Half-sem exam:

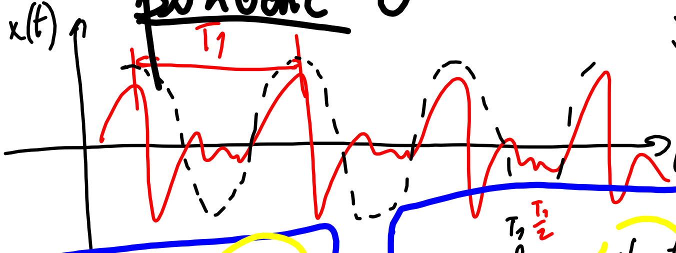
for **all**, next Wed 30.10. 8:00 – 9:00

D105, D0207, E105

- first two (colourful ...) lectures
- first two num. exercises.

Continuous-time signals - frequency analysis

periodic



$f_1 = \frac{1}{T_1}$ [Hz]

$\omega_1 = \frac{2\pi}{T_1}$ [rad/s]

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_1 t}$$

synthesis $f \rightarrow t$

$$c_k = \frac{1}{T_1}\int_0^{T_1} x(t) e^{-jk\omega_1 t} dt \quad \left(\int_{-T_1/2}^{T_1/2}\right)$$

analysis $t \rightarrow f$

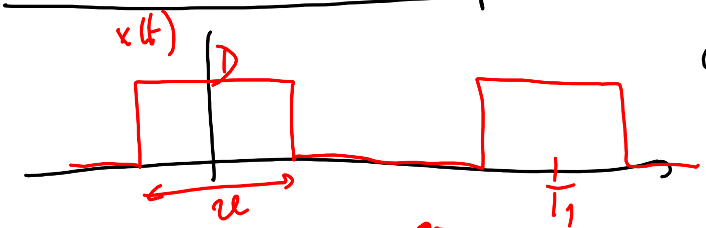
complex numbers

Fourier series

→ Where (which frequencies) at $k\omega_1$

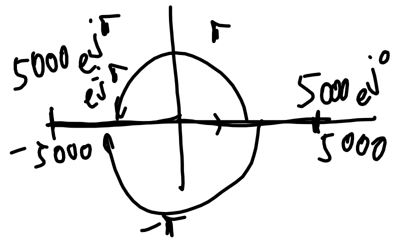
→ How much $|c_k|$

→ How shifted $\arg c_k$



$\omega_1 = \frac{2\pi}{T_1}$

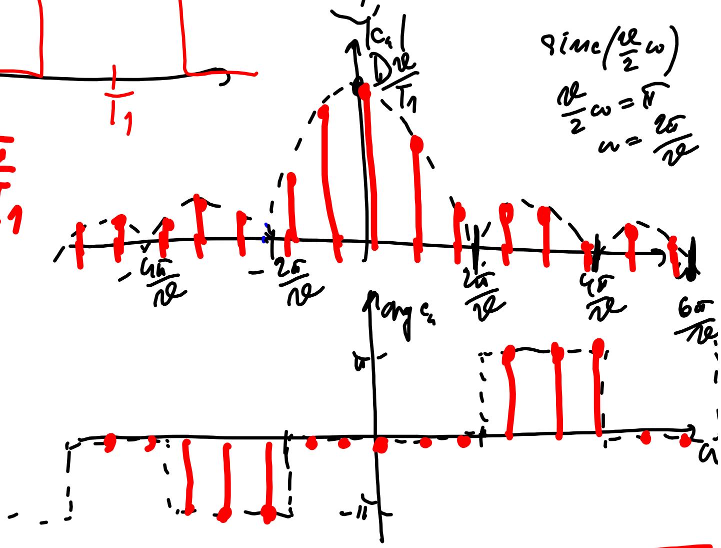
$c_k = \frac{D\vartheta}{T_1} \operatorname{sinc}\left(\frac{\vartheta}{2} k\omega_1\right)$

