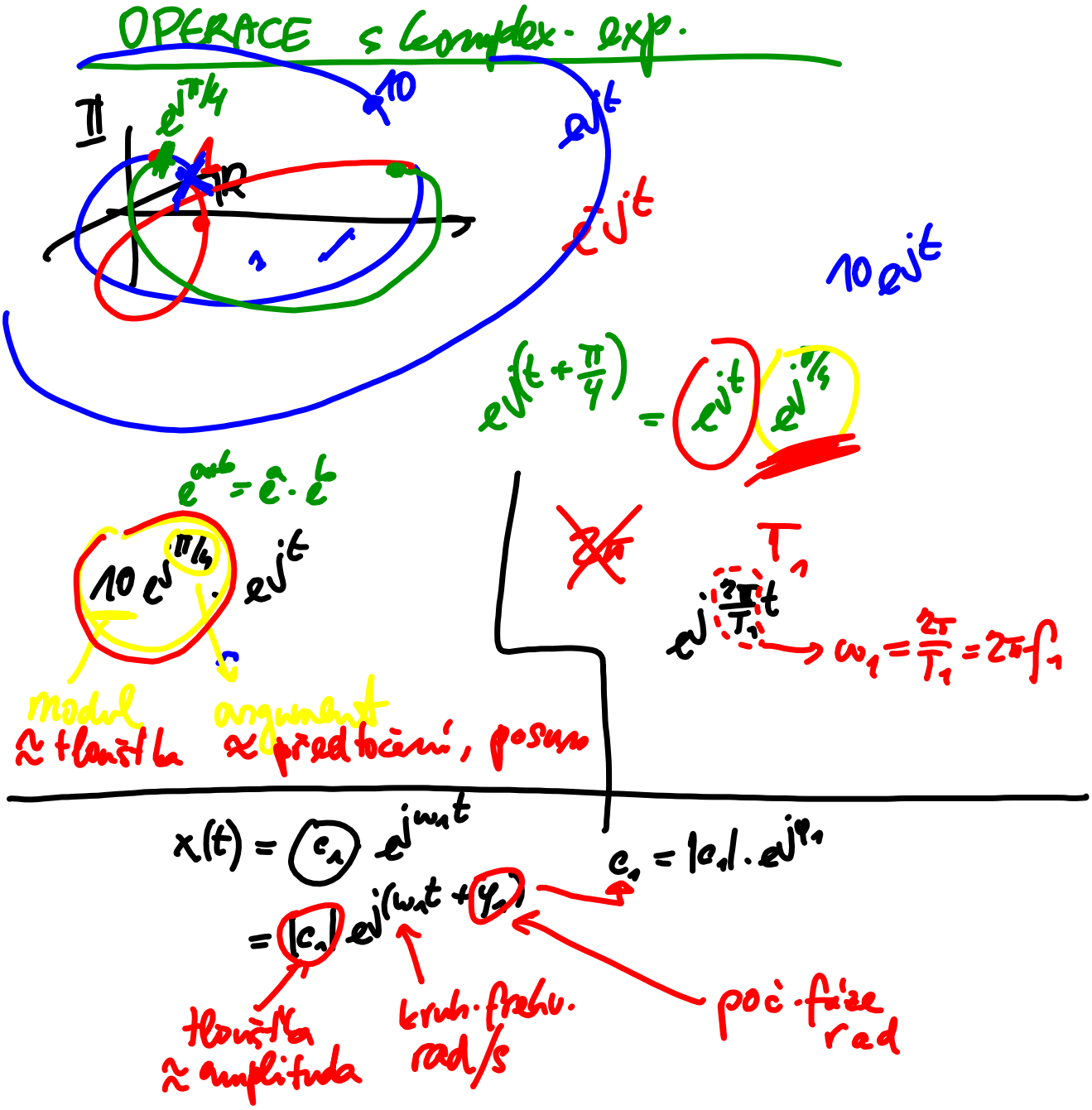


$z + z^* = 2\Re(z) = 2a$

$z \cdot z^* = r^2 = |z|^2$

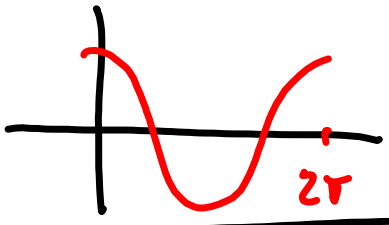



$e^{j\varphi}$ jednotková kružnice
sin φ
1
φ
cos φ
2 cos φ
cos φ
$e^{j\varphi} + e^{-j\varphi} = 2\cos\varphi$
$\cos\varphi = \frac{e^{j\varphi} + e^{-j\varphi}}{2}$
Komplexní exponenciála
e^{jt}
$e^{-\frac{3}{4}\pi j}$
$e^{\frac{\pi}{2}j}$
$e^{-\frac{\pi}{4}j}$
$e^{\pi j}$
$e^{0}=1$
$e^{2\pi j}$
Im
Re
t = 0
t = π/4
t = π/2
t
$\Re(e^{jt}) = \cos t$
$\Im(e^{jt}) = \sin t$


OPERACE s komplex. exp.
10
$e^{j\pi/4}$
e^{jt}
$10e^{jt}$
$e^{j(t+\frac{\pi}{4})} = e^{jt} e^{j\pi/4}$
$e^{a+b} = e^a \cdot e^b$
$10e^{j\pi/4} \cdot e^{jt}$
2π
T_1
$e^{j\frac{2\pi}{T_1}t}$
$\omega_1 = \frac{2\pi}{T_1} = 2\pi f_1$
modul
≈ hlasitka
argument
≈ předtočení, posun
$x(t) = c_1 e^{j\omega_1 t}$
$= |c_1| e^{j(\omega_1 t + \varphi_1)}$
$c_1 = |c_1| \cdot e^{j\varphi_1}$
hlasitka
≈ amplituda
kruh. frekv.
rad/s
poč. fáze
rad

cos. pomocí compl. exp.

