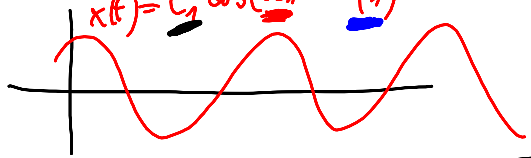
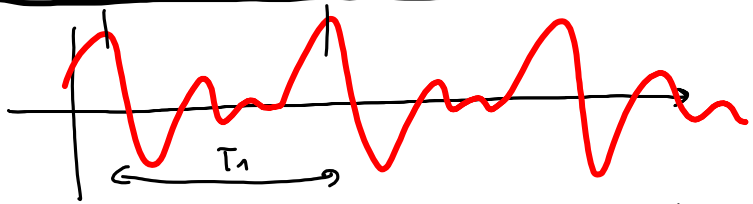
Půlsemestrálka St. 30. 10.
poslední h. přednášky $9^{45} - 10^{45}$ FOR ALL.
- první 2 "backup" přednášky
- první 2 cvika

- 1, co je tam za frekvence ?
- 2, kolik ?
- (3, jak je to posunuté)

Fourierova řada pro periodické sig. se spojitým časem

$x(t) = C_1 \cos(\omega_1 t + \varphi_1)$





$\omega_1 = \frac{2\pi}{T_1}$ $\quad f_1 = \frac{1}{T_1}$

→ jaké frekvence ?

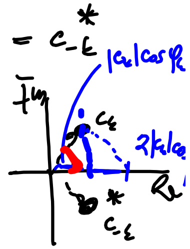
→ kolik ?

→ jak posunuté

$$x(t) = C_0 + C_1 \cos(\omega_1 t + \varphi_1) + \dots + C_k \cos(k\omega_1 t + \varphi_k)$$

$$x(t) = c_0 + \underbrace{c_1 e^{j\omega_1 t} + c_{-1} e^{-j\omega_1 t}}_{C_1 \cos(\omega_1 t + \varphi_1)} + \dots + \underbrace{c_k e^{jk\omega_1 t} + c_{-k} e^{-jk\omega_1 t}}_{C_k \cos(k\omega_1 t + \varphi_k)}$$

$c_k = c_{-k}^*$



$|c_k| = |c_{-k}|$

$\arg c_k = -\arg c_{-k}$

$$|c_k| = |c_{-k}| = \frac{C_k}{2}$$

$$\arg c_k = -\arg c_{-k} = \varphi_k$$

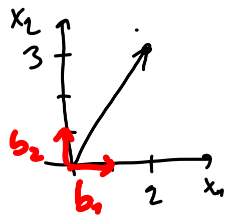
$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_1 t}}$$

Fourierova řada (syntéza pomocí F.Ř.)

↳ NAJÍT koeficienty c_k !!!!

$$x(t) = \sum_{\text{přes báze}} c_k \cdot \text{báze}_k(t)$$

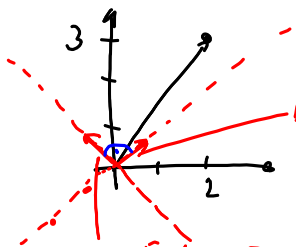
Opakování - báze!



$c_k = b_k^T x$

$c_1 = [1 \quad 0]\begin{bmatrix}2\\3\end{bmatrix} = 2$

$c_2 = [0 \quad 1]\begin{bmatrix}2\\3\end{bmatrix} = 3$



$b_1 = \begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix}$

$b_2 = [-\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}]$

$c_1 = [\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}]\begin{bmatrix}2\\3\end{bmatrix} = 3{,}53$

$c_2 = [-\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}]\begin{bmatrix}2\\3\end{bmatrix} = 0{,}707$

jsou kolmé?

$[\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}]\begin{bmatrix}-\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix} = 0$ jsou!

jsou jednotkové?

$\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$

ORTONORMÁLNÍ BÁZE!