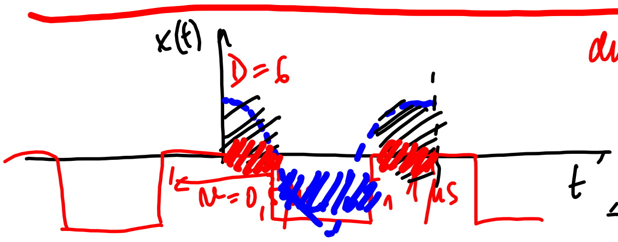


$\operatorname{sinc}\left(\frac{\vartheta}{2}\omega\right)$

$\frac{\vartheta}{2}\omega = \pi$

$\omega = \frac{2\pi}{\vartheta}$

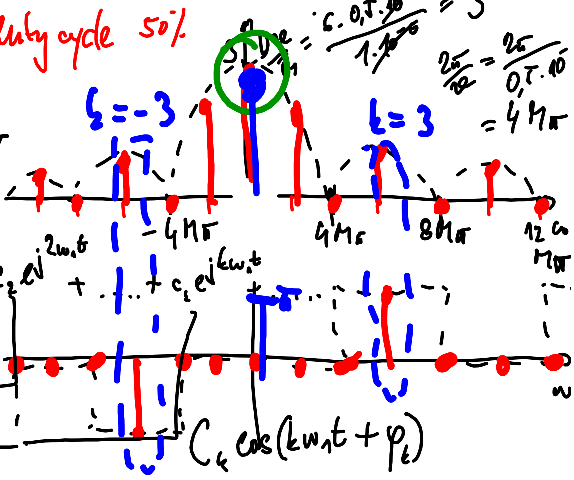




duty cycle 50%

$\frac{D\vartheta}{T_1} = \frac{6 \cdot 0{,}5 \cdot 10^{-6}}{1 \cdot 10^{-6}} = 3$

$\frac{2\pi}{\vartheta} = \frac{2\pi}{0{,}5 \cdot 10^{-6}} = 4M\pi$



$c_{-k}e^{-jk\omega_1 t} \ldots + c_{-2}e^{-j2\omega_1 t} + c_{-1}e^{-j\omega_1 t} + c_0 + c_1 e^{j\omega_1 t} + c_2 e^{j2\omega_1 t} + \ldots + c_k e^{jk\omega_1 t}$

$C_1 \cos(\omega_1 t + \varphi_1)$

$C_2 \cos(2\omega_1 t + \varphi_2)$

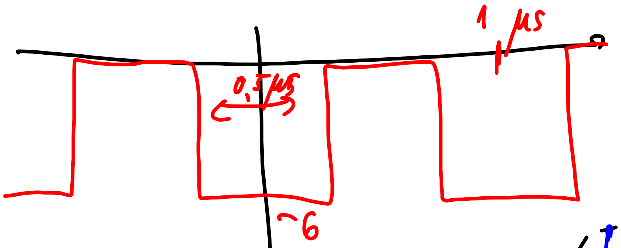
$C_k \cos(k\omega_1 t + \varphi_k)$

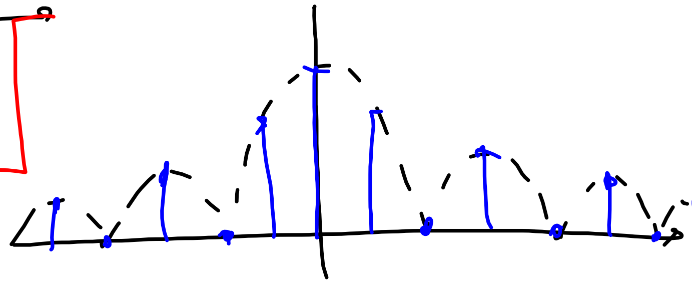
$C_k = 2|c_k|$

$\varphi_k = \arg c_k = -\arg c_{-k}$

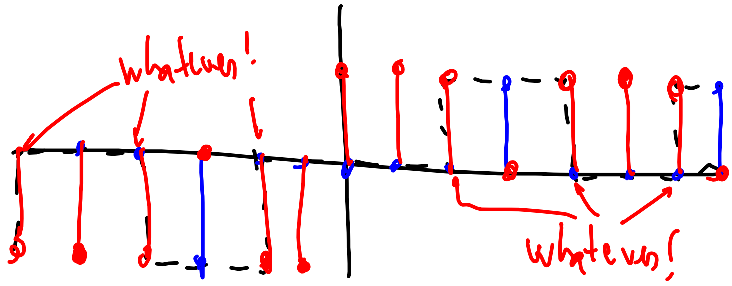
---

c_0 ? $\bar{x} = \frac{1}{T_1}\int_0^{T_1} x(t)\, dt = \frac{1}{T_1}\left(D\frac{\vartheta}{2} + D\frac{\vartheta}{2}\right) = \frac{D\vartheta}{T_1}$ OK, mean value of signal

