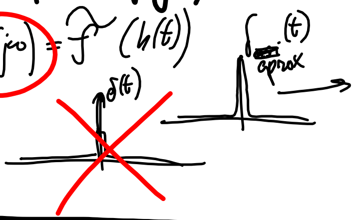
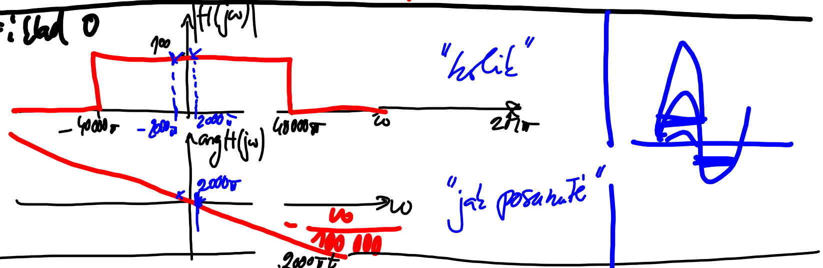
$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$

Systémy se spojitým časem.

komplexní frekv. char. (frequency response) $H(j\omega) = \mathcal{F}(h(t))$



Příklad 0



Příklad 1:

$x(t) = 1 \cdot e^{j2000\pi t}$

$y(t) = 100 \cdot e^{j(2000\pi t - 0{,}02\pi)}$

Příklad 2:

$x(t) = 5 \cdot \cos(2000\pi t + 0{,}02\pi) =$ $\qquad \cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$

$= \frac{5}{2} e^{j0{,}02\pi} \cdot e^{j2000\pi t} + \frac{5}{2} e^{-j0{,}02\pi} \cdot e^{-j2000\pi t}$

následně $H(j2000\pi)$, $H(-j2000\pi)$

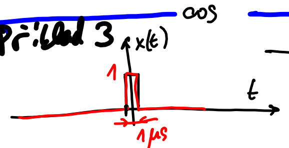
$y(t) = 100\, e^{-j0{,}02\pi} \cdot \frac{5}{2} e^{j0{,}02\pi} \cdot e^{j2000\pi t} + 100\, e^{j0{,}02\pi} \cdot \frac{5}{2} e^{-j0{,}02\pi} \cdot e^{-j2000\pi t} =$

$= \underbrace{250\, e^{j2000\pi t} + 250 \cdot e^{-j2000\pi t}}_{\cos} = 500 \cos(2000\pi t + 0)$

Příklad 3

$x(t) \rightarrow \boxed{H(j\omega)} \rightarrow y(t) = ?$

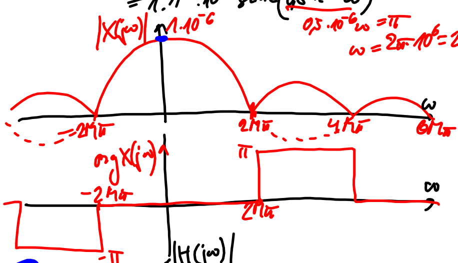
① $x(t) \rightarrow X(j\omega)$
② $Y(j\omega) = X(j\omega) \cdot H(j\omega)$
③ $Y(j\omega) \rightarrow y(t)$



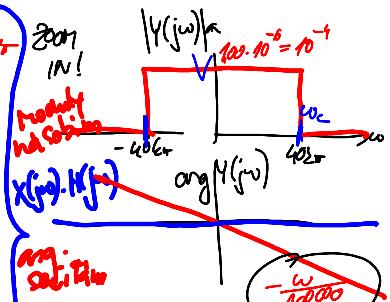
$\frac{\vartheta}{2} = 0{,}5\,\mu s$

① $X(j\omega) = \mathcal{F}(x(t)) = D\vartheta\, \mathrm{sinc}\left(\frac{\vartheta}{2}\omega\right) =$
$= 1 \cdot 1 \cdot 10^{-6}\, \mathrm{sinc}(0{,}5 \cdot 10^{-6}\omega)$

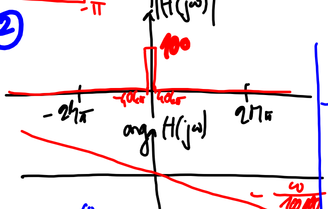
$0{,}5 \cdot 10^{-6}\omega = \pi$
$\omega = 2\pi \cdot 10^{6} = 2M\pi$



zoom in!



