Fourth-order Time-stepping Scheme in Simulation of Ultrasound Propagation

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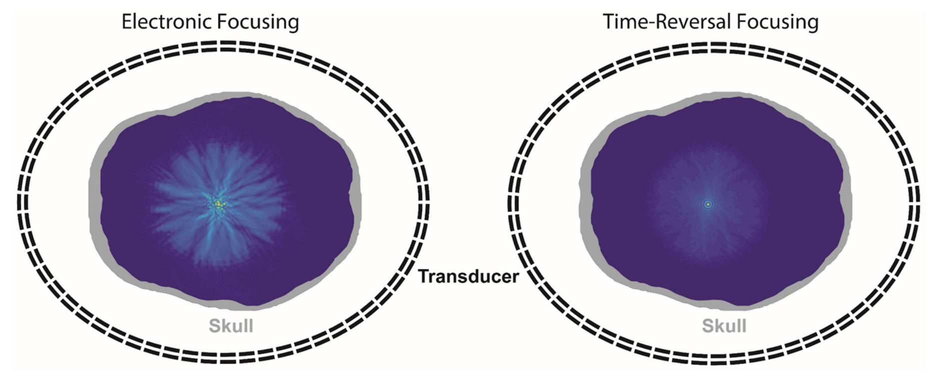
Overview

The simulation of elastic wave propagation has many applications in ultrasonics, including the classification of bone diseases and non-destructive testing. In biomedical ultrasound in particular, elastic wave models have been used to investigate the propagation of ultrasound in the skull and brain, and to optimize the delivery of therapeutic ultrasound through the thoracic cage.



Proposed Method

The numerical model is based on the explicit solution of coupled PDEs derived from an wave equation using the Fourier pseudospectral method. This uses the Fourier collocation spectral method to compute spatial derivatives, which are highly accurate provided the Nyquist–Shannon theorem is satisfied. However, to integrate in time the model uses a staggered grid version of second-order backward difference (LFS).

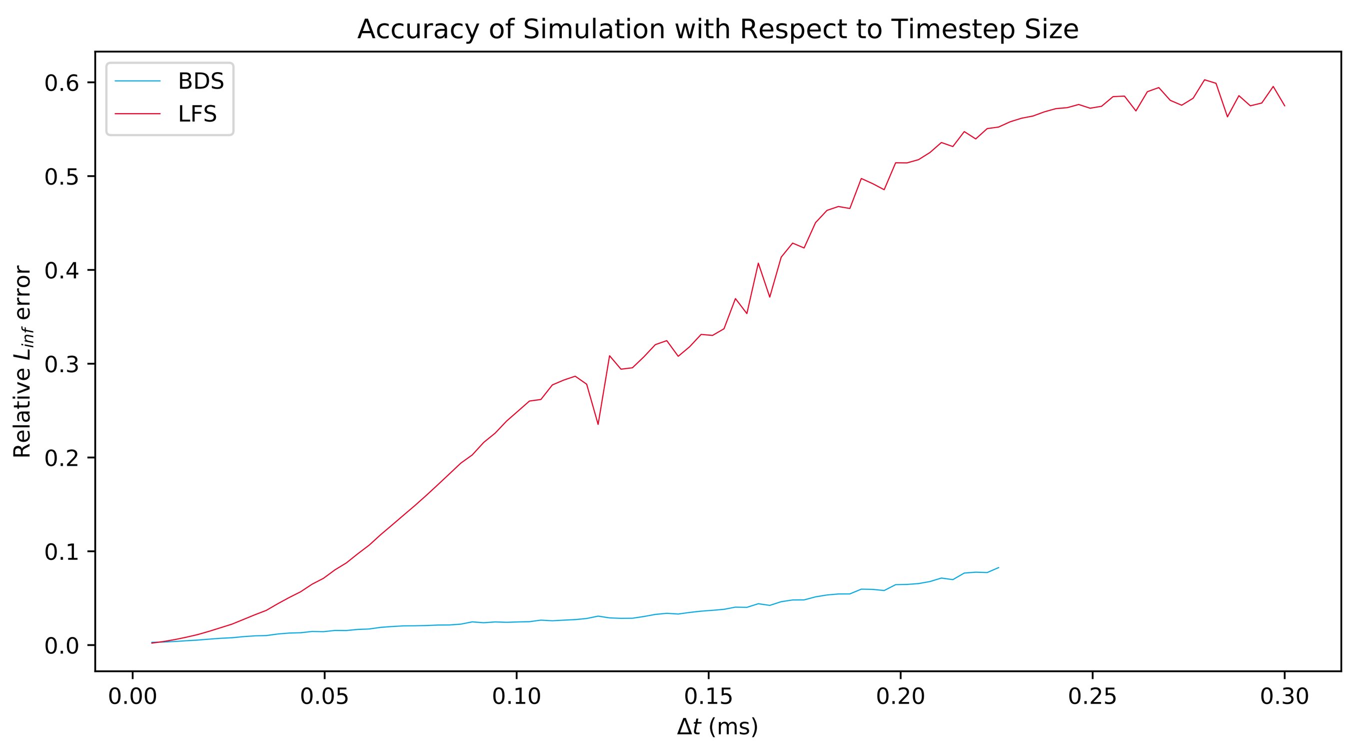
$$p^{n+1} = p^n + \Delta t \rho c^2 \mathbb{F}^{-1}\left(ike^{ik\Delta x/2}\mathbb{F}\left(u^{n+\frac{1}{2}}\right)\right)$$

