Evaluation of Libraries for One-dimensional Sparse Fourier transform

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Overview

This work is focused on comparison of several Sparse Fourier Transform (SFT) libraries. Comparisons were made on signals obtained from a one-dimensional ultrasound propagation simulation made by the k-Wave toolbox. Since focused ultrasound simulations are very sparse in frequency domain we added a randomly generated signal with large number of coefficients to show behaviour of the libraries on more dense signals.

Runtime complexities of the Sparse FFT libraries overview

N	- Size of input domain
k	- Number of significant coefficients

FFTW3[1]

$O(N \cdot \log N)$

Gopher Fast Fourier Transform (GFFT)[2]

- GFFT-det-slow - $O(N \cdot k \cdot \log^2 N)$
- GFFT-det-fast - $O(k^2 \cdot \log^4 N)$
- GFFT-rand-fast - $O(k \cdot \log^5 N)$
- GFFT-rand-slow - $O(N \cdot \log N)$

Discrete Michigan State Fourier Transform (DMSFT)[3]

$O(k^2 \cdot \log^{\frac{11}{2}} N)$

Multiscale Sub-Linear Sparse Fourier Algorithm (MSFFT)[4]

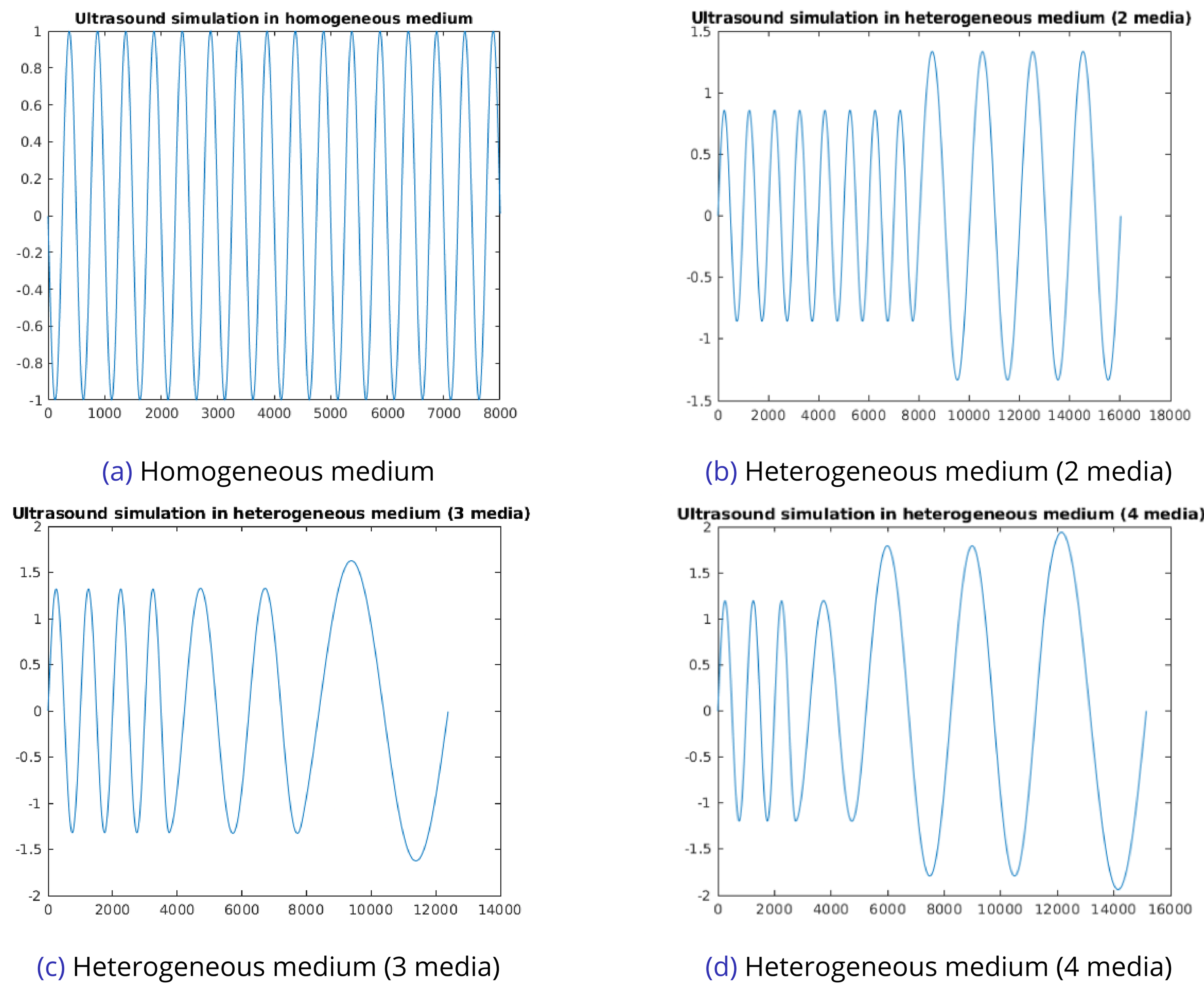
$O(k \cdot \log k \cdot \log \frac{N}{k})$