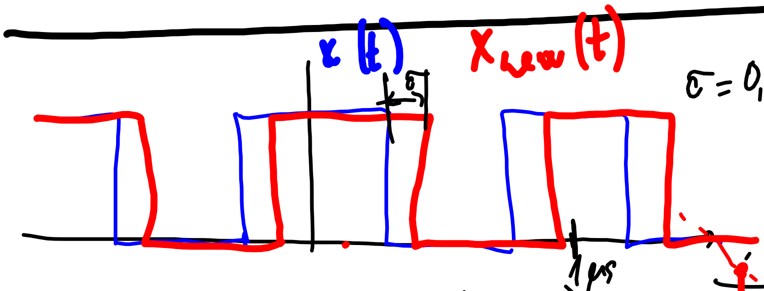




$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_1 t}$$

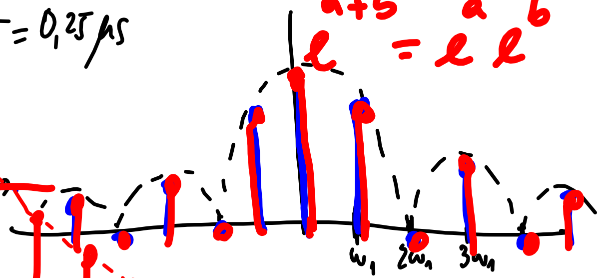


- where
- how much
- how shifted **WORK!**



$\tau = 0{,}25\,\mu s$

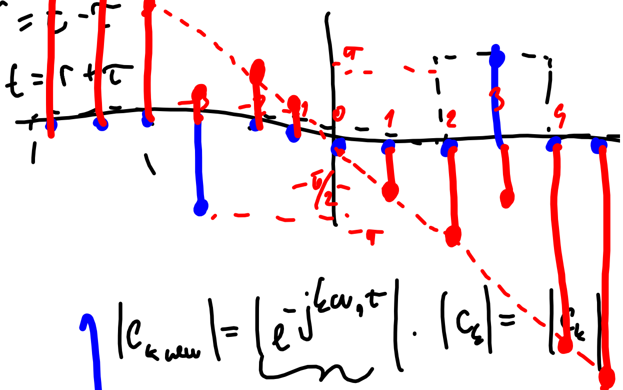
$$e^{a+b} = e^a e^b$$



$$x(t) \to x_{new}(t) = x(t-\tau)$$

$r = t - \tau$

$t = r + \tau$



$$c_{new\,k} = \frac{1}{T_1}\int_{T_1} x(\underbrace{t-\tau}_{r}) e^{-jk\omega_1 t}\,dt =$$

$$= \frac{1}{T_1}\int_{T_1} x(r) e^{-jk\omega_1 (r+\tau)}\,dr =$$

$$= \frac{1}{T_1}\int_{T_1} x(r) e^{-jk\omega_1 \tau} e^{-jk\omega_1 r}\,dr =$$

$$= e^{-jk\omega_1 \tau} \underbrace{\frac{1}{T_1}\int_{T_1} x(r) e^{-jk\omega_1 r}\,dr}_{c_k} = e^{-jk\omega_1 \tau} \cdot c_k$$

$$|c_{k\,new}| = \underbrace{|e^{-jk\omega_1 \tau}|}_{1} \cdot |c_k| = |c_k|$$

$$\text{ang}\, c_{k\,new} = \text{ang}\, e^{-jk\omega_1\tau} + \text{ang}\, c_k =$$

