

# VECTOR-BASED MEDICAL IMAGE SEGMENTATION USING ADAPTIVE DELAUNAY TRIANGULATION

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## ABSTRACT

The image segmentation plays an important role in medical image processing. Many segmentation algorithms exist. Most of them produce raster data which is not suitable for further 3D geometrical modeling of tissues. In this paper, a vector segmentation algorithm based on an adaptive Delaunay triangulation is proposed. Triangular meshes are used to divide an image into several non-overlapping regions whose characteristics are similar. Novel methods for improving quality of the mesh and its adaptation to the image structure are also presented.

## KEY WORDS

CT/MR data, image segmentation, 3D geometrical model, vector-based segmentation, Delaunay triangulation.

## 1 Introduction

Medical imaging become one of the most important diagnostic tools in medicine. Imaging devices like the Computer Tomography (CT) and Magnetic Resonance (MR) produce image data detailing human tissues within a scanned patient body part. The most frequent way of medical treatment, investigation of grayscale images, is not sufficient for many applications. However, the CT/MR data make possible to explore other ways of clinical applications.

Our research is focused on 3D modeling of tissue geometry for implants design, surgery planning and simulation (see Figure 1). In conjunction with St. Anne's University Hospital in Brno, a number of clinical applications in aesthetic surgery, orthopaedics and dental surgery are investigated. Therefore, knowledge-based methods of tissue modeling which uses atlas of human anatomy are not suitable.

In general, traditional part of all image processing algorithms is segmentation that separates objects, e.g. tissues of particular types. The segmentation provides crucial information for higher level vision tasks such as 3D tissues modeling.

Due to a complexity and variability of human tissues, it is more reliable to segment an image by the human eye than by sophisticated computer algorithm. Hence, the man-

ual verification and correction of the segmentation is always needed.

The segmented CT/MR data can be used for creation of three dimensional models of tissues. Advantage of a 3D surface representation of anatomy is that it gives a three-dimensional view from any angle. As mentioned above, it is an improvement over investigation of the original two-dimensional grayscale images.

**Raster-based Segmentation Techniques** Many segmentation algorithms can be found in the literature [11, 2]. Most of them produce segmented raster data (thresholding, clustering, Watershed transform, neural networks and etc.). Common raster-based segmentation methods generate data which are not suitable for fast geometrical 3D tissues modeling (see Figure 2). Further vectorization and decimation of the model is required. The 3D geometrical model can be created from raster data using an algorithm such as Marching Cubes [6].

**Vector-based Segmentation** Most widely used vector segmentation methods are based on deformable models [14]. Deformable models include curves, surfaces or solids deformed under influence of external and internal forces derived from image characteristics.

The deformable models are robust against noise and boundary gaps. These models are also capable of adjusting itself to significant variability of human anatomy. Main disadvantage is that they require manual initialization and interaction during the segmentation. Moreover, extension of the deformable models to 3D is not trivial task.

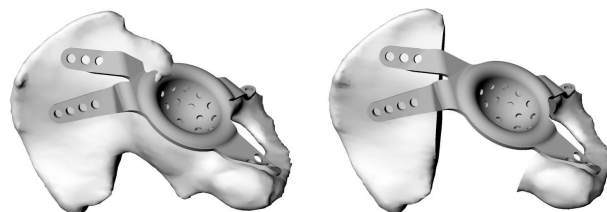


Figure 1. Implant design realized in cooperation with Czech company Beznoska a.s., the producer of orthopedic implants and instruments.

