Delaunay-based Vector Segmentation of Volumetric Medical Images

Michal Španěl, Přemysl Kršek, Miroslav Švub, Vít Štancl and Ondřej Šiler

Department of Computer Graphics and Multimedia
Faculty of Information Technology
Brno University of Technology, Czech Republic
e-mail: {spanel, krsek, svub, stancl, siler}@fit.vutbr.cz

Abstract. The image segmentation plays an important role in medical image processing. Many segmentation algorithms exist. Most of them produce raster data which is not suitable for 3D geometrical modeling of human tissues. In this paper, a vector segmentation algorithm based on a 3D Delaunay triangulation is proposed. Tetrahedral mesh is used to divide a volumetric CT/MR data into non-overlapping regions whose characteristics are similar. Novel methods for improving quality of the mesh and its adaptation to the image structure are also presented.

1 Introduction

Medical imaging devices like the Computer Tomography (CT) and Magnetic Resonance (MR) produce volumetric image data detailing human tissues within a scanned patient body part. Subsequent image segmentation separates objects, e.g. tissues of particular types, in the image. The segmented CT/MR data can be used for creation of geometrical models of tissues. Advantage of a 3D representation of human anatomy is that it gives a better view from any angle. Recent research is also focused on 3D modeling of tissue geometry for implants design, surgery planning and simulation.

Raster-based Segmentation Techniques. Many 2D and 3D segmentation algorithms can be found in the literature. Most of them produce segmented raster data (thresholding, clustering, Watershed transform, neural networks, etc.). Most often, an algorithm such as Marching Cubes [1] is applied to reconstruct surfaces (3D geometrical models) from the raster segmented data and further decimation of the model is required and may not be elementary.

Vector-based Segmentation. Most widely used vector segmentation methods are based on deformable models [2]. Deformable models include curves or solids deformed under influence of external and internal forces derived from image characteristics. Numerous researchers have explored application of deformable surface models to volumetric medical images [3, 4]. Deformable models are robust against noise and boundary gaps. These models are also capable of adjusting themselves to significant variability of human anatomy. Main disadvantage is that they require manual initialization and interaction during the segmentation.

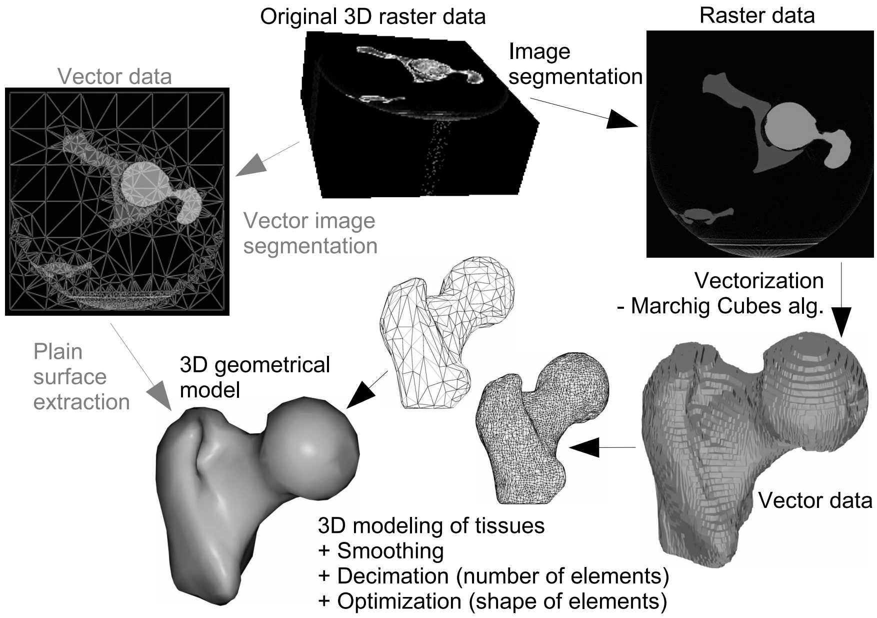


Fig. 1. Comparison of the traditional raster-based segmentation (black labeling) and the proposed vector segmentation method (underlined).

Delaunay-based Segmentation. It has been shown that 2D *Delaunay triangulation (DT)* can be used to effectively partition the image [5, 6], while mesh of the DT is adapted to the image structure by combining region and edge information.

