Techniques for Efficient Fourier Transform Computation in Ultrasound Simulations

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Motivation

- Simulating ultrasound wave propagation in media is computationally expensive and time-consuming. Simulation time and computational resource requirements increase significantly with the size of the simulation and can reach days even on HPC clusters.
- Simulating ultrasound wave propagation involves solving a set of equations. The k-Wave toolbox employs the pseudo-spectral method with Fourier basis functions to solve these governing equations. This approach leads to approximately 60% of the simulation time being spent on computing the Fast Fourier Transform (FFT).
- These simulations are typically used in the preoperative treatment planning of ultrasound surgery, where a narrow-band limited transducer is employed.
- This work investigates incorporating some sort of the Sparse Fourier transform (SFT) to achieve a reduction in the time spent on computation of the Fourier transform.

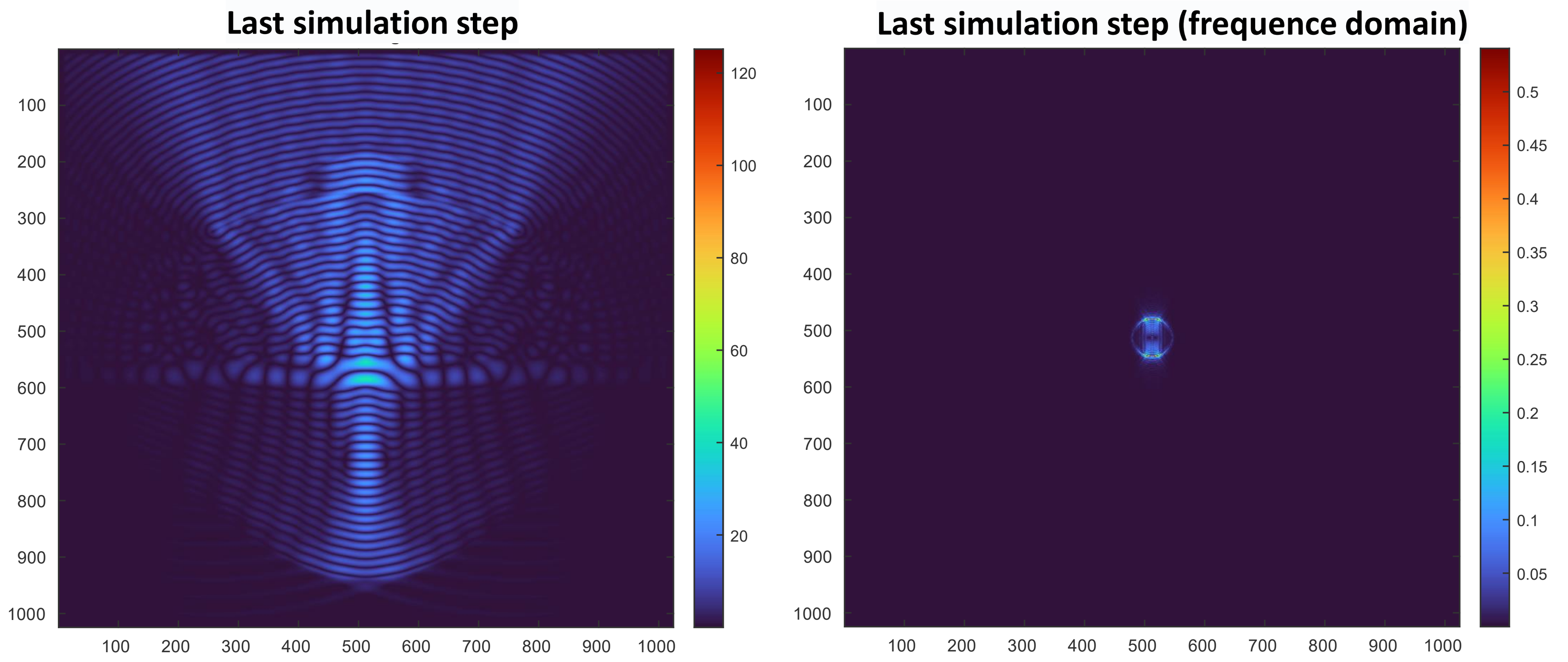


Figure 1: Acoustic pressure distribution at the final simulation step of ultrasound wave propagation (left) and its corresponding frequency spectrum (right). The simulation consists of four media (water, skin, skull, brain), a transducer, and the final focus.

