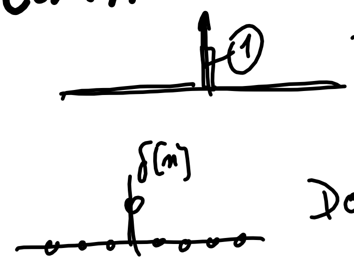
CONTINUOS TIME SYSTEMS



Theory! (Approx)

Doable

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$y(t) = x(t) * h(t)$$

$$y(t) = \int x(\tau)\, h(t-\tau)\, d\tau$$

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$y[n] = x[n] * h[n]$$

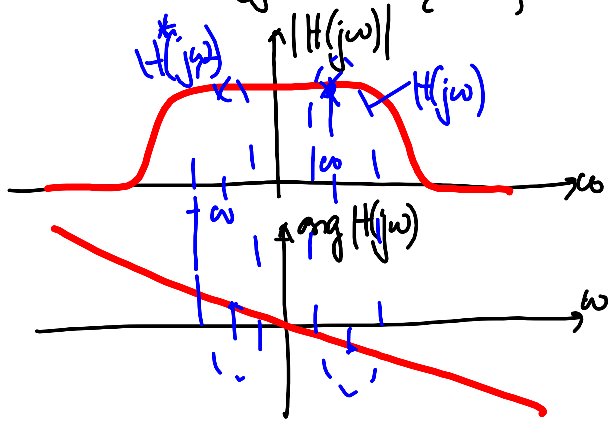
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

(COMPLEX) FREQUENCY RESPONSE

$$H(j\omega) = \mathcal{F}\{h(t)\}$$

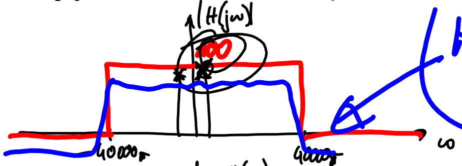
$$H(j\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt$$

real

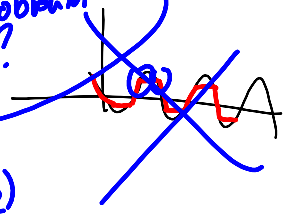


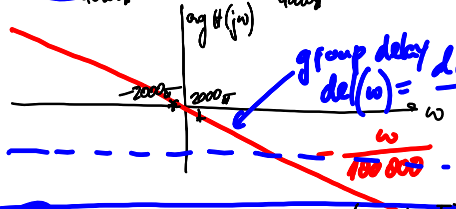
$$H(j\omega) = H^*(-j\omega)$$

SUPER HI-FI AMPLIFIER



How to obtain H(jω)?





group delay $del(\omega) = \dfrac{d \arg H(j\omega)}{d\omega}$

$-\dfrac{\omega}{100\,000}$

$-\dfrac{2000\pi}{100\,000} = -0.02\pi$

Ex 1 Easy one: $x(t) = 5 \cdot e^{j(2000\pi t + \frac{\pi}{2})}$

$$|H(j2000)|\, 5\, e^{j(2000\pi t + \frac{\pi}{2} - \arg H(j2000\pi))} = 100 \cdot 5\, e^{j(2000\pi + \frac{\pi}{2} - 0.02\pi)} =$$
$$= 500\, e^{j(2000\pi + 0.48\pi)}$$

Ex 2 $x(t) = 5\cos\left(2000\pi t + \frac{\pi}{2}\right)$ $\quad \cos\alpha = \dfrac{e^{j\alpha} + e^{-j\alpha}}{2}$

$$x(t) = \frac{5}{2} e^{j\frac{\pi}{2}} e^{j2000\pi t} + \frac{5}{2} e^{-j\frac{\pi}{2}} e^{-j2000\pi t}$$