$$\text{ang}\, c_k - k\omega_1\tau$$

$$= \text{ang}\, c_k - k\, 2\pi \cdot 10^6 \cdot 0{,}25 \cdot 10^{-6} =$$

$$= \text{ang}\, c_k - k\frac{\pi}{2}$$

$z = r \cdot e^{j\varphi}$ $\quad \tilde{f} = \tilde{g} \cdot e^{jh_0}$

$$|z \cdot \tilde{f}| = r \cdot \tilde{g}$$

$$\text{ang}(z \cdot \tilde{f}) = \varphi + h_0$$

F. S. summary

input: continuous-time periodic signal

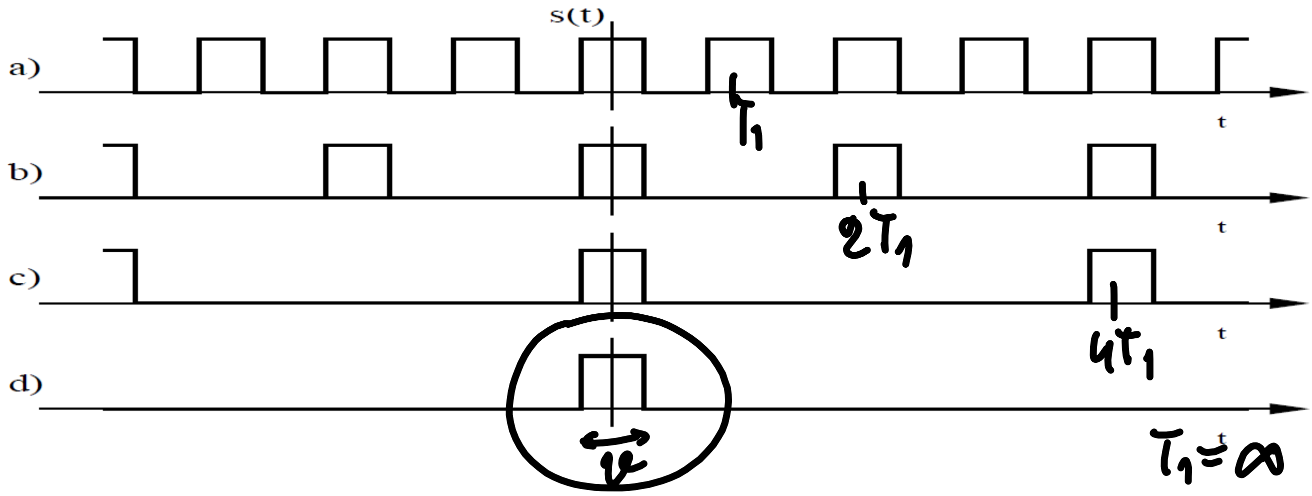
output: coefficients c_k
- → where? $k\omega_1$
- → how much? $|c_k|$
- → how shifted? $\arg c_k$