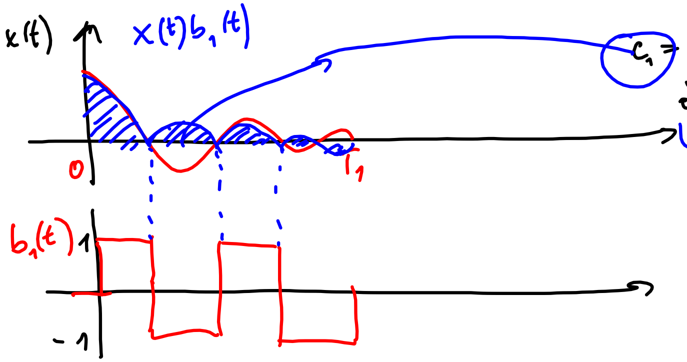
$x = [3 \ 2 \ 1 \ 0 \ 1 \ 2 \ 3 \ 4]^T$

$b_1 = \frac{1}{\sqrt{8}}[1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1]^T$

$c_1 = b_1^T x = 5{,}65$

$b_2 = \frac{1}{\sqrt{4}}[0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1 \ 1]$

$b_3 = \frac{1}{\sqrt{4}}[1 \ 1 \ 1 \ {-1} \ 0 \ 0 \ 0 \ 0]$

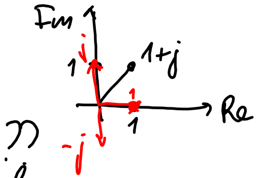


$c_1 = \int_0^{T_1} \underbrace{\underbrace{x(t)\, b_1(t)}_{\text{násobení}}\, dt}_{\text{sčítání všeho.}}$

HLEDÁME c_k ! (známe $x(t)$ ...)

$$x(t) = \sum_{k=-\infty}^{\infty} c_k \underbrace{e^{jk\omega_1 t}}_{\text{báze}}$$

$$c_k = \int x(t)\, e^{-jk\omega_1 t}\, dt$$



$Re(1.(1+j)) = 1$ ✓

$Re(j(1+j)) =$

$Re(j-1) = -1$

$Re(j^*(1+j)) = Re(-j(1+j)) =$

$Re(-j+1) = +1$ ✓

ORTONORMÁLNÍ SYSTÉM ??

$$\int_0^{T_1} e^{jk\omega_1 t}\, e^{-jl\omega_1 t}\, dt = 0 \quad k \neq l$$

$$\boxed{\int_0^{T_1} e^{j(k-l)\omega_1 t}\, dt = 0}$$

$k-l=2$

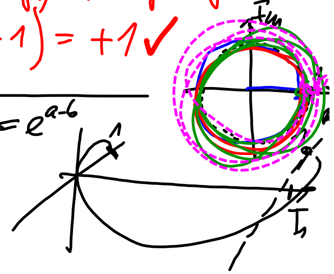
$k-l=4$

$k-l=-3$

ORTOGONÁLNÍ !

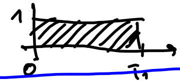
$e^a \cdot e^{-b} = e^{a-b}$

$$\int_0^{T_1} e^{j\omega_1 t}\, dt = 0$$



NORMÁLNÍ ? $\sqrt{b_1^2 + b_2^2 + b_3^2 \cdots} = 1$

$$\int_0^{T_1} |\underbrace{e^{jk\omega_1 t}}_{1}|^2 dt = 1 \qquad \int_0^{T_1} 1\, dt = T_1$$



báze: $\frac{1}{T_1} e^{jk\omega_1 t}$

F.Ř. ANALÝZA

$$\boxed{c_k = \frac{1}{T_1}\int_0^{T_1} x(t)\, e^{-jk\omega_1 t}\, dt}$$

F.Ř. syntéza

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} c_k e^{+jk\omega_1 t}}$$

STEJNÉ AŽ NA ZNAMÉNKO

$f \to t$

$t \to f$

výstup = možná každá konst. sumační operátor vstup $\overset{+}{\underset{-}{e}}{}^{j\,\text{frekvence} \cdot \text{čas}}$

$\omega_1 = \frac{2\pi}{T_1}$

Jak to bylo u disk. signálů (z. barevné přednáška)

syntéza

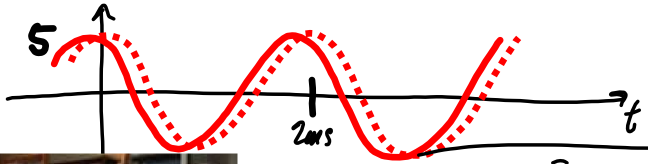
$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{+j\frac{2\pi}{N}k \cdot n}$$

analýza

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}k\, n}$$

FŘ cosinusovky

$x(t) = 5\cos\left(1000\pi t + \frac{\pi}{4}\right)$

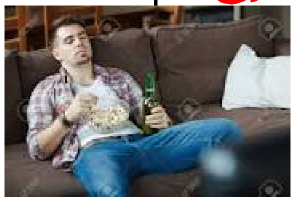


FŘ ?

