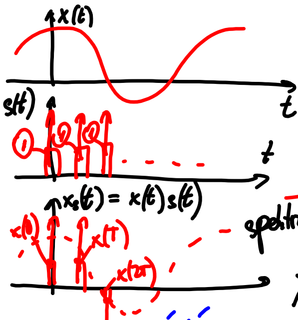
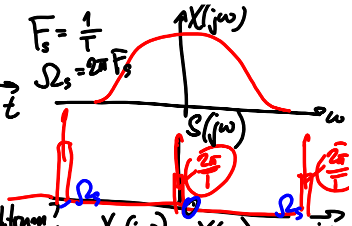



$F_s = \frac{1}{T}$

$\Omega_s = 2\pi F_s$




spektrum:

$X_s(j\omega) = X(j\omega) * S(j\omega)$

$X_s(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\alpha) S(\omega-\alpha)\, d\alpha$

VZORKOVÁNÍ $\longleftrightarrow$ PERIODISACE SPEKTRA

$X(\omega)$

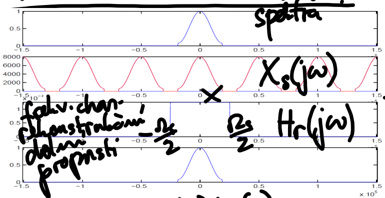
$$\boxed{X_s(\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(\omega - k\Omega_s)}$$

spek. funkce vzorkovaného signálu.

$$\boxed{\Omega_s > 2\,\omega_{max}}$$

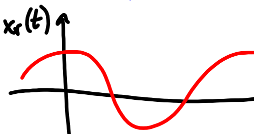
Rekonstrukce v časové oblasti?

co se děje v čase?





$X_r(j\omega) = X_s(j\omega) H_r(j\omega)$

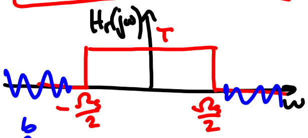



$x_r(t) = x_s(t) * h_r(t)$

$$\boxed{H(j\omega) \longleftrightarrow h(t)}$$

$h_r(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_r(j\omega) e^{+j\omega t} d\omega =$

$= \frac{T}{2\pi}\int_{-\frac{\Omega_s}{2}}^{\frac{\Omega_s}{2}} e^{+j\omega t} d\omega =$




$\int_{-b}^{b} e^{+jxy} dy = 2b\,\mathrm{sinc}(bx)$

$= \cancel{\frac{T}{2\pi}} \cdot \cancel{\frac{2\pi}{T}}\, \mathrm{sinc}\left(\frac{\Omega_s}{2} t\right) = \mathrm{sinc}\left(\frac{\Omega_s}{2} t\right)$

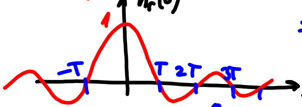
$h_r(t)$

$F_s = \frac{1}{T}$

$\Omega_s = \frac{2\pi}{T}$

$\frac{\Omega_s}{2} t = \pi$

$t = \frac{2\pi}{\Omega_s} = \frac{2\pi}{\frac{2\pi}{T}} = T$




Rekonstrukce v časové oblasti

$x_r(t) = \sum_{k=-\infty}^{\infty} x(kT)\, \mathrm{sinc}\left(\frac{\Omega_s}{2}(t - kT)\right)$


$$\boxed{x_s(t) = \sum_{n=-\infty}^{\infty} x(nT) \cdot \delta(t - nT)}$$

e. e $x[n]$

$x(nT) \longrightarrow x[n]$

[s] []