Computing *K* most significant Fourier coefficients

Sparse Fourier Transform algorithms efficiently compute the *K* most significant frequency coefficients of a signal, rather than computing all *N* coefficients. This makes them well-suited for acoustic simulations where pressure distributions tend to be sparse in the frequency domain (Fig. 1). However, directly zeroing out a specified percentage of the smallest coefficients to emulate the behaviour of an SFT algorithm leads to increased errors in the reconstructed pressure.

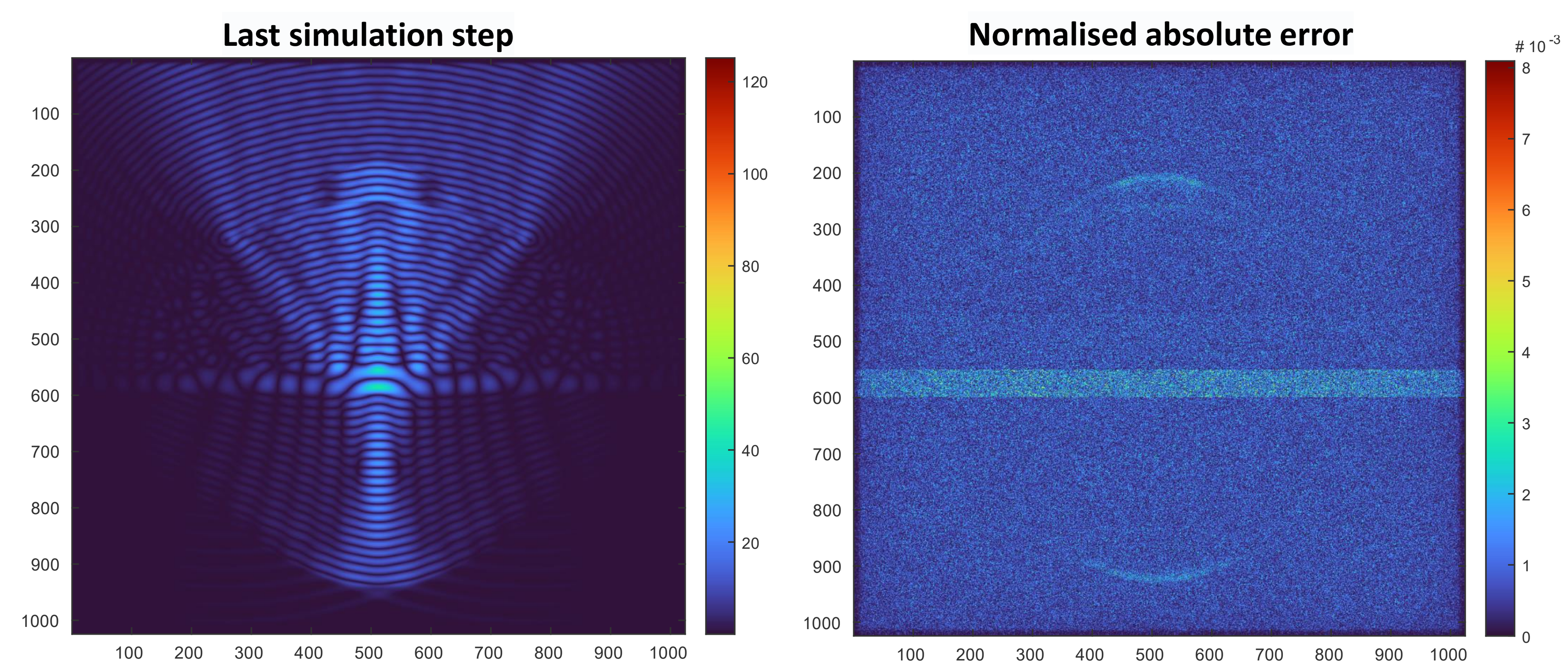


Figure 2: Acoustic pressure distribution at the final simulation step of ultrasound wave propagation with 30% lowest coefficient filtered out (left) and its normalised absolute error to reference simulation (right). The maximum error in focus is around 0.8%.

