A. Draw a Bayesian Network with 3 nodes A, B, and C, where random variables A and B are marginally independent, but they are conditionally dependent given variable C (i.e. so-called "explaining away"). Explain intuitively (e.g. on an example) what and how causes the dependency.

B. Assume some arbitrary joint distribution of 6 random variables $p(x_1, x_2, x_3, z_1, z_2, z_3)$. What operation will give us (in general for any such distribution) marginal joint probability $p(x_1, x_2, x_3)$?

C. Consider the factorization

$$p(x_1, x_2, x_3, x_4, x_5) = p(x_1)p(x_2)p(x_3|x_1)p(x_4|x_1)p(x_5|x_4,x_2).$$

Consider that all the random variables x_i are discrete and that we know all the distributions corresponding to the individual factors (i.e. we have the corresponding tables with probabilities).

Let the symbol $\sum_{x_i}$ represent the sum over all possible values of the random variable x_i. Using mathematical notation, express how can the following probabilities be inferred most efficiently (i.e. use the right order of sums and/or brackets). Notice that not all the factors (probability tables) are necessary to evaluate the following probabilities.

(a) $p(x_1)$
(b) $p(x_3)$
(c) $p(x_5)$
(d) $p(x_1|x_2)$
(e) $p(x_3|x_1)$
(f) $p(x_1|x_3)$

D. Write an equation expressing that variables a and b are conditionally independent given variable c.

E. Draw the Bayesian Network corresponding to the factorization:
$p(x_1, x_2, x_3, x_4, x_5) = p(x_1)p(x_2)p(x_3|x_2)p(x_4|x_1,x_2)p(x_5|x_4)$

F. For each of the following statements, say whether the statement holds true for the factorization (and the Bayesian Network) from the previous questions and explain why (e.g. using d-separation).

(a) $p(x_1, x_2) = p(x_1)p(x_2)$

(b) $p(x_1, x_4) = p(x_1)p(x_4)$

(c) $p(x_1, x_5|x_3) = p(x_1|x_3)p(x_5|x_3)$

(d) $p(x_1, x_5|x_4) = p(x_1|x_4)p(x_5|x_4)$

(e) $p(x_3, x_5|x_4) = p(x_3|x_4)p(x_5|x_4)$

(f) $p(x_1, x_2|x_5) = p(x_1|x_5)p(x_2|x_5)$