Spectral Domain Decomposition Using Local Fourier Basis:
Application to Ultrasound Simulation on a Cluster of GPUs

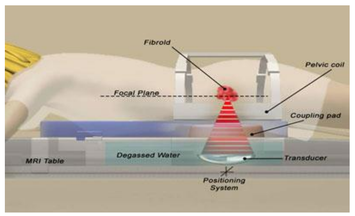
Jiri Jaros[1], Filip Vaverka[1], Bradley E. Treeby[2]

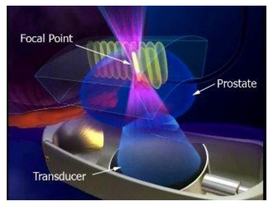
[1]Faculty of Information Technology, Brno University of Technology, CZ
[2]Department of Medical Physics and Bioengineering, University College London, UK

k-Wave
A MATLAB toolbox for the time-domain simulation of acoustic wave fields

1 Overview

High-intensity focused ultrasound (HIFU) is an emerging non-invasive cancer therapy that uses tightly focused ultrasound waves to destroy tissue cells through localised heating. The treatment planning goal is to select the best transducer position and transmit parameters to accurately target the tumour. The path of the ultrasound waves can be predicted by solving acoustic equations based on mass, momentum, and energy conservation. However, this is a computationally difficult problem because the domain size is very large compared to the acoustic wavelength.





2 Nonlinear Ultrasound Wave Propagation in Tissue

The governing equations must account for the nonlinear propagation of ultrasound waves in tissue, which is a heterogeneous and absorbing medium. Accurately accounting for acoustic absorption is critical for predicting ultrasound dose under different conditions.
The required acoustic equations can be written as:

$$\frac{\partial \mathbf{u}}{\partial t} = -\frac{1}{\rho_0}\nabla p \qquad \textit{momentum conservation}$$

$$\frac{\partial \rho}{\partial t} = -\left(2\rho + \rho_0\right)\nabla \cdot \mathbf{u} - \mathbf{u} \cdot \nabla \rho_0 \qquad \textit{mass conservation}$$

$$p = c_0^2\left(\rho + \mathbf{d} \cdot \nabla \rho_0 + \frac{B}{2A}\frac{\rho^2}{\rho_0} - \Pi\rho\right) \qquad \textit{pressure-density relation}$$

These equations are discretised using the *k*-space pseudo-spectral method and solved iteratively. This reduces the number of required grid points per wavelength by an order of magnitude compared to finite element or finite difference methods. For uniform Cartesian grids, the gradients can be calculated using the fast Fourier transform (FFT), e.g.

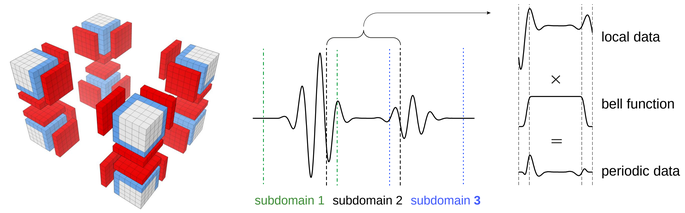
$$\frac{\partial p}{\partial x} = \mathbb{F}^{-1}\left\{ik_x \mathbb{F}\left\{p\right\}\right\}$$

3 Local Fourier Basis Decomposition

Local domain decomposition reduces the communication burden by partitioning the domain into a grid of local subdomains where gradients are calculated locally and the global communication is replaced by the nearest-neighbor halo exchange.