$\omega_1 = 1000\pi$ rad/s

$f_1 = 500$ Hz

$T_1 = \frac{1}{500} = 2\,ms$



$c_k = \frac{1}{T_1}\int_0^{T_1} x(t)\, e^{-jk\omega_1 t}\, dt$

$x(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{jk\omega_1 t}$

$\cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$

$e^{a+b} = e^a \cdot e^b$

$x(t) = \frac{5}{2} e^{j(1000\pi t + \pi/4)} + \frac{5}{2} e^{-j(1000\pi t + \pi/4)} =$

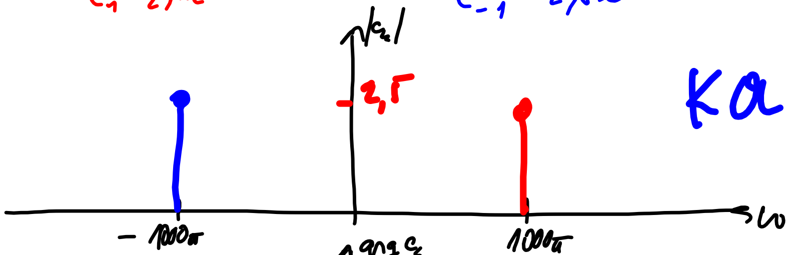
$= 2{,}5\, e^{j\pi/4}\, e^{j1000\pi t} + 2{,}5\, e^{-j\pi/4}\, e^{-j1000\pi t}$

$= c_1$ $\quad = c_{-1}$

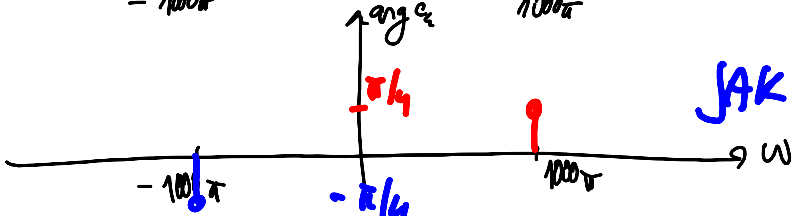
$c_1\, e^{j1\omega_1 t}$?

$c_1 = 2{,}5\, e^{j\pi/4}$

$c_{-1} = 2{,}5\, e^{-j\pi/4}$



KOLIK ?



JAK POSUNUTÉ ?

$$x(t) = 8\cos\left(1000\pi t + \pi/4\right) + 2\cos\left(3000\pi t - \frac{\pi}{2}\right)$$

F.Ř.?