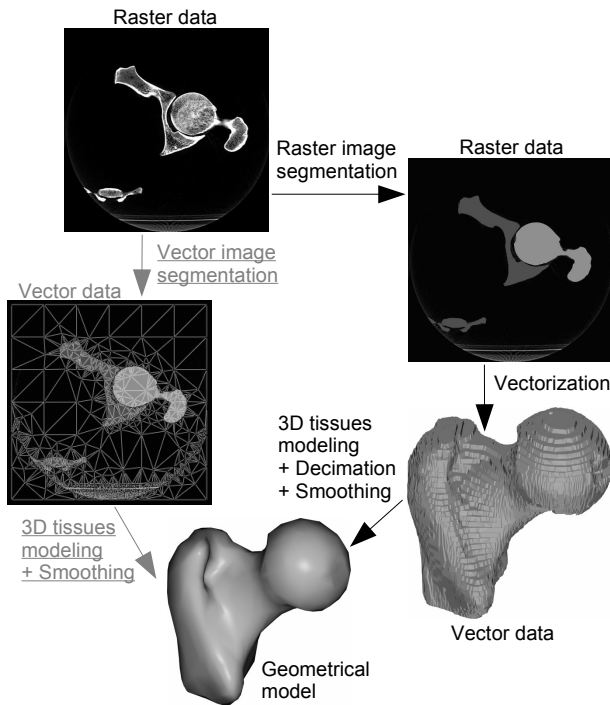


Figure 2. Typical application of the traditional raster segmentation (black labeling) and the proposed method of vector segmentation (underlined). It is assumed that 2D case of the algorithm presented here is going to be extended to 3D.

**Segmentation based on Delaunay triangulation** In this paper, a novel vector segmentation method by virtue of an *Adaptive Delaunay Triangulation (ADT)* is proposed. Triangular mesh is used to divide an image into several disjoint regions whose characteristics, such as intensity and texture, are similar [5]. This concept of the vector-based segmentation has a number of advantages:

- Direct vector representation of image regions eliminates difficult process of raster data vectorization (see Figure 2).
- Easy manual corrections of the segmentation (adding/removing vertices and manual assignment of tissue types to triangles).
- The presented two-dimensional case can be extended to the 3D space.
- Effective representation of image structure (reduced number of triangles instead of particular pixels).

## 2 Adaptive Delaunay Triangulation for Image Segmentation

The principle task of the segmentation is image partitioning into a set of non-overlapping regions so that the variation of some property (mean pixel value, variance, and etc.)

within each region  $R_k$  is either constant, or follows a simple model.

**Delaunay Triangulation** It has been shown that Delaunay triangulation (DT) can be used to effectively partition the image [3] and simultaneously the tessellation grid of the DT can be adapted to the structure of the image by combining region and edge information.

The Delaunay triangulation of a set of points generates regularly shaped triangles and is preferred over alternative triangulations for image segmentation. The DT can be constructed by several methods. Most common is the *Incremental Method* [4].

Adaptive segmentation scheme is based on the Delaunay triangulation described above, while the mesh of the DT is adapted to the underlying structure of the image data. During construction of the Delaunay triangulation, the image is divided into a number of non-overlapping triangles  $t_i$ . These triangles are not segments of the image by itself, but they belong to image regions  $R_k = \{t_1, t_2, \dots, t_n\}$ .

This relationship can be expressed by a region membership function. *Hard assignment* means that this function assigns exactly one region to a given triangle. In practice, membership function of the form  $m(t_i, R_k) = p(t_i | R_k)$  making a *soft assignment* of triangles overcomes the hard one and leads in better results. Soft membership function is usually a likelihood function assigning each triangle into every image region with some certainty. The value is higher as the similarity of the triangle and the region increases. Based on the introduced principles, the adaptive image segmentation is proposed as follows:

1. *Edge and corner detection* - Candidate vertices lying on regions boundaries, meaningful edges and corners are located.
2. *Initial Delaunay triangulation* - Triangular mesh is constructed from the preselected set of candidate vertices.
3. *Triangles grouping* - Initial classification of triangles. Region membership function is initialized looking at the result of some data clustering method.
4. *Iterative adaptation* - The triangulation is adapted to the underlying image structure by means of edge splitting, triangles splitting and edge suppression, thus the overall segmentation improves. The region membership function is re-estimated after each step.