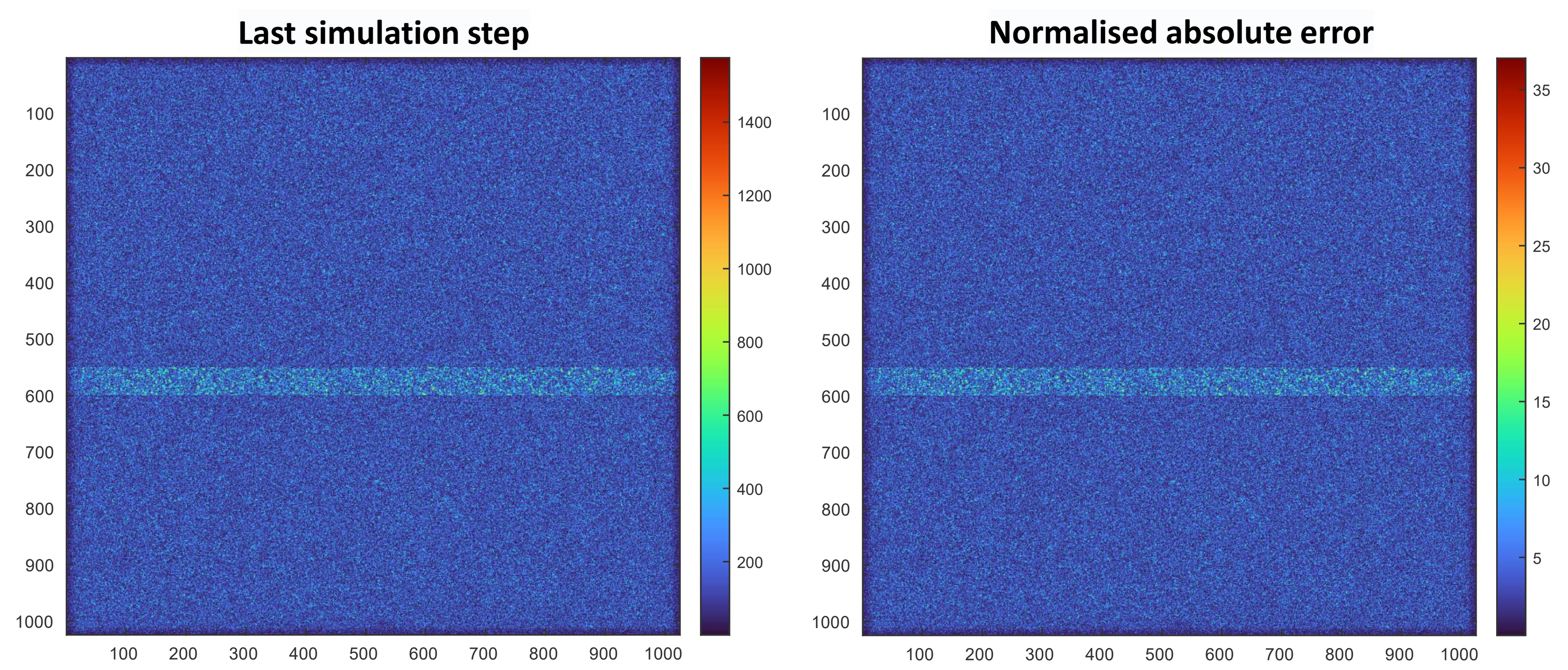