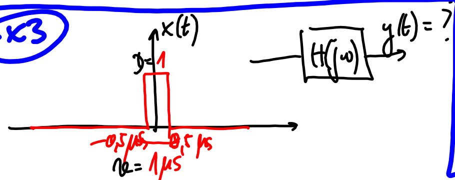
$H(j2000\pi) = 100\, e^{-j0.02\pi}$

$H(-j2000\pi) = 100\, e^{j0.02\pi}$

$$y(t) = \frac{5}{2} \cdot 100\, e^{j\frac{\pi}{2}} e^{-j0.02\pi} e^{j2000\pi} + \frac{5}{2} \cdot 100\, e^{-j\frac{\pi}{2}} e^{+j0.02\pi} e^{-j2000\pi} =$$

$$= 250\, e^{j0.48\pi} e^{j2000\pi} + 250\, e^{-j0.48\pi} e^{-j2000\pi} = 500\cos(2000\pi t + 0.48\pi)$$

Ex 3



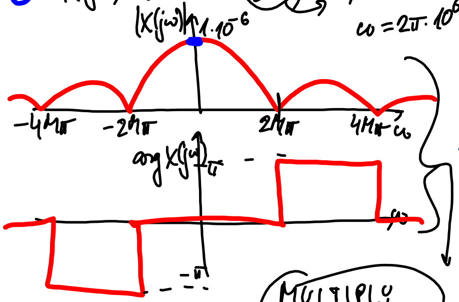
(1) $X(j\omega) = \mathcal{F}(x(t))$

(2) $Y(j\omega) = X(j\omega) \cdot H(j\omega)$

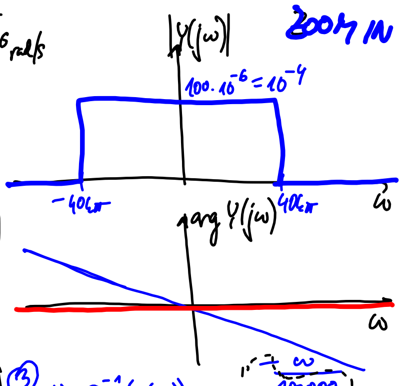
(3) $y(t) = \mathcal{F}^{-1}(Y(j\omega))$

(1) $X(j\omega) = D\vartheta\, \mathrm{sinc}\left(\frac{\vartheta}{2}\omega\right)$ $\quad 0.5 \cdot 10^{-6}\omega = \pi$

$\omega = 2\pi \cdot 10^{6}$ rad/s



ZOOM IN



MULTIPLY

(2)

$-\dfrac{\omega}{100\,000}$

$$\int_{-b}^{b} e^{\pm jxy}\, dy = 2b\, \mathrm{sinc}(bx)$$

(3) $y(t) = \mathcal{F}^{-1}(Y(j\omega))$

$$y_z(t) = \mathcal{F}^{-1}(Y_z(j\omega)) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y_z(j\omega) e^{j\omega t}\, d\omega = \frac{10^{-4}}{2\pi}\int_{-40k\pi}^{40k\pi} e^{+j\omega t}\, d\omega =$$

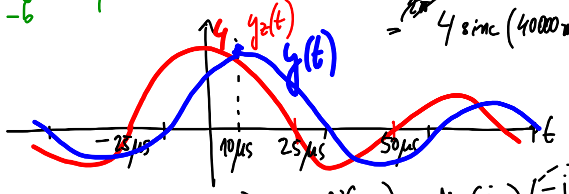
$$= \frac{10^{-4}}{2\pi}\, 2 \cdot 40 \cdot 10^{3}\pi\, \mathrm{sinc}(40 \cdot 10^{3}\pi t) =$$

$$= 4\, \mathrm{sinc}(40000\pi t)$$

$40000\pi t = \pi$

$t = \dfrac{1}{40000} = 0.25 \cdot 10^{-4}$

$= 25 \cdot 10^{-6}$



$y(t) = y_z(t - \tau)$ $\qquad Y(j\omega) = Y_z(j\omega)\, e^{-j\omega\tau}$ → magnitudes copy-paste