②



$\int_{-b}^{b} e^{\pm jxy}\, dy = 2b\, \mathrm{sinc}(bx)$

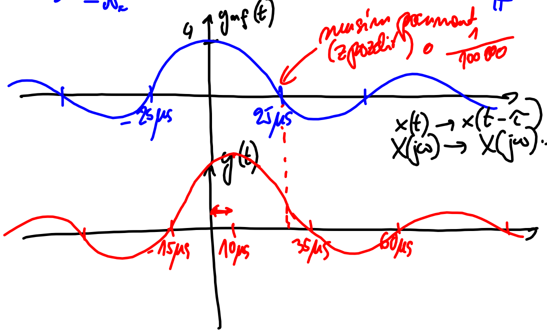
③ zpět do času:

$y(t) = \mathcal{F}^{-1}(Y(j\omega)) =$

$y_{mp}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Y_{mp}(j\omega)\, e^{+j\omega t}\, d\omega =$

$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} V e^{+j\omega t} d\omega = \frac{V}{2\pi} 2 \cdot \omega_c\, \mathrm{sinc}(\omega_c t) = \frac{V\omega_c}{\pi}\, \mathrm{sinc}(\omega_c t)$ → $\frac{10^{-4} \cdot 40000\pi}{\pi}$ → $(4)\, \mathrm{sinc}(40000\pi t)$

$= \frac{\pi}{\omega_c} = \frac{1}{40000} = 0{,}025\, ms = 25\,\mu s$



$x(t) \rightarrow x(t - \tau)$
$X(j\omega) \rightarrow X(j\omega) \cdot e^{-j\omega\tau}$ → modul ditto

→ arg. pos. o $-\omega\tau$

Frekv. charakteristika přímo z popisu systému

TEORIE

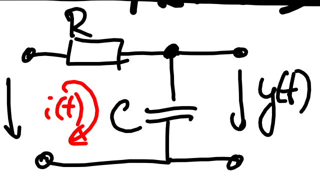
$$b_0x(t)+b_1\frac{dx(t)}{dt}+\dots b_Q\frac{d^Qx(t)}{dt^Q}=a_0y(t)+a_1\frac{dy(t)}{dt}+\dots a_P\frac{d^Py(t)}{dt^P}$$

$$\boxed{\sum_{n=0}^{Q}b_n\frac{d^nx(t)}{dt^n}=\sum_{m=0}^{P}a_m\frac{d^my(t)}{dt^m}}$$

DIFERENCIÁLNÍ ROVNICE – differential eq.

VS. DIFERENČNÍ ROVNICE (diskrétní čas!) – diference eq.

PŘÍKLAD



$$i(t)=\frac{x(t)-y(t)}{R} \qquad \frac{x(t)-y(t)}{R}=C\frac{dy(t)}{dt}$$

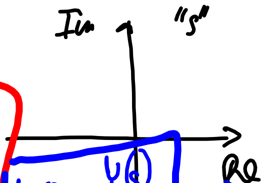
$$i(t)=C\frac{dy(t)}{dt} \qquad \boxed{x(t)=y(t)+RC\frac{dy(t)}{dt}}$$