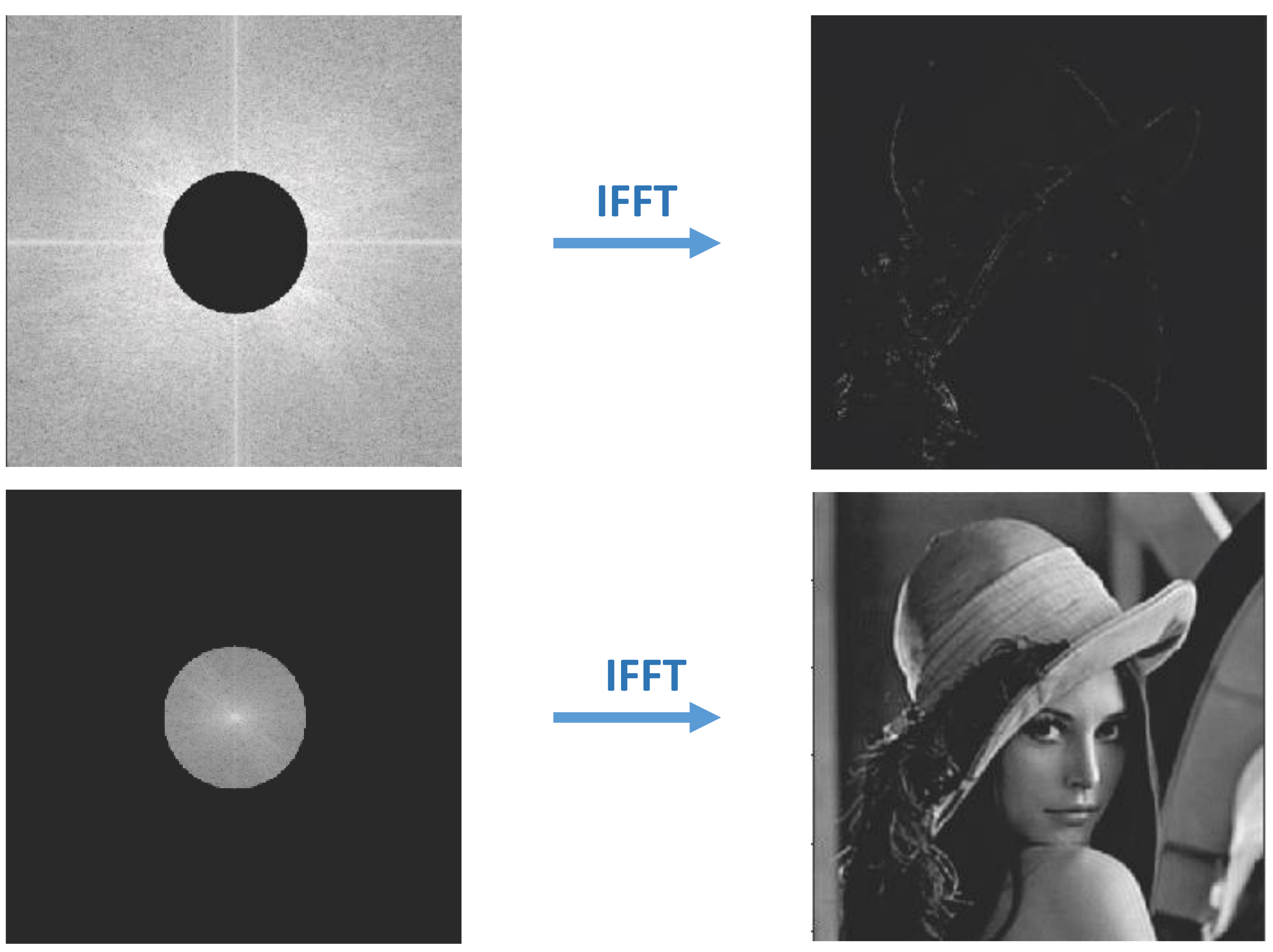