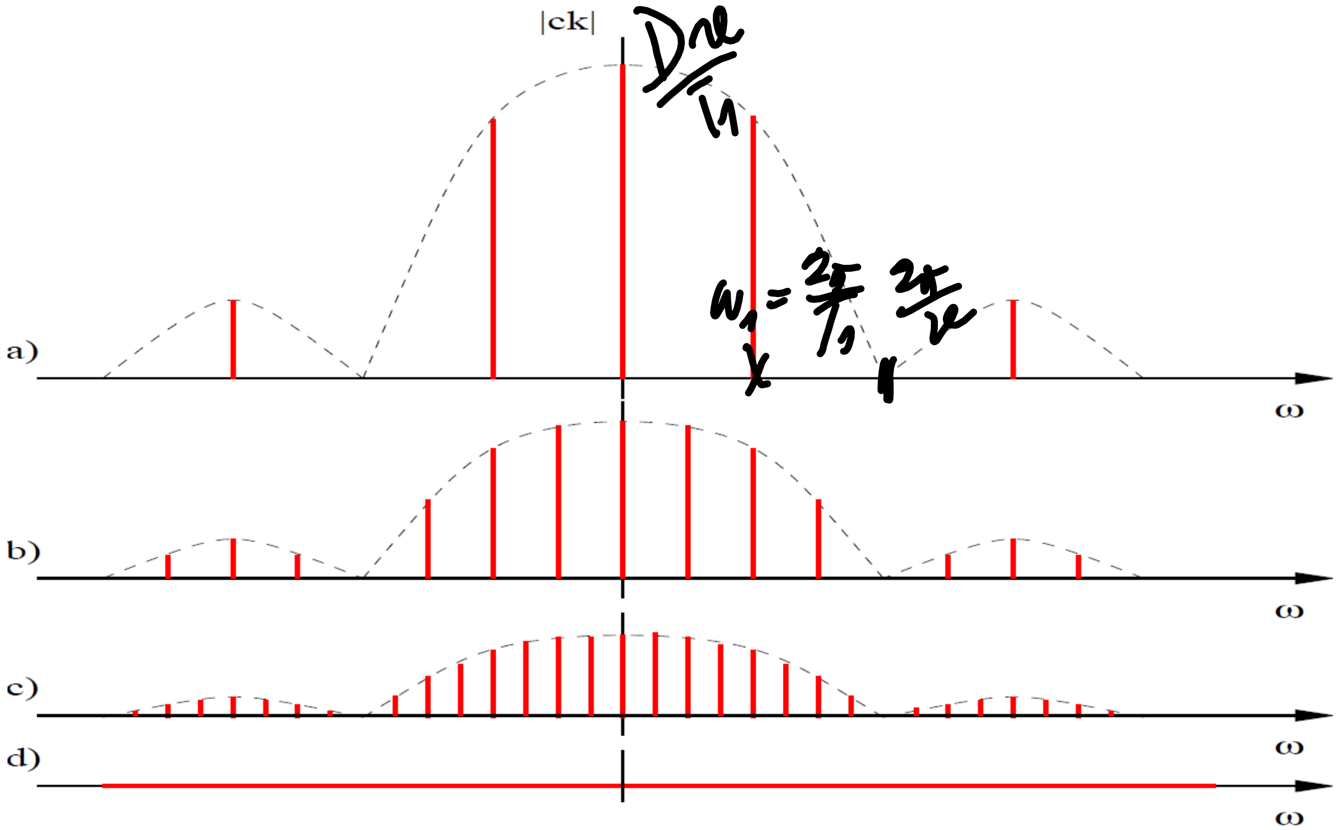
} spectrum

Shifted signal?
- where - same
- how much - same
- how shifted - $\arg c_k$

~~delayed~~ ~~advanced~~

FOURIER TRANSFORM.





$$c_k = \frac{1}{T_1}\int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} x(t)\, e^{-jk\omega_1 t}\, dt$$

$T_1 \to \infty$ $\quad$ ~~$\omega_1 = \frac{2\pi}{T_1} = \emptyset$~~

$\omega_1 = \frac{2\pi}{T_1} = d\omega$ $\quad$ $T_1 = \frac{2\pi}{d\omega}$

$\frac{1}{T_1} = \frac{d\omega}{2\pi}$

$$dc = \frac{d\omega}{2\pi}\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(j\omega) \quad \text{~~}2\pi\frac{dc}{d\omega}\text{~~} = \int x(t)\, e^{-j\omega t}\, dt$$

spectral function

Fourier transform

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$t \to f$ "Analysis"

$X(j\omega) = X^*(-j\omega)$

F.S.:

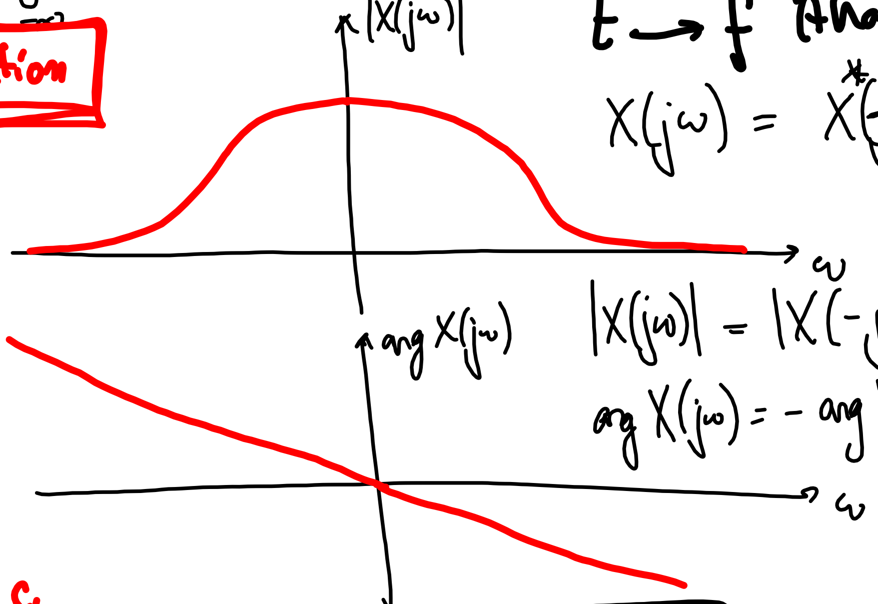
$c_k = c_{-k}^*$

$|c_k| = |c_{-k}|$

$\arg c_k = -\arg c_{-k}$

$\mathrm{Re}\, c_k = \mathrm{Re}\, c_k$

$\mathrm{Im}\, c_k = -\mathrm{Im}\, c_k$



$|X(j\omega)| = |X(-j\omega)|$

$\arg X(j\omega) = -\arg X(-j\omega)$

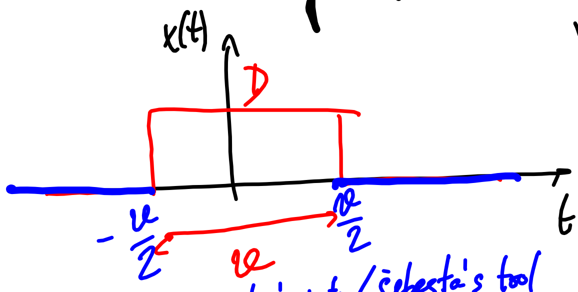
Fourier transform

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

Inverse F.T.

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{+j\omega t}\, d\omega$$

F.T. example 1 :



$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt =$$

$$= D \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-j\omega t}\, dt = D\, 2 \cdot \frac{\tau}{2}\, \text{sinc}\left(\frac{\tau}{2}\omega\right)$$

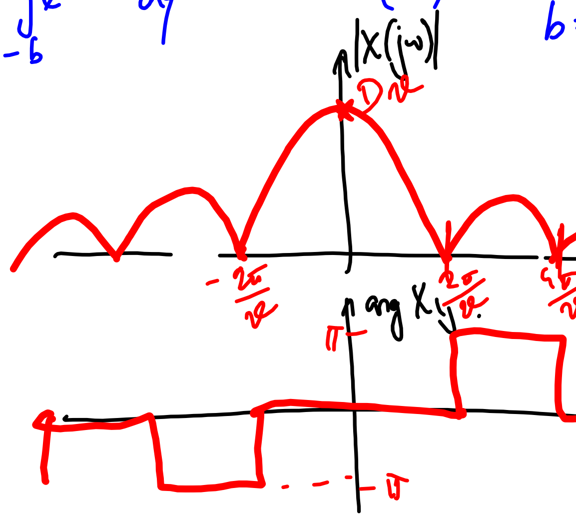
$$= D\tau\, \text{sinc}\left(\frac{\tau}{2}\omega\right)$$

Euler's int./Šebesta's tool

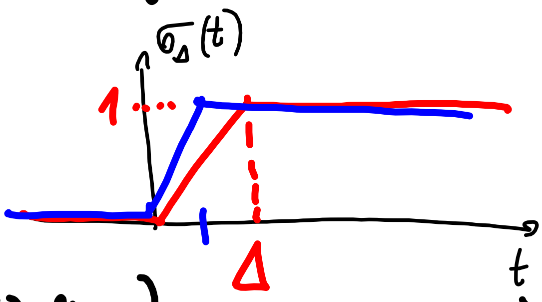
$$\int_{-b}^{b} e^{\pm jxy}\, dy = 2b\, \text{sinc}(bx)$$