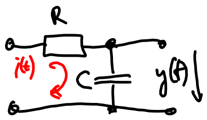
angles $-\omega\tau$

$\tau = \dfrac{1}{100\,000} = 10\,\mu s$

$\mathcal{L}$ How to determine $H(j\omega)$?

$$b_0 x(t) + b_1 \frac{dx(t)}{dt} + \dots + b_Q \frac{d^Q x(t)}{dt^Q} = a_0 y(t) + a_1 \frac{dy(t)}{dt} + \dots + a_P \frac{d^P y(t)}{dt^P}$$

$$\sum_{k=0}^{Q} b_k \frac{d^k x(t)}{dt^k} = \sum_{k=0}^{P} a_k \frac{d^k y(t)}{dt^k}$$



$$i(t) = \frac{x(t) - y(t)}{R} \qquad i(t) = C \frac{dy(t)}{dt}$$

$$\frac{x(t) - y(t)}{R} = C \frac{dy(t)}{dt}$$

$$x(t) = y(t) + RC \frac{dy(t)}{dt}$$

$b_0 = 1$ $a_0 = 1$ $a_1 = RC$

$\mathcal{L}(\quad)$

Laplace transform $\approx$ generalization of Fourier transform

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$\boxed{X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt}$$ Laplace transf.

living in the whole complex plane!

$$X(s) = Y(s) + RC\, Y(s) s$$

$$X(s) = Y(s)(1 + RCs)$$

$$\frac{Y(s)}{X(s)} = \boxed{H(s) = \frac{1}{1 + RCs}}$$

$$H(j\omega) = \frac{1}{1 + RCj\omega}$$

"Rewriting rules"

$$x(t) \to X(s)$$
$$a\,x(t) \to a\,X(s)$$
$$\frac{dx(t)}{dt} \to s\,X(s)$$
$$\frac{d^k x(t)}{dt^k} \to s^k X(s) \quad \text{WHY?}$$

$$b_0 X(s) + b_1 X(s) s + \dots + b_Q X(s) s^Q = a_0 Y(s) + a_1 Y(s) s + \dots + a_P Y(s) s^P$$

$$H(s) = \frac{1}{RC\left(s + \frac{1}{RC}\right)} = \frac{1}{RC} \frac{1}{\left(s - \left(-\frac{1}{RC}\right)\right)}$$

$s + \frac{1}{RC} = 0$

$s = -\frac{1}{RC}$

TRANSFER FUNCTION

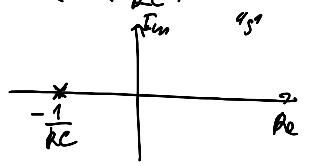
$ax^2 + bx + c$

x_1, x_2 "roots" $a(x - x_1)(x - x_2)$

$$H(s) = \frac{Y(s)}{X(s)}$$

$$X(s)\left(b_0 + b_1 s + \dots + b_Q s^Q\right) = Y(s)\left(a_0 + a_1 s + \dots + a_P s^P\right)$$

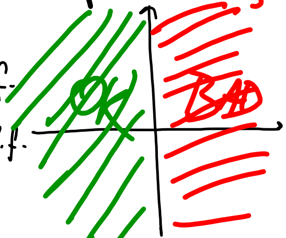
$$\frac{Y(s)}{X(s)} = \boxed{H(s)} = \frac{b_0 + b_1 s + \dots}{a_0 + a_1 s + \dots} = \boxed{\frac{\sum_{k=0}^{Q} b_k s^k}{\sum_{k=0}^{P} a_k s^k}}$$



FREQUENCY RESPONSE

$$H(j\omega) = H(s)\Big|_{s=j\omega} = \frac{\sum_{k=0}^{Q} b_k (j\omega)^k}{\sum_{k=0}^{P} a_k (j\omega)^k}$$

$$\mathrm{Re}(p_i) < 0 \quad \text{"s"}$$



$$H(s) = \text{const} \frac{(s - m_1)(s - m_2) \dots (s - m_Q)}{(s - p_1)(s - p_2) \dots (s - p_P)}$$

determined by b_k

m_i - roots of numerator $\to$ zeros of transf. f.

