A. Draw the Markov Random Field graphical model for the distribution defined as:

$$P(x_1, x_2, x_3, x_4, x_5) = \frac{1}{Z}\psi_a(x_1, x_2)\psi_b(x_1, x_3)\psi_c(x_2, x_4)\psi_d(x_3, x_4, x_5)$$

where are some positive (potential) functions and Z is an appropriate normalizing constant.

B. For each of the following statements, say whether the statement holds true for the Markov Random Field from the previous questions and explain why.

(a) $p(x_1, x_5|x_3) = p(x_1|x_3)p(x_5|x_3)$

(b) $p(x_1, x_5|x_3, x_4) = p(x_1|x_3, x_4)p(x_5|x_3, x_4)$

(c) $p(x_3, x_4|x_1, x_5) = p(x_3|x_1, x_5)p(x_4|x_1, x_5)$

(d) $p(x_2, x_3|x_1, x_5) = p(x_2|x_1, x_5)p(x_3|x_1, x_5)$

C. For the Markov Random Field below, express the joint probability $p(x_1, x_2, x_3, x_4, x_5)$ in terms of potential functions and in terms of energy functions. What are the potential and energy functions? What are their properties? How are they related?