Ann Arbor Fast Fourier Transform (AAFFT)[5]

$O(k \cdot polylog N)$

Benchmark data

The comparison of the Sparse Fourier Transform libraries was performed on a set of pressure distribution simulations signals with different number of media where each has different properties. Number of elements in each testing signal are $N = 2^{20}, 2^{22}, 2^{24}, 2^{26}, 2^{28}$. Input file sizes are from 16MB to 4096MB.



(a) Homogeneous medium
(b) Heterogeneous medium (2 media)
(c) Heterogeneous medium (3 media)
(d) Heterogeneous medium (4 media)

Figure 1: Last step of linear wave propagation simulation performed by k-Wave toolbox

Results

Following figures present results of SFT libraries computation time on mentioned signals.

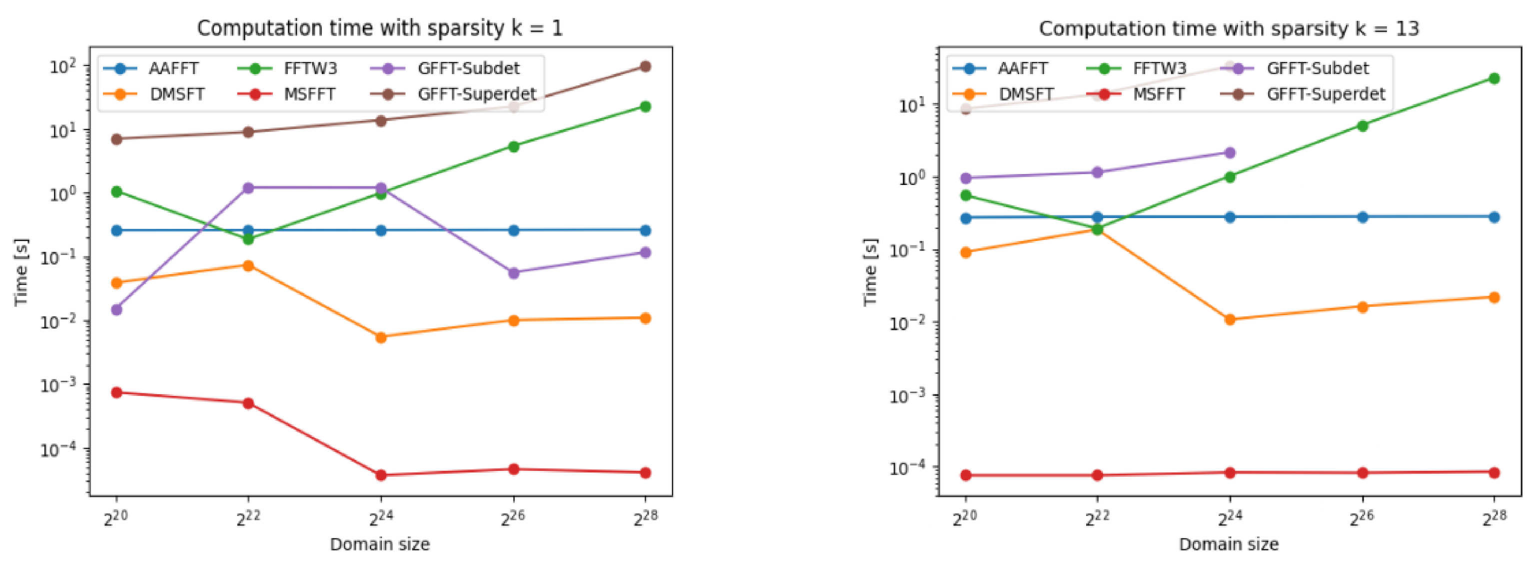


Figure 2: Results for homogeneous and heterogeneous medium (2 media)