### 2.1 Initial Delaunay Triangulation

To construct the image partition, the triangulation starts with a set of candidate points/vertices distributed over the entire image. These candidates can be found by various edge and corner detection algorithms [13]. Afterwards, the edge pixels are ordered by their significance. While the initial DT is being constructed by the incremental method,

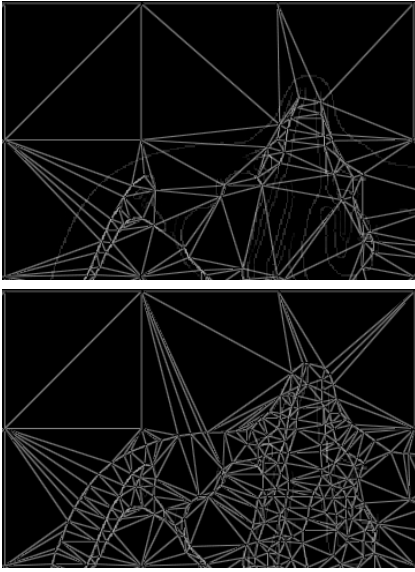


Figure 3. Incremental construction of initial Delaunay triangulation. The number of vertices increases from top to down.

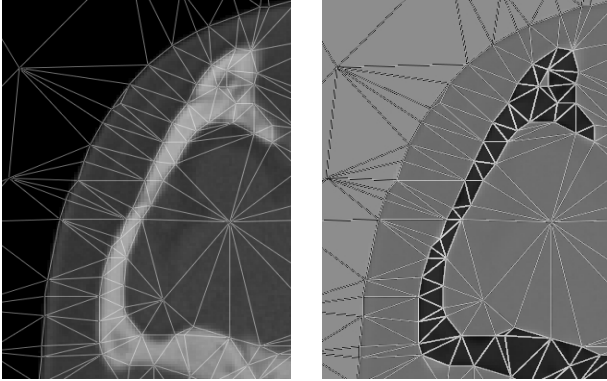


Figure 4. Detail of the classified triangular mesh (right) and the original image (left).

vertices located on strong edges are taken the first (see Figure 3).

## 2.2 Triangles Grouping

After the initial Delaunay triangulation, the opening set of triangles is grouped into disjoint regions. The region membership function is initialized. Ideally, the number of regions is equal to the number of tissues so that the relationship between groups of triangles and tissue types is one to one.

Every triangle  $t_i$  of the mesh is characterized by a *feature vector*  $\vec{f}_i = (f_0, \dots, f_m)$ , where features are extracted from underlying image. Values of the features detail image structure of the triangle and its close neighbourhood. In fact, the first two components are mean value (1) and in-

tensity variance (2) of the pixels inside the triangle. Others may cover image texture [12, 1, 9], for example

- Autocorrelation function.
- Features derived from gray-level co-occurrence matrices.
- Local moments of the image function.
- Gabor wavelet filters.
- Local binary patterns [8].

and spatial configuration of adjacent triangles.

$$\mu(t_i) = \frac{1}{N} \sum_{x,y \in t_i} I(x,y) \quad (1)$$

$$\sigma(t_i) = \frac{1}{N} \sum_{x,y \in t_i} |\mu(t_i) - I(x,y)|^2 \quad (2)$$

In the sense of the feature vectors, definition of the region membership function can be reformulated in the following manner  $m(t_i, R_k) = p(\vec{f}_i | R_k)$ .

## 2.3 Iterative Adaptation

Fundamental part of the proposed segmentation method is the adaptation of the mesh to cover the underlying image representing anatomy of human tissues. Following four main steps are repeated until the triangulation and overall segmentation satisfies some convergence criterion.

1. Triangular mesh optimization.
2. Edge splitting.
3. Triangles splitting.
4. Triangles merging by means of edge reduction.
5. Re-estimation of the membership function.

**Triangular Mesh Optimization** First step of the iterative adaptation improves triangles quality with respect to the shape. Because thin triangles are ineffective for further processing, they have to be eliminated.