$$c_k = \frac{1}{T_1}\int x(t)\, e^{-jk\omega_1 t}\, dt \quad \ldots$$

c_1, c_{-1}

$c_1 = 4 \cdot e^{j\pi/4}$

$c_{-1} = 4 \cdot e^{-j\pi/4}$

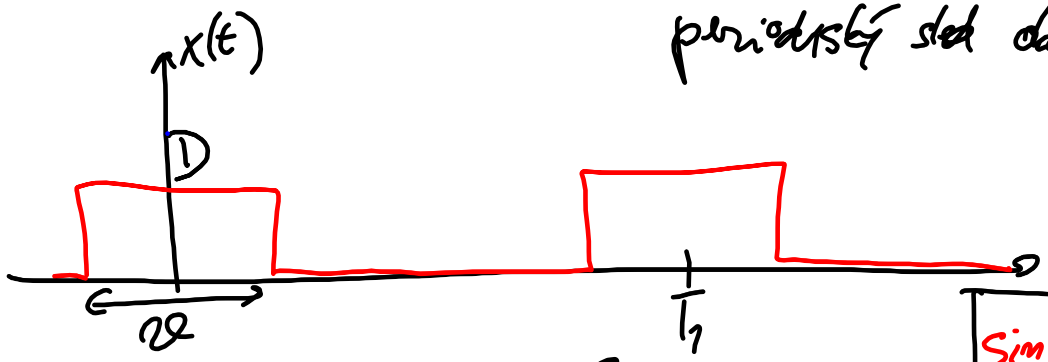
c_3, c_{-3}

$c_3 = 1 \cdot e^{-j\pi/2}$

$c_{-3} = 1 e^{+j\pi/2}$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_1 t} = 4 \cdot e^{j\pi/4} \cdot e^{j1000\pi t} + 4 \cdot e^{-j\pi/4} e^{-j1000\pi t} + 1 \cdot e^{-j\pi/2} e^{j3 \cdot 1000\pi t} + 1 \cdot e^{+j\pi/2} e^{-j3 \cdot 1000\pi t}$$

periodický sled obdh. impulsů



FR ?

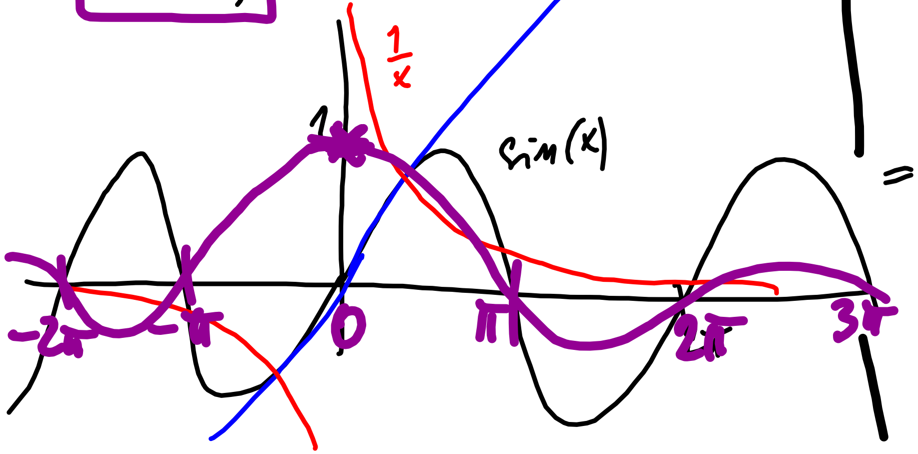
$\sin x = \frac{e^{jx} - e^{-jx}}{2j}$

$\frac{d \frac{e^{jxy}}{jx}}{\partial dy} =$

PŘÍPRAVA

kardinální sinus

$\boxed{\mathrm{sinc}(x)} = \frac{\sin(x)}{x}$

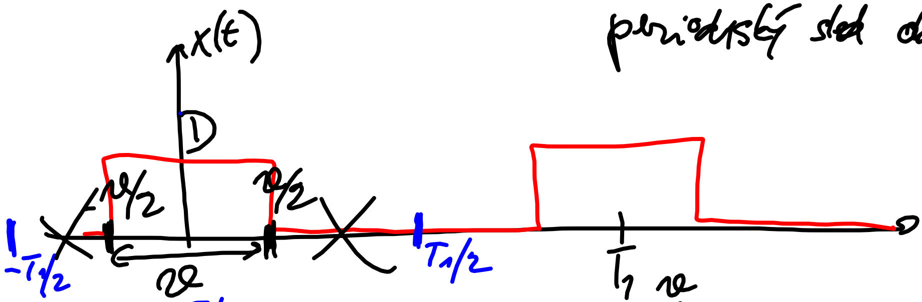


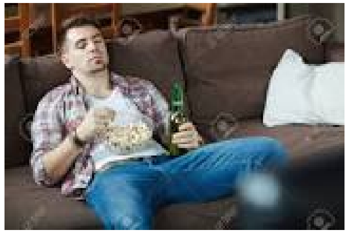
$$\int_{-b}^{b} e^{+jxy}\,dy = \left[\frac{e^{jxy}}{jx}\right]_{-b}^{b} =$$