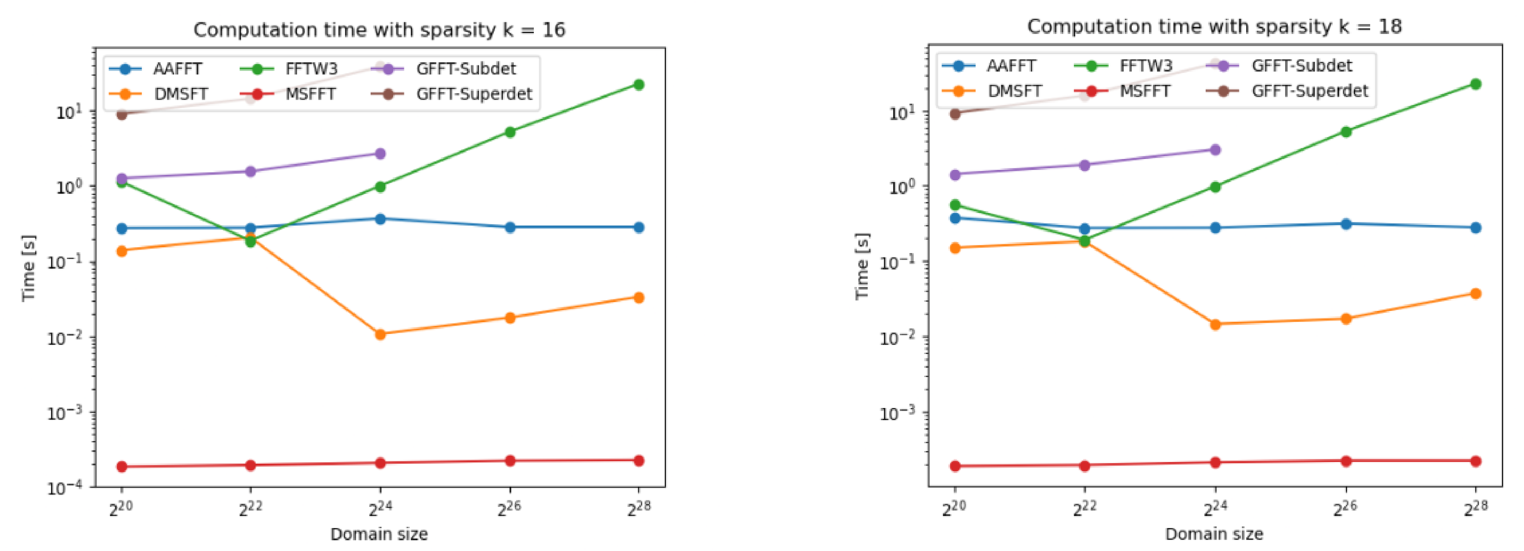


Figure 3: Results for heterogeneous (3 and 4 media) medium.

Next figure shows L_2 and L_{inf} errors under different signal sparsity for a domain size of $N = 2^{28}$. The accuracy of the computation depends on setting of input parameters, selected to achieve balance between speed and accuracy. We omit the L_2 and L_{inf} error for the GFFT library on domain with size $N = 2^{28}$ due to complexity of the computation.