An ideal triangle, having the best quality, is equilateral. There are many measures of triangle quality regarding the ideal shape. The most general one is ratio of the longest triangle edge and the radius of its inscribed circle. In practice, the normalized form [6] is preferred. The ideal triangle has the normalized quality equal to 1. Any other triangle has the value greater.

First, the quality of all triangles is estimated. Then, triangles having the normalized quality below a given threshold are optimized. The new point in the center of circumscribed circle is added to the mesh. The effect of the optimization is shown in the Figure 5.

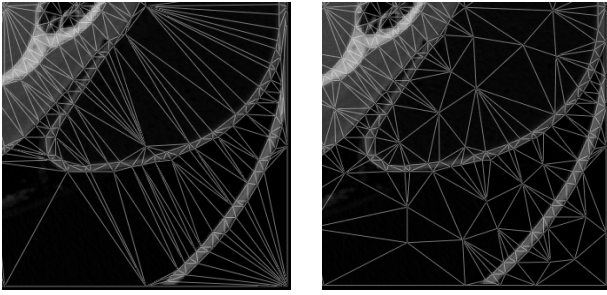


Figure 5. Result of the triangular mesh optimization (right) and the original mesh (left).

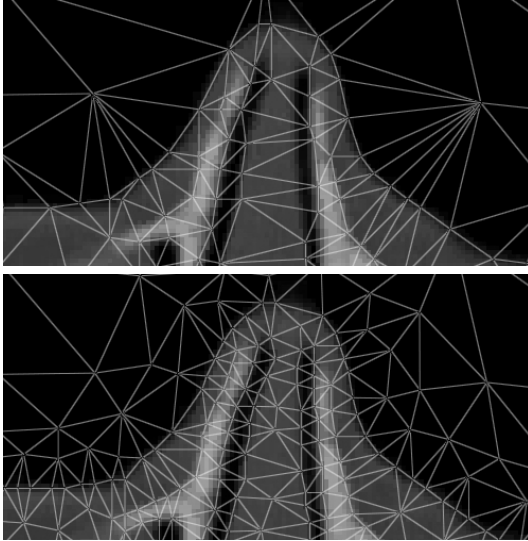


Figure 6. Original triangular mesh (left) and mesh after edge splitting and subsequent optimization.

**Edge Splitting** Another well known technique of triangular mesh optimization is edge splitting of maximal/longest edges [6]. Similar approach can be used to adjust triangular mesh to the image structure. Instead of maximal edges, those edges crossing significant image edges are divided. The new vertex is inserted to the mesh in the point of intersection of both edges.

To prevent over-segmentation, the angle between the triangle edge and the image edge is computed. The splitting operation is performed only if the angle is greater than a given threshold. Edges that are almost parallel with the image edge remain unchanged.

**Triangles Splitting** In this phase, non-homogeneous triangles are divided. First, the algorithm examines all triangles by computing the *homogeneity measures*. If the homogeneity predicate is false, edge pixels in  $t_i$  are classified based on their significance. Then, the homogeneity measures is used to evaluate an error. The edge point with the lowest error is taken and entered into the mesh. The splitting phase continues until all triangles satisfy the homogeneity

measure. Basic homogeneity measure is the mean intensity variance (3).

**Edge Reduction** This process consists of replacing an edge  $e_i$  with only one vertex  $A$  connected to all the vertices previously connected to the initial edge. The resulting point is located between endpoints of the original edge. The edge elimination leads in a simplification of the triangular mesh and reduction of the number of triangles. From a practical point of view, it is necessary to verify if all triangles resulting from reduction, and including the point  $A$  as a vertex, are valid.

Also, the homogeneity of all triangles sharing the edge is checked. Function  $S(t_j, t_i)$ , the *similarity measure*, provides the criterion by which adjacent triangles could be merged into one. If the value is below a given merge threshold for every possible pair of triangles, the edge can be eliminated. The basic similarity measure is mean intensity value (4).

$$H(t_i) = \exp(-\sigma(t_i)/2\rho^2) \quad (3)$$

$$S(t_j, t_i) = \exp(-\max_{t \in t_i} |\mu(t) - \mu(t_j)|^2 / 2\rho^2) \quad (4)$$

The parameter  $\rho$  affects the sensitivity of the measure. The variable  $\mu(t_j)$  is the mean value of all pixels within a given triangle  $t_j$  and  $\sigma(t_i)$  is the variance of the intensity. Both the  $\mu(t_j)$  and the  $\sigma(t_j)$  are components of the triangle feature vector.