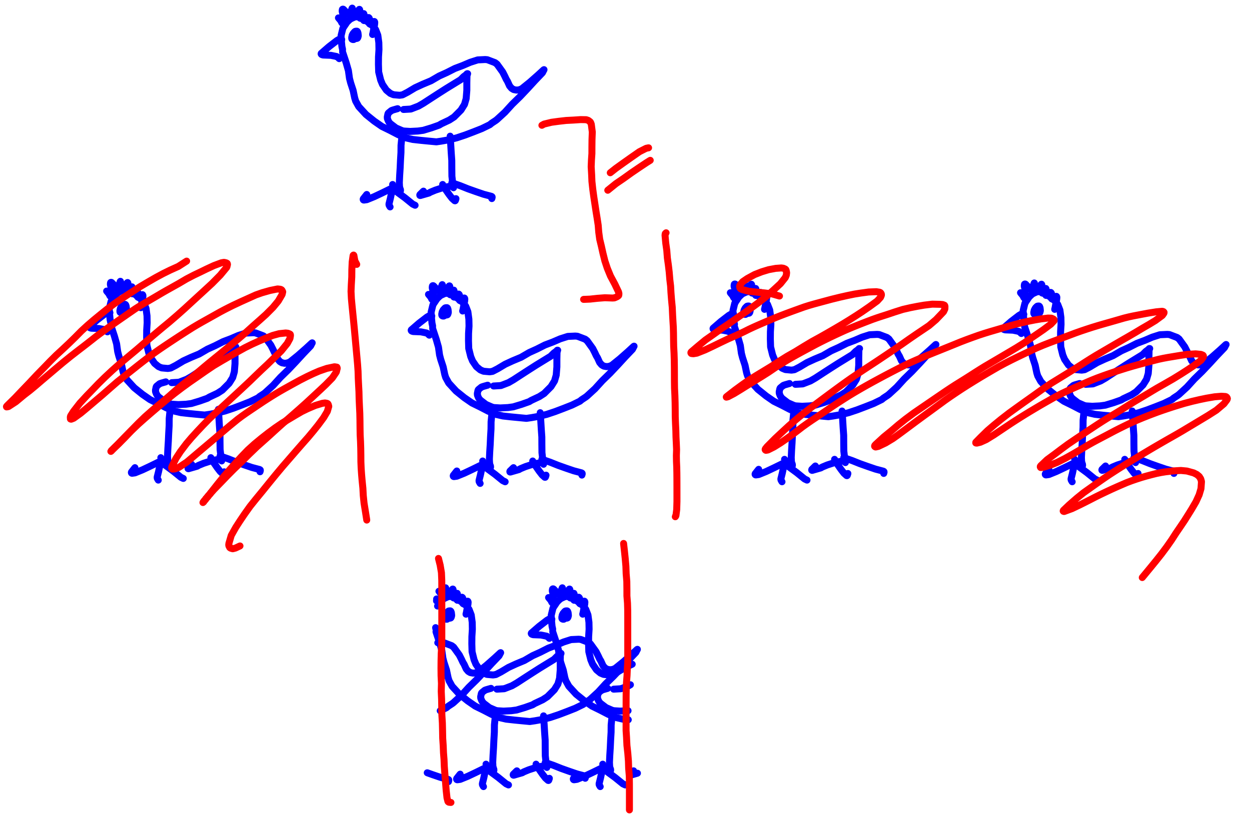
$n = \frac{nT}{T}$

index vzorku ≈ normovaný čas! bezrozměrný!

$x[n]$ musíme dodat $F_s$!

$f' = \frac{f}{F_s}$ normované frekvence

$\omega' = \frac{\omega}{F_s}$ normované kruhové frekvence



# Diskrétní signály

$x[n]$  $\omega = 2\pi f$, $f = \frac{\omega}{2\pi}$

$x[n] = C_1 \cos(\omega n + \varphi_1)$  rad

normovaná kruh. frekvence

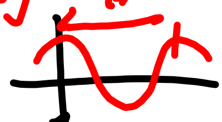
$\boxed{\omega N_1 = 2\pi}$  $N_1 = \frac{2\pi}{\omega}$

musí být celé!