$x(t) = \cos t$



$= \frac{e^{jt} + e^{-jt}}{2}$

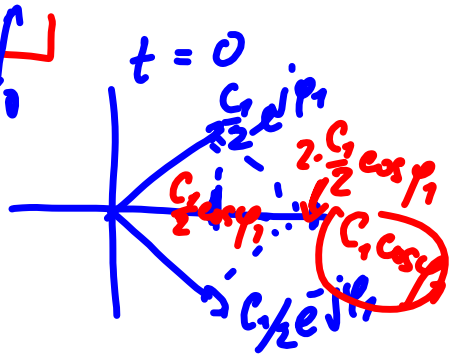
$e^{a+b} = e^a \cdot e^b$

$C_1 \cos(\omega_1 t) = \frac{C_1}{2} e^{j\omega_1 t} + \frac{C_1}{2} e^{-j\omega_1 t}$ ✓

$C_1 \cos(\omega_1 t + \varphi_1) = \frac{C_1}{2} e^{j(\omega_1 t + \varphi_1)} + \frac{C_1}{2} e^{-j(\omega_1 t + \varphi_1)} =$

$= \frac{C_1}{2} e^{j\varphi_1} e^{j\omega_1 t} + \frac{C_1}{2} e^{-j\varphi_1} e^{-j\omega_1 t}$

0 modul $\frac{C_1}{2}$ arg. φ_1 modul $\frac{C_1}{2}$ arg $-\varphi_1$

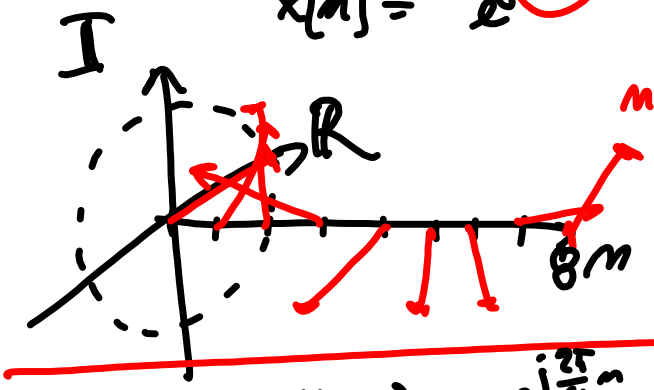


1) komplexní lášev !
2) Py nb ...
3) ISS demo Tona Kaňoka

KOMPLEX. EXP. S DISK. ČASEM.

$\cos(\omega_1 t)$ - - - - $\cos\left(\frac{2\pi}{N}n\right)$

$x[n] = e^{j\frac{2\pi}{N}n}$



$$\cos\left(\frac{2\pi}{N}n\right) = \frac{e^{j\frac{2\pi}{N}n} + e^{-j\frac{2\pi}{N}n}}{2}$$

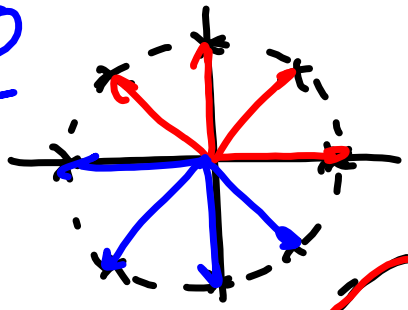
$$C_1\cos\left(\frac{2\pi}{N}n + \varphi_1\right) = \frac{C_1}{2}e^{j\varphi_1}e^{j\frac{2\pi}{N}n} + \frac{C_1}{2}e^{-j\varphi_1}e^{-j\frac{2\pi}{N}n}$$

$\frac{C_1}{2}$ hlavička
φ_1 úhel

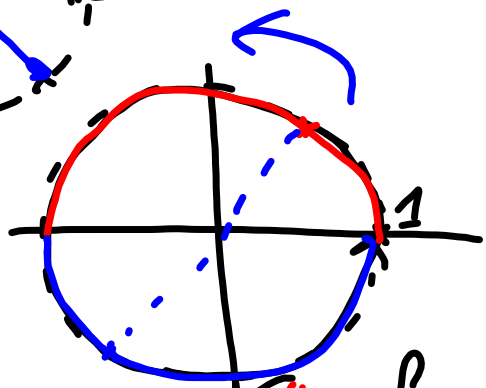
$\frac{C_1}{2}$ hlavičk
$-\varphi_1$ úhel

SUMA / INT přes 1 periodu

$$\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}n} = 0$$

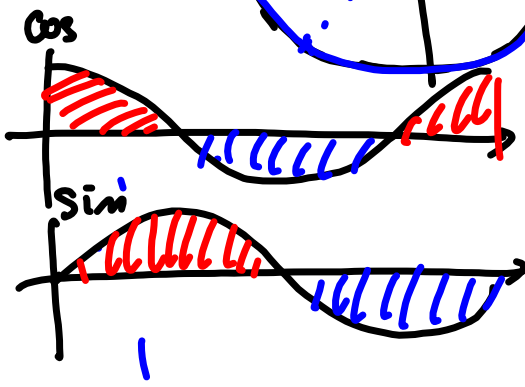


$N = 8$



$$\int_{T_1} e^{j\omega_1 t}\,dt = 0$$

$$\omega_1 = \frac{2\pi}{T_1}$$



$$\int_{T_1} \cos = 0$$

$$\int_{T_1} \sin = 0$$