Moreover, the reduction phase must preserve strong edges in the triangulation. Thus, if some adjacent triangles sharing the initial edge are joint by an image edge, the edge can not be eliminated.

The above mentioned measures are very simple acting as an example. More sophisticated ones have to be investigated.

## 2.4 Extension to 3D space

After all, the proposed 2D case of the segmentation algorithm is not suitable for 3D tissues modeling. Slice by slice segmentation of volume CT/MR data causes that a number of artifacts appear in the created model. Vectorization and smoothing of such 3D model is still necessary. Hence, the entire algorithm have to be extended to the 3D.

Such extension can be easily developed by replacing 2D triangle with 3D tetrahedron. Unfortunately, it is not trivial to extend many parts of the algorithm in practice. For example the edge and corner detection algorithms in 3D, algorithms for fast rasterization of tetrahedrons, 3D texture features detailing the tetrahedron structure and so on.

## 3 Results

The 2D segmentation algorithm was tested on real CT/MR data. The primary aim of the testing was to prove expected

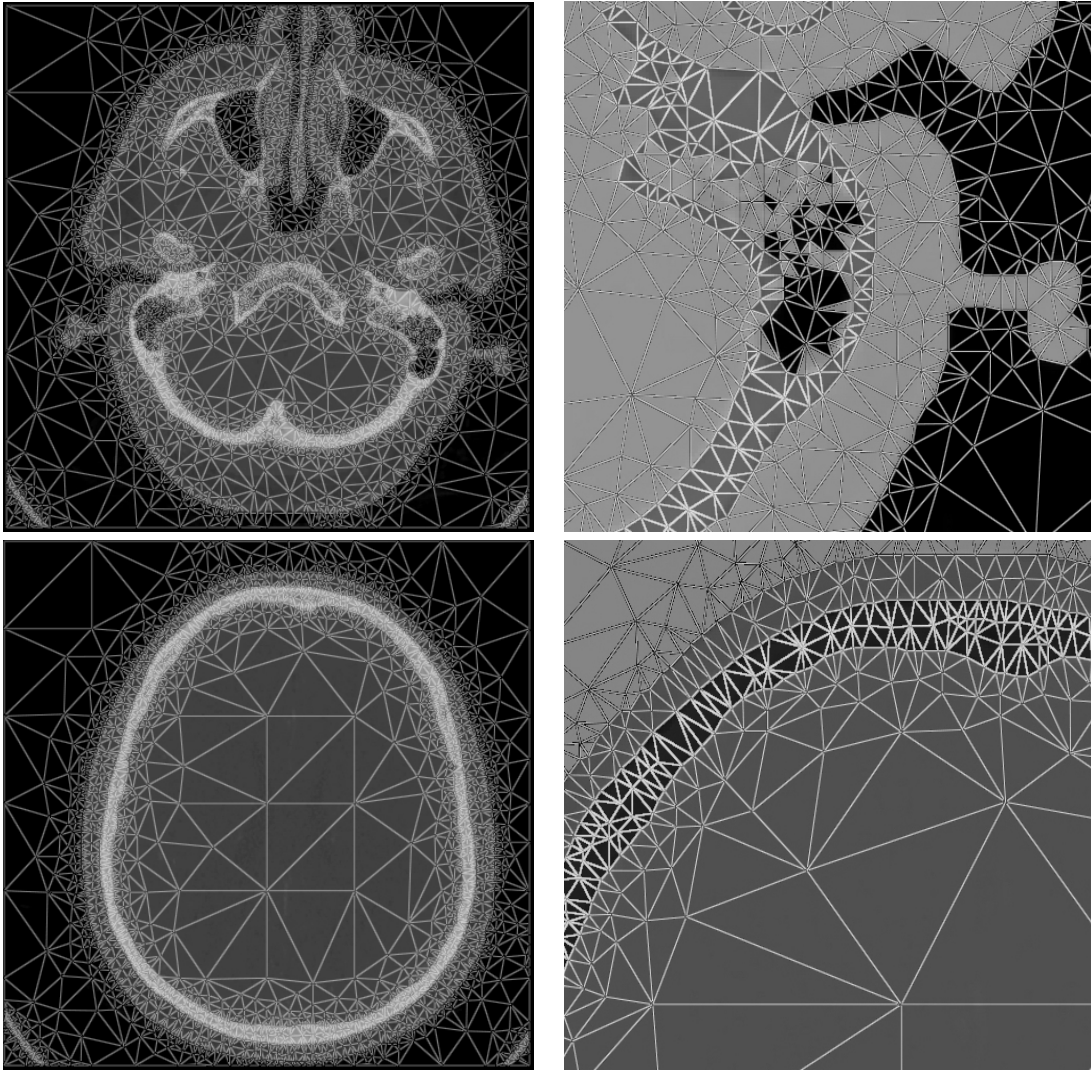


Figure 7. Triangular mesh after iterative adaptation (left) and detail of the final classification of triangles (right).

qualities of the segmentation algorithm before its extension to the 3D space.

In our experiments, two classifiers were designed for the clustering of feature vectors into image segments. First one was the *Fuzzy C-means* (FCM) algorithm [10] and second one the *Gaussian Mixture Model* (GMM) optimized by the well known *Expectation-Maximization* (EM) algorithm [7].

Approximately hundreds of CT slices having resolution mostly 512x512 pixels were segmented during the testing. Initialization of the triangular mesh, its adaptation and classification of triangles take approximately few seconds ( $< 10s$ ) on PC with P4 1.6GHz processor. Sample outputs of the segmentation looking only at the intensity mean value and variance are shown in Figure 7 and concluded in Table 1.

Sample	Initialized, $Q \leq 5$		Adapted, $Q \leq 2.5$	
	NoV <sup>1</sup>	NoT <sup>2</sup>	NoV	NoT