$$= \frac{2}{2}\frac{e^{jxb}}{jx} - \frac{2}{2}\frac{e^{-jxb}}{jx} = \frac{2}{x}\cdot\frac{e^{jxb} - e^{-jxb}}{2j} =$$

$$= \frac{b}{b}\frac{2}{x}\sin(xb) = 2b\,\mathrm{sinc}(xb)$$

periodický sled obdh. impulsů



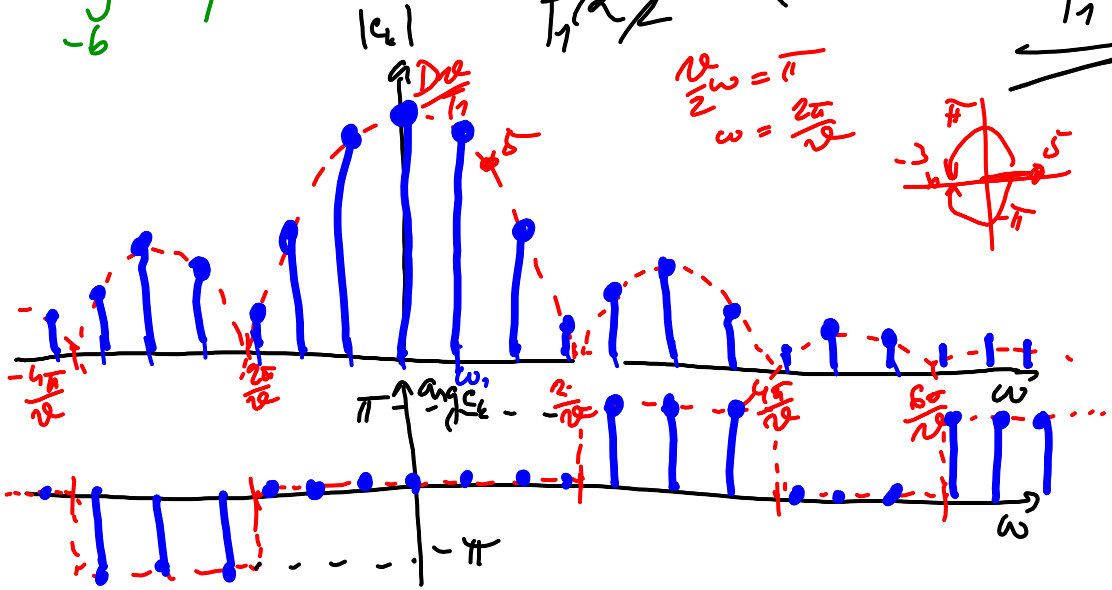


$$c_k = \frac{1}{T_1}\int_{-\frac{T_1}{2}}^{T_1/2} x(t)\, e^{-jk\omega_1 t}\, dt = \frac{1}{T_1}\int_{-\frac{\vartheta}{2}}^{\frac{\vartheta}{2}} D\, e^{-jk\omega_1 t}\, dt = \frac{D}{T_1}\int_{-\frac{\vartheta}{2}}^{\frac{\vartheta}{2}} e^{-jk\omega_1 t}\, dt =$$

$$\int_{-b}^{b} e^{\pm jxy}\, dy = 2b\,\mathrm{sinc}(bx)$$

$$= \frac{D}{T_1}\cancel{2}\frac{\vartheta}{\cancel{2}}\,\mathrm{sinc}\left(\frac{\vartheta}{2}k\omega_1\right) = \frac{D\vartheta}{T_1}\,\mathrm{sinc}\left(\frac{\vartheta}{2}k\omega_1\right)$$

