Investigating the Possibility of Using Pruned FFT in Ultrasound Wave Propagation Simulations

Ondrej Olsak and Jiri Jaros

Faculty of Information Technology, Brno University of Technology, CZ

BRNO UNIVERSITY OF TECHNOLOGY FACULTY OF INFORMATION TECHNOLOGY

Motivation

Ultrasound wave propagation simulation in k-Wave uses a pseudo-spectral method with Fourier basis functions to solve governing equations. Currently, computing Fast Fourier Transforms (FFT) consumes 60% of total simulation time. The work aims to reduce computation time by involving the Pruned Fast Fourier Transform (pruned FFT) applied to acoustic pressure and velocity. To evaluate the impact of this algorithm on the simulation, emulation through spectral filtration is used. The goal is to evaluate the impact of the pruned FFT on the result of the ultrasound wave propagation simulation.

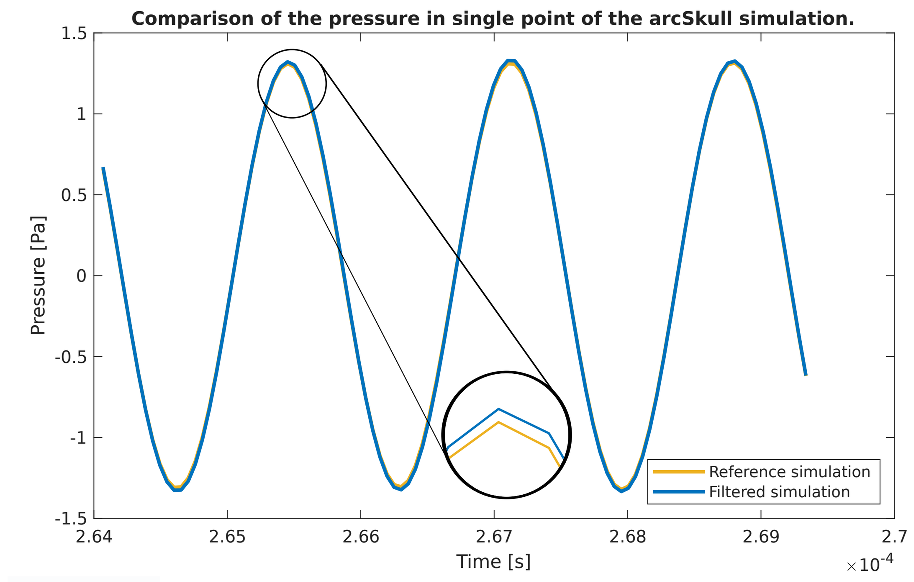


Figure 1: Focal point pressure variation in the final 100 steps of the simulation

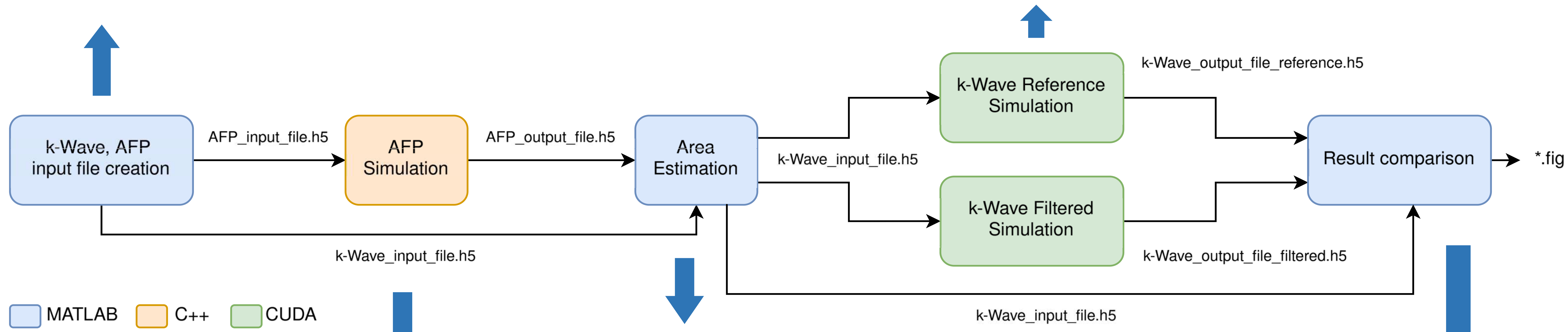
Experimental pipeline

Input files creation

Creation of the k-Wave input file with all simulation properties (media, transducer, simulation time, ...). Based on the simulation properties, the file for Acoustic Field Propagator (AFP) is created.

Ultrasound wave propagation simulation

In this step, two ultrasound wave propagation simulations are executed. The first simulation runs an unmodified (reference) version to establish baseline results. The second simulation incorporates spectrum filtration to emulate pruned FFT behaviour.