In this paper, a novel segmentation technique based on a *3D Delaunay Triangulation* is proposed. Tetrahedral mesh is used to partition the volumetric data into regions. Process of the mesh construction respects significant image edges. Hence, surfaces of image regions are well described and can be easily derived.

This concept of the vector segmentation has a number of advantages: vector representation of image regions eliminates the vectorization process, continuous approximation and smoothing of region boundaries, effective representation of image structure (reduced number of tetrahedra instead of voxels) and easy manual corrections by adding vertices and re-classification of tetrahedra.

2 Delaunay Triangulation and Meshing

Algorithms for unstructured 3D mesh generation have been intensively studied over the last years. A good survey of these methods can be found in [7].

Delaunay Triangulation. Every tetrahedron of DT satisfies the *Delaunay criterion.* This criterion means that circumsphere associated with the tetrahedron does not contain any others vertices. This criterion is a characterization of the Delaunay triangulation and it leads to several other characteristics.

The DT generates regularly shaped tetrahedra and is preferred over alternative triangulations for image segmentation. Delaunay triangulations are also very attractive from a robustness point of view due to simplicity of the Delaunay criterion. The DT can be constructed by a number of methods. Most common is the *Incremental Method*. Detailed study can be found in [8].

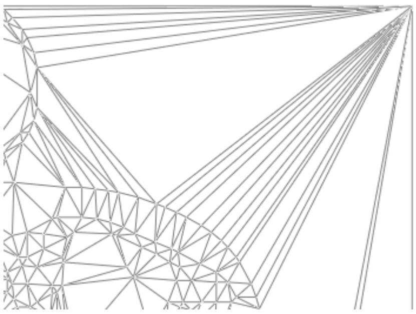
Constrained Delaunay Triangulation. Given a set of *constraints* specified as a set of edges and faces in 3D, *Constrained Delaunay triangulation (CDT)* is obtained by constructing the DT associated with the set of constraints. The CDT has been successfully solved in 2D spaces, while it is still under active investigation in 3D.

Elementary CDT algorithm is a *constraint partitioning method*. Every tetrahedron intersected by a constrained edge/face is divided ensuring that the created sub-edges are in the resulting triangulation

2.1 Isotropic Meshing

Most applications have specific requirements on the size and shape of elements in the mesh. Aim of the isotropic meshing is to locate vertices so that the resulting mesh consists of almost *equilateral* tetrahedra. In addition, the element size is close to a predefined size constraint.

a)



b)

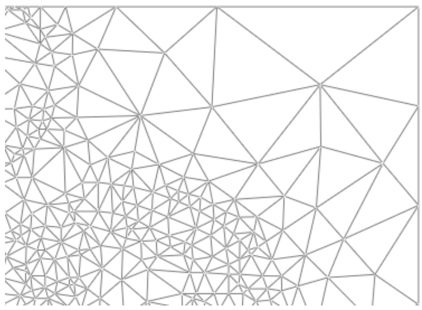


Fig. 2. 2D Delaunay triangulation constructed by the plain incremental method (a) and result of the isotropic meshing (b).

Definition 1. Control space H *(so called* sizing field*) is a function $h(P)$ defined at any point $P(x, y, z)$ of space. This function specifies size of the elements in the mesh [8].*

Let AB be an edge. Length of the edge in the control space metric can be approximately calculated as $l_H(AB) = \frac{1}{2}\|AB\|(1/h(A) + 1/h(B))$, where $\|AB\|$ is the real distance between A and B. The size $h(P)$ is the desired length of all the edges originating from the point P defined by the control space.

Creation of Points Along the Edges. The idea is to create new points along existing edges in the triangulation and obtain nearly equilateral tetrahedra having edges of unit length in the control space. Let T be a threshold value, for instance 0.2. If $l_H(AB) < T$, the edge is not divided, otherwise a new point in the middle of the edge AB is introduced. Produced sub-edges are recursively tested and divided if necessary. Once we have a sequence of points $Q_0 \dots Q_n$ such that $l_H(Q_i, Q_{i+1}) < T$, the final set of points dividing the edge AB can be found.

The smallest index i satisfying the criterion $\sum_{j=0}^{i} l_H(Q_j, Q_{j+1}) > 1$ is found and the point Q_i is introduced to the mesh as new vertex. Iterating this process results in construction of several new points along the edge. Applied to every edge in the current mesh, a large set of points is obtained.