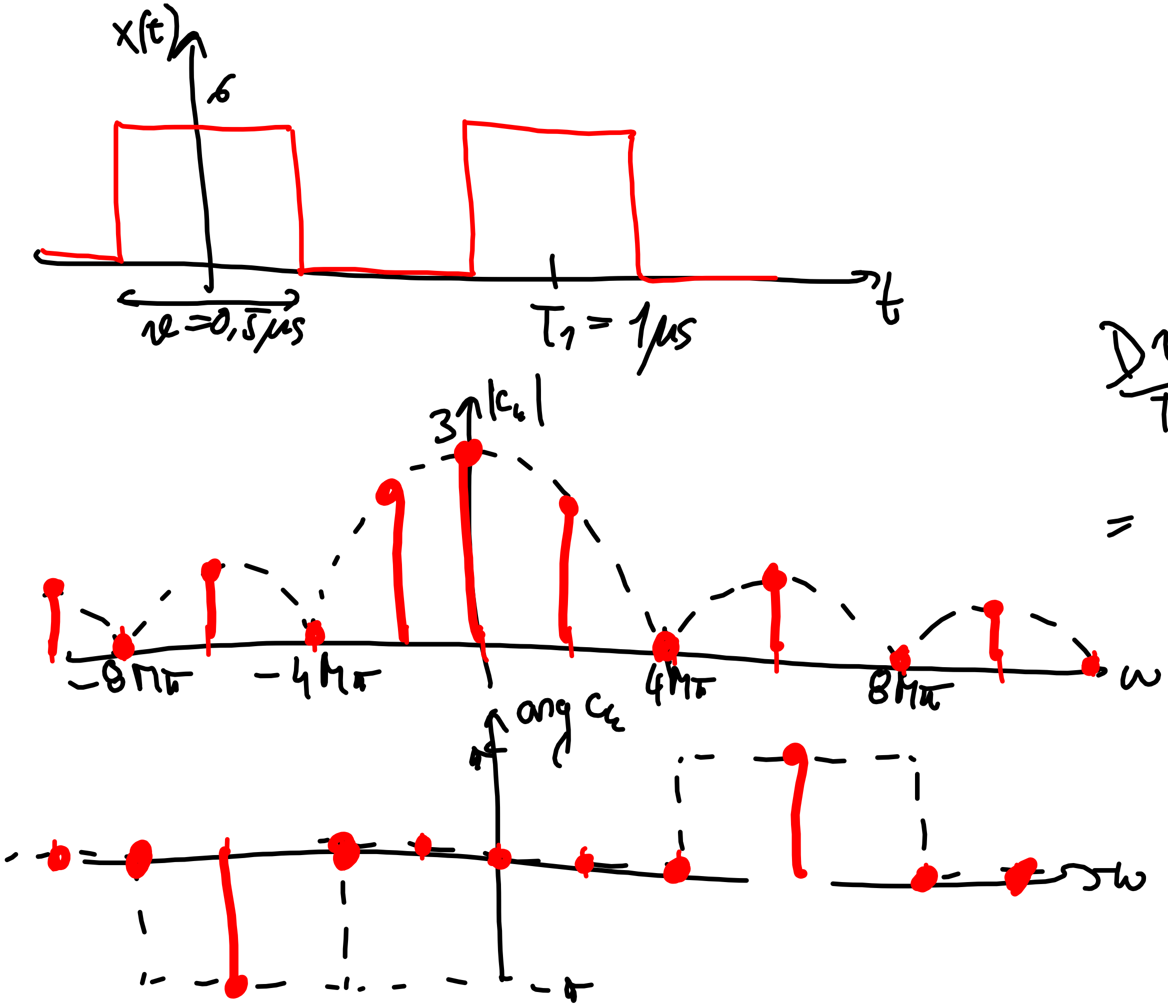
$$\frac{\vartheta}{2}\omega = \pi \qquad \omega = \frac{2\pi}{\vartheta}$$



1) pomocná funkce $\frac{D\vartheta}{T_1}\,\mathrm{sinc}\left(\frac{\vartheta}{2}\omega\right)$

2) nastřílet pod ní koeficienty c_k. na násobky ω_1

$$\omega_1 = \frac{2\pi}{T_1}$$



$$\frac{D\vartheta}{T_1}\,\mathrm{sinc}\left(\frac{\vartheta}{2}\omega\right) =$$

$$= 3 \cdot \mathrm{sinc}\left(0{,}25\cdot 10^{-6}\,\omega\right) =$$

$$\frac{2\pi}{\vartheta} = \frac{2\pi}{0{,}5\cdot 10^{-6}} = 4\,\mathrm{Mega}\,\pi$$

$$\omega_1 = \frac{2\pi}{1\mu s} = 2\,\mathrm{Mega}\,\pi$$