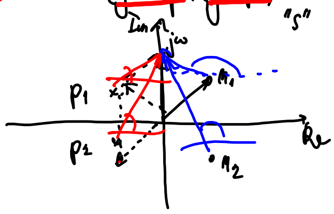
p_i - roots of denominator $\to$ poles of transf. f.

determined by a_k

FREQUENCY RESPONSE FROM ZEROS and POLES

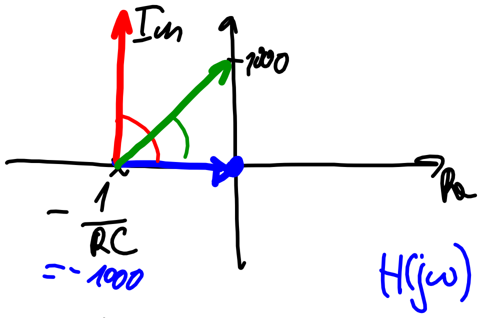
$$H(j\omega) = \text{const} \frac{(j\omega - m_1)(j\omega - m_2)}{(j\omega - p_1)(j\omega - p_2)}$$

$$|H(j\omega)| = \text{const} \frac{\text{multiplication of length of blue vectors}}{\text{multiplication of length of red vectors}}$$



$\arg H(j\omega)$ = Sum of angles of blue vectors − Sum of angles of red vectors.

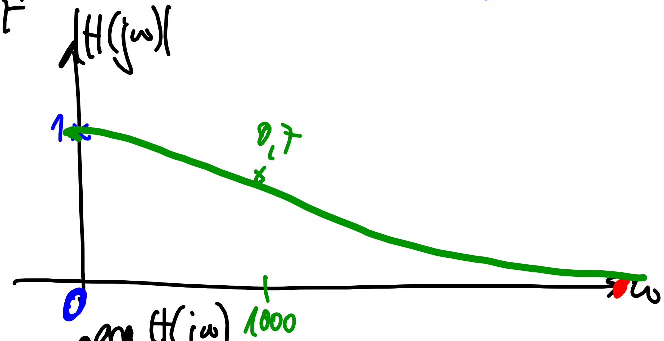
$$H(s) = \frac{1}{RC} \cdot \frac{1}{\left(s-\left(-\frac{1}{RC}\right)\right)}$$



$$\frac{1}{RC} = \frac{1}{1000 \cdot 10^{-6}} = \frac{1}{10^{-3}} = 1000$$

$$H(j\omega) = 1000 \cdot \frac{1}{\text{VECTOR}}$$

$R = 1000$ $\quad C = 1\mu F = 1 \cdot 10^{-6} F$

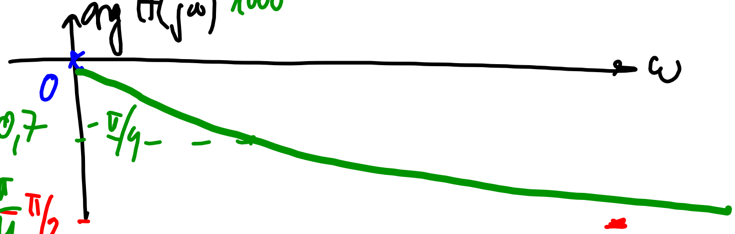


$\omega = 0$ $\quad |H(j0)| = \frac{1000}{1000} = 1$

$\arg H(j0) = 0 - 0 = 0$

$\omega = \infty$ $\quad |H(j0)| = \frac{1000}{\infty} = 0$

$\arg H(j0) = 0 - \frac{\pi}{2} = -\frac{\pi}{2}$



$\omega = 1000$ $\quad |H(j1000)| = \frac{\cancel{1000}}{\sqrt{2}\,\cancel{1000}} = 0{,}7$

$\arg H(j1000) = 0 - \frac{\pi}{4} = -\frac{\pi}{4}$