3 Delaunay-based Vector Segmentation

Proposed segmentation scheme is based on the DT described above. The image is partitioned into regions whose characteristics, such as intensity and texture, are similar, while the mesh is adapted to the underlying structure of the volumetric image data. Each image region r_k consists of a set of tetrahedra $t_1, \dots, t_n$. Based on the introduced principles, the adaptive segmentation is proposed as follows

1. *3D edge and corner detection* - candidate vertices lying on regions boundaries, meaningful edges and corners are located. Candidates can be found by various edge and corner detection algorithms extended to the 3D space. In our experiments, the well known *Canny edge detector* has been used which was designed to be an optimal edge detector according to predefined criterions.
2. *Initial Delaunay triangulation* - tetrahedral mesh is constructed from the preselected set of candidate vertices.
3. *Iterative adaptation* - the tessellation grid is adapted to the image structure.
4. *Tetrahedra grouping* - classification of tetrahedra into image regions.

3.1 Iterative Adaptation

Following three main steps are repeated until the triangulation satisfies some convergence criterion.

Isotropic Edge Splitting. In this phase, the isotropic meshing algorithm creating new points along existing edges and another well known technique of tetrahedral mesh optimization, splitting of maximal edges, are combined together. Instead of maximal edges, those edges crossing significant image edges are divided. A new vertex is inserted to the mesh in the point of intersection This approach is partially similar to the CDT. The whole isotropic edge splitting process can be briefly formulated as follows

1. Sequentially process every edge AB in the current triangulation.

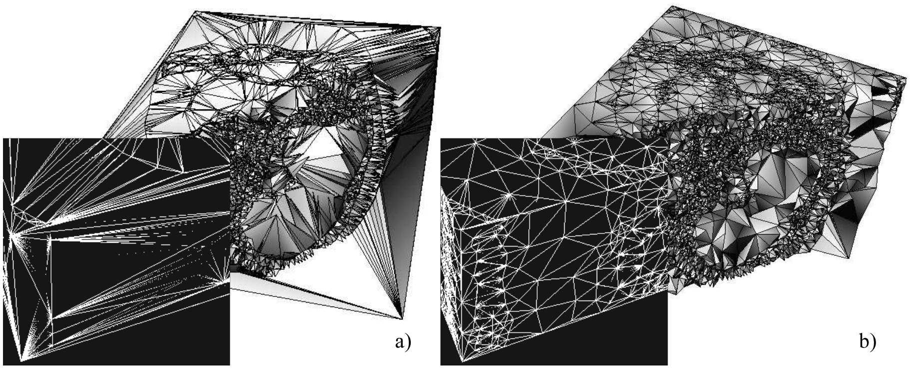


Fig. 3. Initial mesh (a) and result of the iterative adaptation (b).

  - Find intersection points P_i of the edge and all image edges.
  - Introduce the sub-edges $AP_1, P_1P_2, \ldots, P_nB$ and divide them in the sense of isotropic meshing algorithm (Section 2.1).
2. Filter the set of newly created points to discard vertices too close to any other point respecting the control space metric.
3. Insert points to the mesh, continue starting from the step 1 until convergence.

Edges that are almost parallel with the image edge remain unchanged to prevent degradation of the mesh.

The control space H can be defined as a distance function representing proximity of an image edge at the point P. Because the length of any tetrahedron edge emanating from the point P must be smaller, or equal to this value, the edge cannot cross any image edge. Value of the sizing field increases with distance from the closest image edge. Therefore, bigger tetrahedra appear in the center of large homogenous regions. If the point P lies exactly on an image edge, the control space is equal to zero, $h(P) = 0$. Because it is not possible to create edges having zero length, a minimal edge length must be given.

Vertex moving. The vertex moving causes that vertices close to an image edge are attracted directly to this edge. Important effect of this process is that there are many vertices lying on image edges, while no vertices lie in a close vicinity of image edges. Such optimization results in better approximation of region boundaries in the final triangulation.