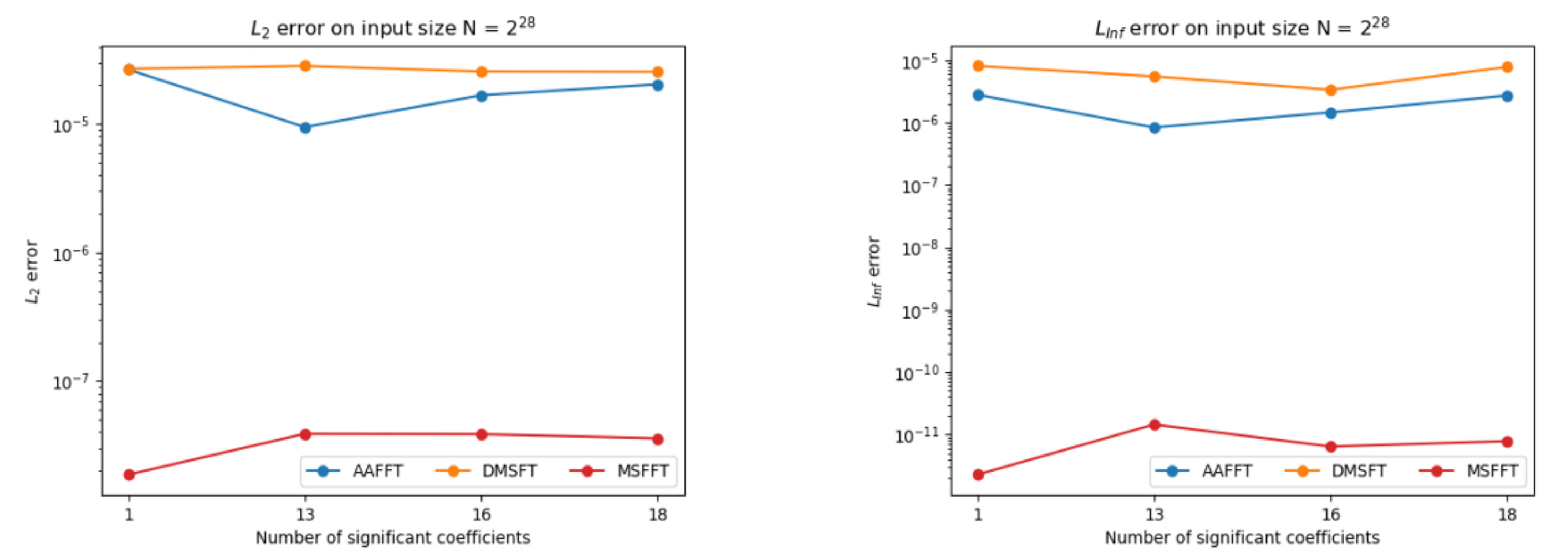


Figure 4: L_2 and L_{inf} error on domain with size $N = 2^{28}$

Following table shows speedup on domain size equal to 2^{28} on each of the mentioned pressure distribution signals against FFTW3. For GFFT library is given speedup on domain with size 2^{26}.

SFT Library	MSFFT	DMSFT	AAFFT	GFFT-SUB	GFFT-SUPER
1 medium, k = 1	554951	2072	86	196	0.2
2 media, k = 13	271342	1051	81	0.46	0.03
3 media, k = 16	102202	685	80	0.36	0.02
4 media, k = 18	104157	616	82	0.32	0.02
Rand, k = 2000	5.4	21.2	62.7	X	X

MSFFT performs extremely well against FFTW3 and other libraries. However, when we increase the number of coefficients then the MSFFT is slower than other SFT libraries, and for some signal sizes, is even slower than FFTW3. Figure 5 shows described situation.

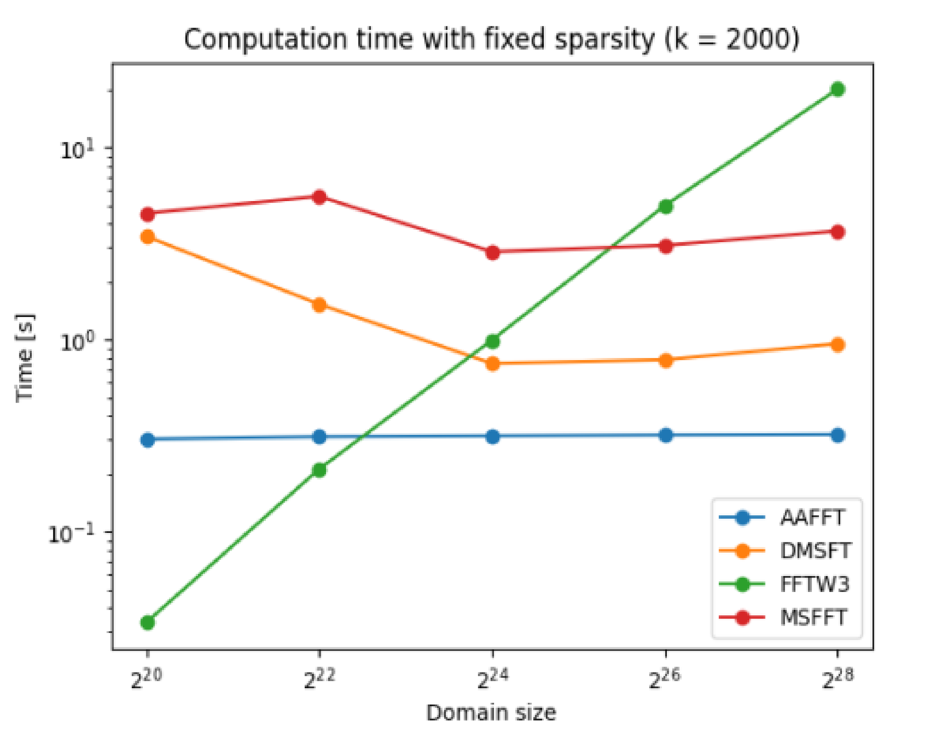


Figure 5: Signal with k = 2000 random coefficients

Conclusion

We compared FFTW3 library for the Fast Fourier Transform against some Sparse Fourier Transform libraries on signals from the linear wave propagation simulation. The results show that MSFFT gives the best results in terms of computation time and accuracy on homogeneous and heterogeneous media. This suggests that MSFFT may be suitable for reducing computation time in ultrasound propagation simulation.

[1] FFTW homepage: https://www.fftw.org/
[2] Improved sparse Fourier approximation results: faster implementations and stronger guarantees
[3] A new class of fully discrete Sparse Fourier Transforms: Faster stable implementation with guarantees
[4] A Multiscale sub-linear time Fourier Algorithm for noisy data
[5] Improved Time Bounds for Near-Optimal Sparse Fourier Representations

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