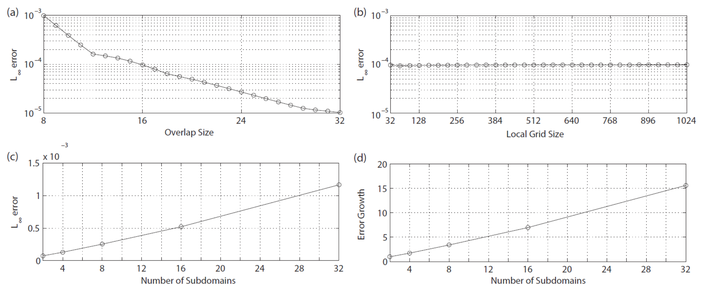
The gradient calculation with the hallo on an *i*-th subdomain reads as follows (*b* is a bell function smoothening the subdomain interface):

$$\frac{\partial p_i}{\partial t} = \mathbb{F}^{-1}\{ik_i\mathbb{F}(b \cdot p_i)\}$$



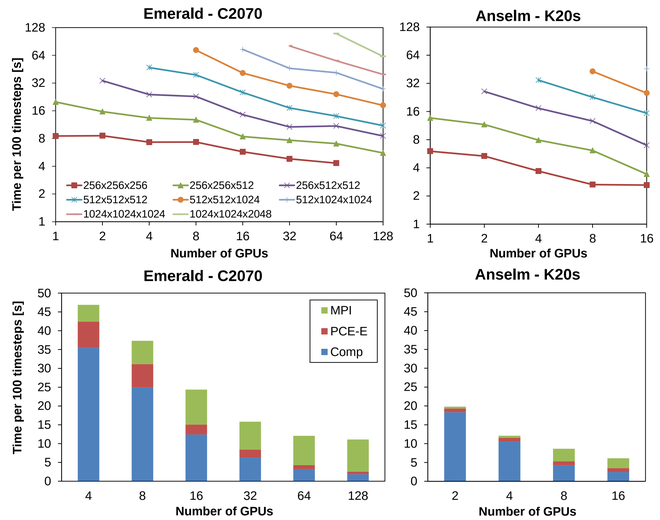
4 Local Fourier Basis Accuracy

Since the gradient is not calculated on the whole data, numeric error is introduced. Its level can be tuned by the thickness of the halo region. For an overlap (halo) size of 16 grid points, the error is on the order of 10^{-3}, which is comparable to the error introduced by the perfectly matched layer (ensuring signal attenuation at the domain boundaries and enforcing periodicity).



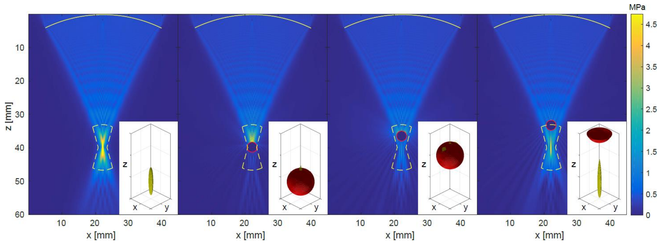
5 Performance Investigation

Strong scaling and simulation time breakdown were investigated on Emerald and Anselm clusters with up to 128 GPUs.



6 Realistic Simulations and Their Costs

Pressure field from a prostate ultrasound transducer simulated using a domain size of 1536 x 1024 x 2048 (45mm x 30mm x 60mm) with 48,000 time steps (60μs).



Compute Resources	Simulation Time	Simulation Cost
96 GPUs	14h 09m	$475
128 GPUs	**9h 29m**	**$426**
128 CPU cores	6d 18h	$1,826
256 CPU cores	3d 0h	**$1,623**
512 CPU cores	**2d 5h**	$2,395

The project is financed from the SoMoPro II programme. The research leading to this invention has acquired a financial grant from the People Programme (Marie Curie action) of the Seventh Framework Programme of EU according to the REA Grant Agreement No. 291782. The research is further co-financed by the South-Moravian Region. This work reflects only the author's view and the European Union is not liable for any use that may be made of the information contained therein.
This work was supported by The Ministry of Education, Youth and Sports from the Large Infrastructures for Research, Experimental Development and Innovations project „IT4Innovations National Supercomputing Center – LM2015070"The authors would like to acknowledge that the work presented here made use of Emerald, a GPU-accelerated High Performance Computer, made available by the Science & Engineering South Consortium operated in partnership with the STFC Rutherford-Appleton Laboratory.