Acoustic Field Propagation (AFP)

The AFP efficiently calculates acoustic pressure fields across all spatial positions in a single computational step. While limited to heterogeneous media with no reflection capabilities, its strength lies in accurately estimating spectral coefficient positions. This targeted functionality makes it an ideal tool for preliminary wave propagation analysis and coefficient mapping.

Area Estimation

A bisection method is applied in both X and Y directions to analyse the shifted acoustic pressure spectrum from AFP. The spectral domain area is adjusted symmetrically based on set thresholds during each iteration. During this process, coefficients outside the defined borders are temporarily set to zero, followed by inverse FFT computation. The transformed result is compared with the original space domain, triggering border adjustments: moving inward when error falls below the threshold or outward when exceeding it. The process creates a binary mask identifying essential coefficients for further calculations.

Simulation Result Analysis

The outputs from both simulations are compared to evaluate the impact of spectrum filtration on the simulation result. This includes analysis of the spatial domain of the acoustic pressure and its spectral domain representations.

Measurement of Skipped Coefficients and Final Simulation Error

The original simulation employs a 576×648 grid with uniform spacing. In the first case, the domain resolution is progressively increased by factors of two, up to eight times the original dimensions, while proportionally reducing grid spacing to maintain consistent physical domain size. In the second case, the physical domain size is expanded while keeping the original grid spacing constant, resulting in a larger physical domain.

The measurement for the various resolution of the original domain for 2% and 3% threshold error.

	2%								3%							
	1x		2x		4x		8x		1x		2x		4x		8x	
	Skip [%]	L_∞ [%]	Skip [%]	L_∞ [%]	Skip [%]	L_∞ [%]	Skip [%]	L_∞ [%]	Skip [%]	L_∞ [%]	Skip [%]	L_∞ [%]	Skip [%]	L_∞ [%]	Skip [%]	L_∞ [%]
Arc	19.10	1.68	22.22	0.33	22.66	0.16	22.79	0.11	28.12	2.64	33.16	0.61	33.94	0.31	34.16	0.12
ArcX	46.18	2.26	72.22	1.47	82.81	1.56	90.10	3.02	57.29	8.12	76.39	3.10	87.07	3.53	92.97	4.84
Piston	3.82	0.73	4.17	0.24	4.17	0.14	4.17	0.15	5.90	0.87	6.08	0.22	6.25	0.13	6.25	0.15
PistonX	19.44	1.14	35.59	0.49	56.25	0.41	75.87	0.48	26.39	1.71	46.35	0.61	68.40	0.64	80.86	1.16
Dot	6.25	3.74	8.51	1.02	10.59	1.44	12.67	3.27	9.03	4.70	12.33	1.24	15.28	1.64	18.06	3.37

The measurement for the various domain sizes of the original domain for 2% and 3% threshold error.

	2%								3%							
	1x		2x		4x		8x		1x		2x		4x		8x	
	Skip [%]	L_∞ [%]	Skip [%]	L_∞ [%]	Skip [%]	L_∞ [%]	Skip [%]	L_∞ [%]	Skip [%]	L_∞ [%]	Skip [%]	L_∞ [%]	Skip [%]	L_∞ [%]	Skip [%]	L_∞ [%]
Arc	19.10	1.68	27.60	1.24	35.33	1.67	44.31	3.19	28.12	2.64	38.37	2.03	45.49	2.65	52.34	4.74
ArcX	46.18	2.26	58.68	5.20	62.50	17.44	64.89	79.78	57.29	8.12	62.67	28.99	67.01	53.21	72.92	86.07
Piston	3.82	0.73	3.82	0.39	3.91	0.23	3.95	0.23	5.90	0.87	5.73	0.39	5.82	0.29	5.90	0.32
PistonX	19.44	1.14	19.62	0.44	19.27	0.34	19.49	0.40	26.39	1.71	26.91	0.64	26.48	0.54	26.74	0.58
Dot	6.25	3.74	6.25	3.27	6.25	1.62	6.25	3.38	9.03	4.70	9.20	4.33	9.11	2.26	9.11	4.92

Conclusion

Our experiments with emulation of the pruned FFT applied to acoustic pressure and velocity have shown that it is possible to reduce the number of computed spectral coefficients, especially in high-resolution domains, while maintaining suitable simulation error. The best results were achieved in high-resolution simulations, where the number of reduced coefficients was up to 90%, with approximately 5% error.

Future research will focus on:

- **Implementation of the pruned FFT and its integration into ultrasound wave propagation simulation**
- **Investigation of the method's stability and its impact on the simulation time**
- **Extension of this approach to three-dimensional simulation**

VSB TUO | IT4I

k-Wave A MATLAB toolbox for the time-domain simulation of acoustic wave fields

European Innovation Council

citrus

This work was supported by the Ministry of Education, Youth and Sports of the Czech Republic through the e-INFRA CZ (ID:90254). This project has received funding from the European Unions Horizon Europe research and innovation programme under grant agreement No 101071008. This work was supported by Brno University of Technology under project number FIT-S-23-8141.