$b_0=1 \quad a_0=1 \quad a_1=RC$

LAPLACEOVA TRANSFORMACE

$$X(j\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt$$

$$\boxed{X(s)=\int_{-\infty}^{\infty}x(t)e^{-st}dt}$$



$x(t)\longrightarrow X(s)$

$a\,x(t)\longrightarrow a\,X(s)$

$\frac{dx(t)}{dt}\longrightarrow s\,X(s) \qquad \frac{d^kx(t)}{dt^k}\longrightarrow s^kX(s)$

$$\boxed{H(s)=\frac{Y(s)}{X(s)}}$$ PŘENOSOVÁ FUNKCE – transfer function

L.T. systému:

$$b_0X(s)+b_1X(s)s+\dots+b_QX(s)s^Q=a_0Y(s)+a_1Y(s)s+\dots+a_PY(s)s^P$$

$$X(s)(b_0+b_1s+\dots b_Qs^Q)=Y(s)(a_0+a_1s+\dots+a_Ps^P)$$

$$\frac{Y(s)}{X(s)}=\boxed{H(s)=\frac{b_0+b_1s+\dots b_Qs^Q}{a_0+a_1s+\dots a_Ps^P}=\frac{\sum_{n=0}^{Q}b_ns^n}{\sum_{m=0}^{P}a_ms^m}}$$

$$X(s)=Y(s)+RC\,Y(s)\cdot s$$

$$X(s)=Y(s)[1+RCs]$$

$$\frac{Y(s)}{X(s)}=H(s)=\frac{1}{1+RCs}$$

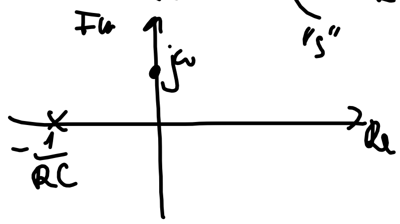
$$H(j\omega)=\frac{1}{1+RCj\omega} \qquad s+\frac{1}{RC}=0$$

PREKV. CHAR.

$$H(j\omega)=\frac{b_0+b_1j\omega+\dots b_Q(j\omega)^Q}{a_0+a_1j\omega+\dots a_P(j\omega)^P}=\frac{\sum b_n(j\omega)^n}{\sum a_m(j\omega)^m}$$

freqs

$$H(s)=\frac{1}{RC\left(s+\frac{1}{RC}\right)}=\frac{1}{RC}\frac{1}{\left(s-\left(-\frac{1}{RC}\right)\right)}$$



FAKTORIZACE PŘENOS. FCE.

$x^2+2x+1=0 \qquad (x-x_1)(x-x_2) \qquad (x+1)(x+1)$

NULOVÉ BODY (NULY) – kořeny čitatele $n_1\dots n_Q$

$$H(s)=konst\frac{(s-n_1)(s-n_2)\dots(s-n_Q)}{(s-p_1)(s-p_2)\dots(s-p_P)}$$

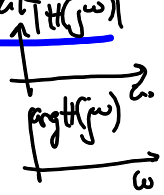
kořeny jmenovatele $p_1\dots p_P$ PÓLY

$R=1k\Omega \quad C=1\mu F$

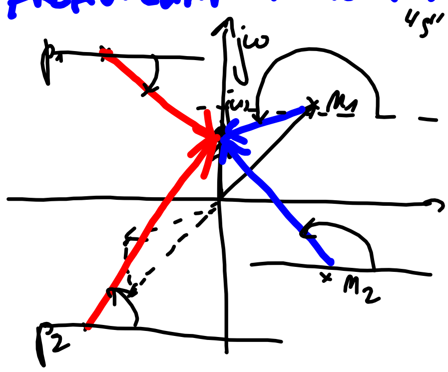
$\omega=0$

$\omega=\infty$

$\omega=\frac{1}{RC}$



PREKV. CHAR. POMOCÍ NUL A PÓLŮ



$$H(j\omega)=konst\frac{(j\omega-n_1)(j\omega-n_2)}{(j\omega-p_1)(j\omega-p_2)}$$

$$|H(j\omega)|=konst\frac{\text{Součin délek modrých vektorů}}{\text{Součin délek červených vektorů}}$$

$$\arg H(j\omega)=\text{součet úhlů modrých vektorů}-\text{součet úhlů červených vektorů.}$$

ε