1	1481	2911	4206	6942
2*	1230	2387	3486	5764
3*	757	1479	2412	4143
4	682	1330	2356	4012
5	876	1690	2460	4343

<sup>1</sup> - Number of vertices. <sup>2</sup> - Number of triangles.  
\* - Sample is shown in Figure 7.

Table 1. Number of vertices and triangles in the mesh after initialization and adaptation.

## 4 Conclusion

The vector segmentation algorithm based on adaptive Delaunay triangulation is proposed in the paper. Triangular

mesh is used to divide an image into several disjoint regions whose characteristics are similar. Certain methods for improving quality of the mesh and its adaptation to the underlying image structure are also presented.

Direct vector representation of image regions makes possible to eliminate difficult process of raster data vectorization. The introduced two-dimensional case can be easily extended to the 3D space by replacing triangles with tetrahedrons. Using the 3D representation, the geometrical model can be created directly from the segmented vector data.

Simple modifications of the triangular mesh, such as adding new vertices, removing old ones, and manual reclassification of triangles, allow easy manual correction of the segmentation. The geometrical model and manual corrections can be made in parallel (and at the same time). All changes in the mesh take effect on the model without delay.

Finally, more effective representation of image structure is obtained which also approximates the raster data. This representation should decrease complexity of classification algorithms working with a reduced number of triangles instead of individual pixels.

## 5 Future Work

First of all, the algorithms have to be extended to the 3D space. This extension is necessary for more complex testing and also for 3D tissues modeling which is the main goal of the proposed segmentation technique.

Further, we would like to correct the misclassification by incorporating more sophisticated segmentation approaches modeling information on texture and spatial properties of particular image regions. Only basic features detailing the triangle structure were used.

## ACKNOWLEDGEMENTS

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