Tetrahedral Mesh Optimization. Previous step of the iterative adaptation may cause that thin tetrahedra ineffective for further processing appear in the mesh. At the beginning, the quality of all tetrahedra is estimated. There are many measures of the quality regarding the ideal shape. The most general one is ratio of the longest tetrahedron edge and the radius of its inscribed circle [1]. Then, those tetrahedra having the normalized quality below a given threshold are optimized. The new point in the center of circumsphere is added to the mesh.

3.2 Tetrahedra Grouping

Every tetrahedron t_i of the mesh is characterized by a *feature vector* $\boldsymbol{f}_i$. Individual features detail image structure of the tetrahedron and its close neighbourhood. In fact, the first two components are mean value $\mu(t_i)$ and intensity variance $\sigma(t_i)$ of voxels inside the tetrahedron. Others features may cover image texture and spatial configuration of adjacent tetrahedra. Feature vectors may be grouped by the help of Any conventional *supervised* or *unsupervised clustering algorithm* that classifies feature vectors into a certain number of classes.

Region Growing and Merging. Topology of the tetrahedral mesh is suitable for region-based image segmentation techniques such as region growing and merging. Instead of pixels and the traditional 4– and 8– pixel connectedness, tetrahedra and graph adjacency are used.

Definition 2. *Let f_i and f_j be two feature vectors extracted for a non-empty group of adjacent tetrahedra. Similarity measure $S(f_i, f_j)$ is a function whose value is larger as the difference between feature vectors decreases.*

Basic similarity criteria are the mean intensity value S_{mean} and statistical test of the similarity based on voxel value variance S_{var}.

$$S_{mean} = \exp(-\frac{1}{2\rho^2}|\mu_{r_i} - \mu_{r_j}|^2) \qquad S_{var} = \frac{\sigma_{r_i}\sigma_{r_j}}{\sigma^2_{r_{i,j}}} \tag{1}$$

The parameter ρ affects sensitivity of the measure and $\sigma_{r_{i,j}}$ is variance of the intensity in a joint region $r_i \cup r_j$.

Region growing starts with seed tetrahedra and grows the regions from them. Adjacent tetrahedra are added to the region if they satisfy a chosen criteria of similarity. In region merging, neighboring regions are examined to determine if they can be merged together. All adjacent regions are examined and similarity of both feature vectors is estimated. If the measure is greater than a given experimentally chosen threshold, both regions are merged and feature vector for the newly formed region is calculated. The merging phase continues until until no more regions are merged, or some stopping criterion is met.

4 Experimental Results

Two classifiers were designed for the *unsupervised* clustering of feature vectors into image segments. First, the *Fuzzy C-means (FCM)* algorithm [9], and second, the *Gaussian Mixture Model (GMM)* optimized by the popular *Expectation-Maximization (EM)* algorithm [10]. In addition, the region growing initially reduces a large number of tetrahedra in the mesh, and after the clustering phase, the region merging algorithm is applied to get the final segmentation.

The segmentation algorithm was tested on a number of real CT imaging data having resolution mostly 512x512 pixels per slice. Initialization of the tetrahedral mesh, its iterative adaptation and classification take approximately $10-15$ minutes on standard PC with P4 1.6GHz processor depending on concrete number of image slices (approximately $120-150$ slices were analyzed).