Figure 3: Acoustic pressure distribution at the final simulation step of ultrasound wave propagation with 70% lowest coefficient filtered out (left) and its normalised absolute error to reference simulation (right).

Efficient Fourier Transform Computation via Filtering

This approach is inspired by filtration used in image compression or edge detection. In acoustic simulations, we can leverage filtering to compute only coefficients within a specific area of the spectrum. This approach achieves more accurate results compared to filtering based on coefficient amplitude. Despite the fact that about 80% of the coefficients are filtered out in this method (Fig. 5), it achieves a significantly better accuracy comparable to results in Fig. 3.



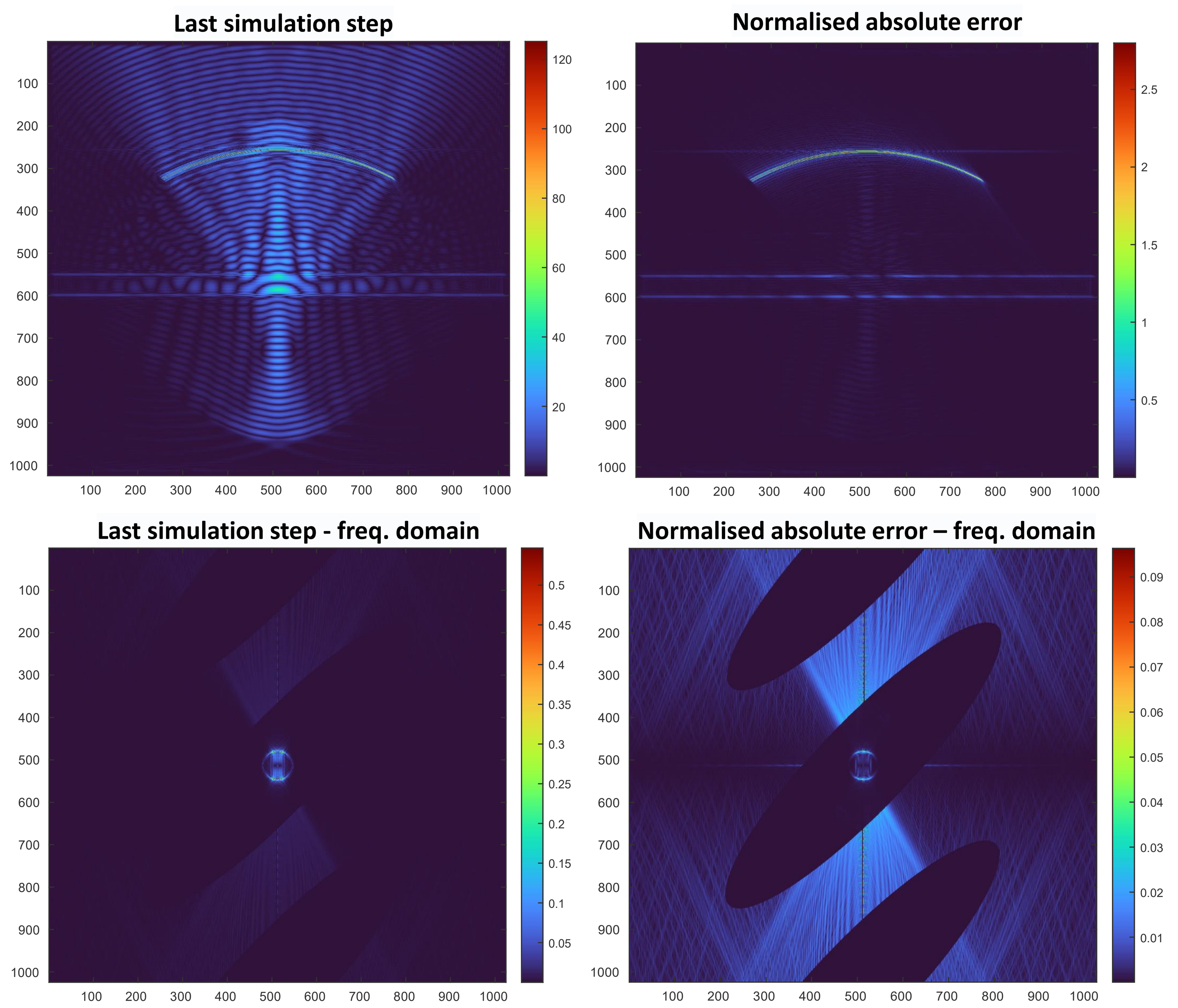


Figure 4: Acoustic pressure distribution at the final simulation step of ultrasonic wave propagation using filtration using mask. Left column contains space and frequency domain, right column normalised absolute error in space and frequency domain. The maximum error in focus in space domain is around 8.9%.

Conclusion

Our experiments demonstrate that computing only a specific region (pattern) of Fourier coefficients significantly reduces the number of coefficients that need to be computed compared to the standard FFT algorithm. This approach has the potential to decrease the overall simulation time by reducing the computing time of the Fourier transform.
Future research will focus on:

- **Mitigating errors:** Minimizing the errors observed near the transmitter and material boundaries.
- **Adaptive filtering:** Developing a method to determine the optimal filtering area based on simulation properties like transmitter driving signal, medium density, and sound speed.

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k-Wave A MATLAB toolbox for the time-domain simulation of acoustic wave fields

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