~~pozor možný průser!~~

$x(t) = C_1 \cos(\omega t + \varphi_1)$  [rad/s]

$T_0$



$T_1 = \frac{2\pi}{\omega} = \frac{1}{f} = \frac{1}{\frac{\omega}{2\pi}} = \frac{2\pi}{\omega}$

$\omega N_1 = k 2\pi$

$N_1 = \frac{k\, 2\pi}{\omega}$  k ladím tak, aby $N_1$ bylo celé

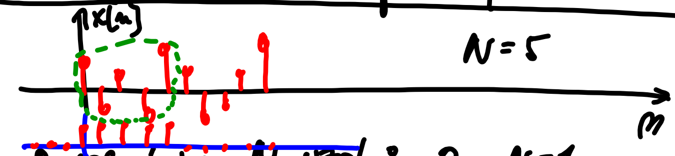
$\omega = \frac{\pi}{8}$  $N_1 = \frac{k\, 2\cancel{\pi}}{\frac{\cancel{\pi}}{8}} = k\, 16$  $k = 1$  $N_1 = 16$

$\omega = \frac{8\pi}{31}$  $N_1 = \frac{k \cdot 2\cancel{\pi}}{\frac{8\cancel{\pi}}{31}} = k \frac{31}{4}$  $k = 4$  $N_1 = 31$

$\omega = \frac{1}{6}$  $N_1 = \frac{k \cdot 2\pi}{\frac{1}{6}} = k\, 12\pi$  $k = ?$ NENDE!

NENÍ PERIODICKÁ !!!

$x[n]$ – signál, posloupnosti



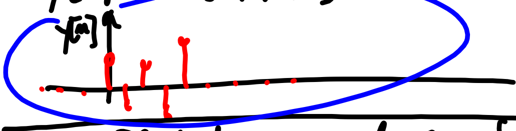
$N=5$

Omezení na $N$ vzorků $0 \ldots N-1$

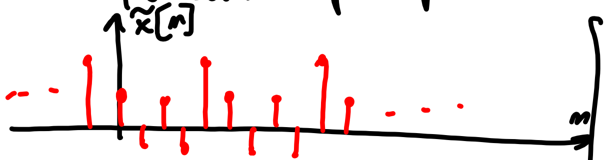
obdélníková funkce

$$R_N = \begin{cases} 1 & \text{od } 0 \ldots N-1 \\ 0 & \text{jinde} \end{cases}$$

$$y[n] = x[n] \cdot R_N[n]$$



Periodisace posloupnosti délky $N$



```
n0 = 0;
for (n=0; n<52; n++) {
    xtil[n] = y[n0];
    n0++;
    if (n0 == N) {
        n0 = 0
    }
}
```

$$\tilde{x}[n] = y[\mathrm{mod}_N(n)]$$

Periodisace s posunutím

| $n$ | −3 | −2 | −1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x[n]$ | | | | 1 | −1 | 1 | −1 | 3 | | | | | | |
| $y[n]$ | | | | 3 | 1 | −1 | 1 | −1 | | | | | | |

$$y[n] = x[\mathrm{mod}_N(n-1)]$$

Kruhový posunutí –

$$y[n] = R_N[n]\, x[\mathrm{mod}_N(n-1)]$$

Konvoluce

$(x^3+2x^2+3x+4)(x^3+x^2-x-1) = \ldots$ ?

$N$ délka

$x_1 = [1 \;\; 2 \;\; 3 \;\; 4]$

$x_2 = [1 \;\; 1 \;\; -1 \;\; -1]$

Lineární konvoluce

$$y[n] = x_1[n] * x_2[n] = \sum_{k=-\infty}^{\infty} x_1[k]\, x_2[n-k]$$

délka výsledku $2N-1 = 7$

Cyklická konvoluce

$$y[n] = x_1[n] \circledast x_2[n] = \sum_{k=0}^{N-1} x_1[k]\, x_2[\mathrm{mod}_N(n-k)]$$

výsledek pro $\forall n$ !

periodické s $N$ vzorky!

Kruhová konvoluce

$$y[n] = x_1[n] \, \textcircled{N} \, x_2[n] = \boxed{R_N[n]} \sum_{k=0}^{N-1} x_1[k]\, x_2[\mathrm{mod}_N(n-k)]$$