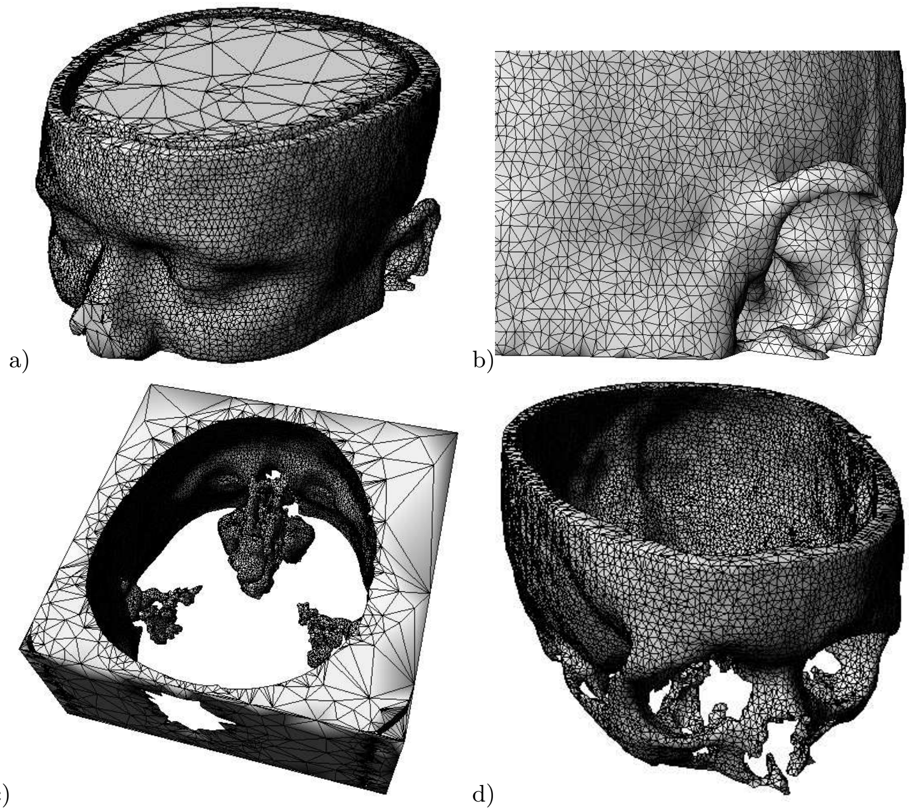


Fig. 4. Result of the vector segmentation. Polygonal surfaces of meaningful image segments extracted directly from the classified tetrahedral mesh: soft tissues (a,b), hard tissues (c), and bacground/air (d).

5 Conclusion

The vector segmentation algorithm based on adaptive Delaunay triangulation is proposed in the paper. Tetrahedral mesh is used to divide a volumetric image into several disjoint regions whose characteristics are similar. Certain methods for improving quality of the mesh and its adaptation to the underlying image structure are also described.

Direct vector representation of image regions makes possible to eliminate difficult process of raster data vectorization. The 3D geometrical model can be created directly from the segmented vector data. More effective representation of the image structure is obtained. Tetrahedral mesh approximates the raster data. This representation should decrease complexity of any classification algorithm that processes a reduced number of tetrahedra instead of voxels.

Another advantage of the vector representation, important in medical imaging, is comfortable manual correction of the final segmentation. Simple modifications of the tetrahedral mesh, such as adding new vertices, removing old ones, and manual reclassification of tetrahedra, allow easy correction of the segmentation. The geometrical model and manual corrections can be made in parallel All changes in the mesh take effect on the model without any longer delay.

6 Future Work

Since the classification is performed within local vicinity of processed tetrahedron, improvements can be made by incorporating global principles. Viewing the mesh as undirected graph with nodes and edges weighted according to feature vectors would allow to use graph algorithms (graph cuts, path algorithms, etc.) for the segmentation.

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Summary Page

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Authors	Michal Španěl, Přemysl Kršek, Miroslav Švub, Vít Štancl and Ondřej Šiler
Affiliation	Department of Computer Graphics and Multimedia, Faculty of Information Technology, Brno University of Technology
Address	Božetěchova 2, 612 66 Brno, Czech Republic
Tel.	+420 54114-1294
Fax	+420 54114-1270
E-mail	spanel@fit.vutbr.cz
Keywords	*Medical image segmentation, Delaunay-based volumetric segmentation, 3D geometrical modeling of tissues*

1. What is the original contribution of this work?
   Effective image partitioning method based on the Delaunay triangulation and common image segmentation methods are combined into vector segmentation technique working directly in 3D space. Implicit volume and surface representation of image regions is obtained.

2. Why should this contribution be considered important?
   Proposed method of "constrained" Delaunay triangulation, its construction and adaptation to volumetric image data can be applied not only to image segmentation.

3. What is the most closely related work by others and how does this work differ?
   The most closely related topics are iso-surface reconstruction methods based on octree, Delaunay triangulation and advancing front methods. This work attempts to overcome the simple iso-surface definition with more complex segmentation technique.

4. How can other researchers make use of the results of this work?
   Basic principles of the presented vector segmentation, especially the mesh construction and adaptation methods, may be adopted. More sophisticated classification algorithms upon the tetrahedral mesh topology should be investigated.

5. Has this work been presented/submitted elsewhere?
   The experimental two-dimensional implementation of the algorithm has been already presented.

6. Which form of presentation is preferred: Oral or Poster?
   The oral presentation is preferred nor necessary.