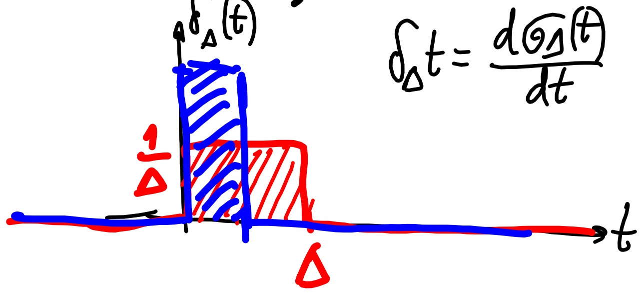
$$b = \frac{\tau}{2}$$

$$\frac{\tau}{2}\omega = \pi \qquad \omega = \frac{2\pi}{\tau}$$



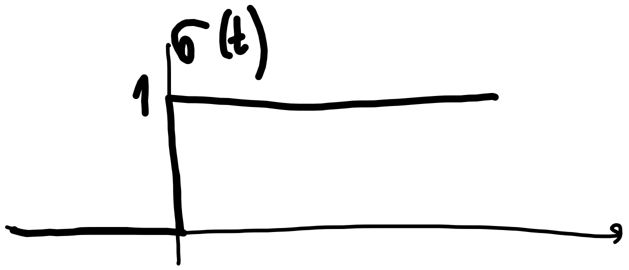
UNIT STEP and UNIT (DIRAC) IMPULSE





$$\delta_\Delta t = \frac{d\sigma_\Delta(t)}{dt}$$

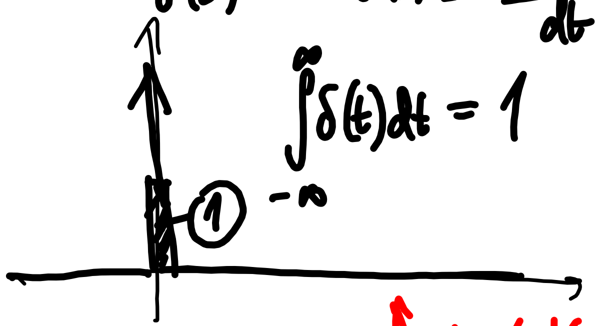
$$\sigma(t) = \lim_{\Delta \to 0} \left(\sigma_\Delta(t) = \begin{cases} 0 & \text{for } t < 0 \\ \frac{1}{\Delta} t & \text{for } t \in \langle 0, 1 \rangle \\ 1 & \text{for } t > 1 \end{cases} \right)$$



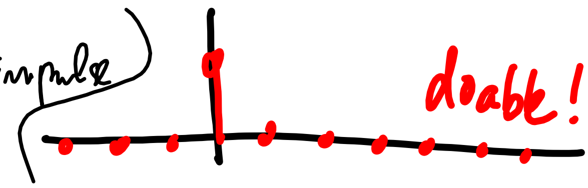
Dirac impulse

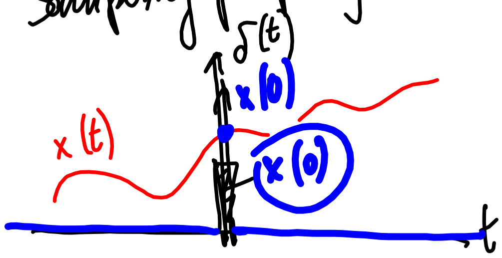
$$\delta(t) = \frac{d\sigma(t)}{dt}$$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$



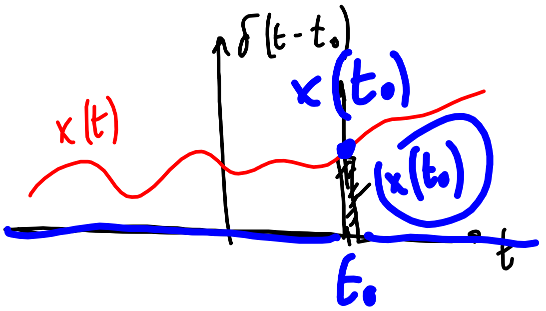
Sampling property of Dirac impulse





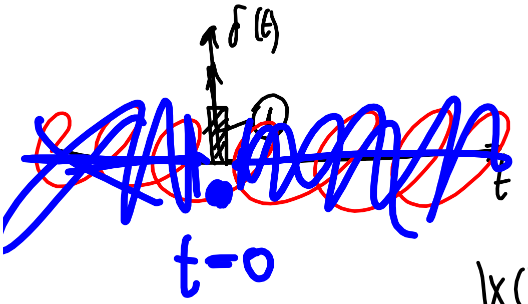
$$\int_{-\infty}^{\infty} x(t)\delta(t)\,dt = x(0)$$

Dirac sampled the signal at $t=0$



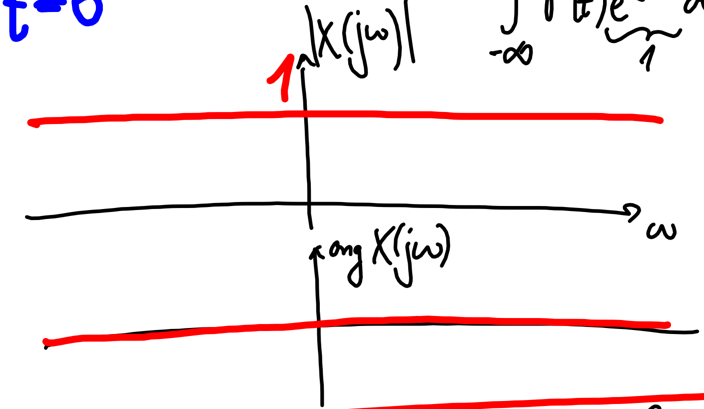
$$\int_{-\infty}^{\infty} x(t)\delta(t-t_0)\,dt = x(t_0)$$

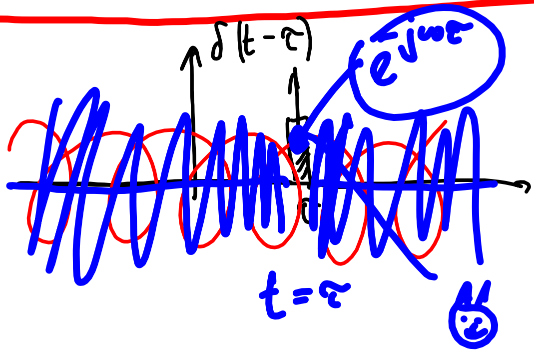
Dirac sampled the signal at $t=t_0$



$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt =$$

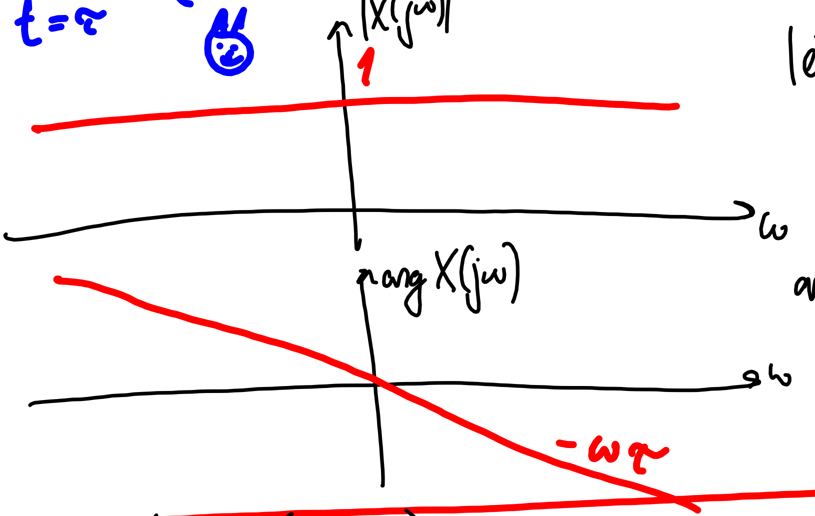
$$\int_{-\infty}^{\infty} \delta(t) \underbrace{e^{-j\omega 0}}_{1} dt = \int_{-\infty}^{\infty} \delta(t) dt = 1$$





$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t-\tau) e^{-j\omega t} dt =$$

$$= e^{-j\omega\tau}$$



$|e^{-j\omega\tau}|$

$\text{ang } e^{-j\omega\tau} = -\omega\tau$

$x(t) \longrightarrow y(t) = x(t-\tau)$

$X(j\omega) \longrightarrow Y(j\omega) = X(j\omega) \cdot e^{-j\omega\tau}$

→ mag. do not change

→ angle go → downhill for $\tau > 0$

→ uphill for $\tau < 0$