We proposed that by using a higher order method (BDS), we would be able to increase the accuracy of the simulation, and therefore, reduce the time to solution within an error constraint.

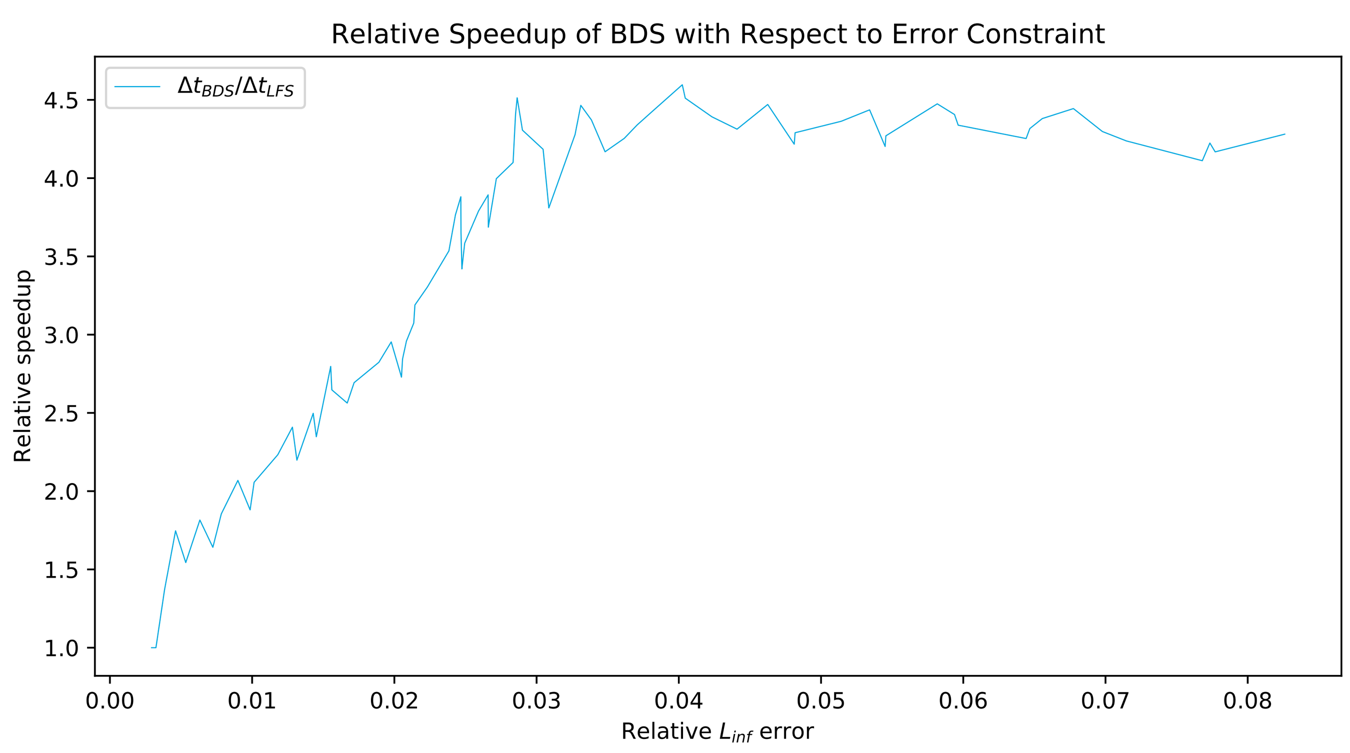
$$p^{n+1} = \frac{17}{22}p^n + \frac{9}{22}p^{n-1} - \frac{5}{22}p^{n-2} + \frac{1}{22}p^{n-3} + \frac{12}{11}\Delta t \rho c^2 \mathbb{F}^{-1}\left(ike^{ik\Delta x/2}\mathbb{F}\left(u^{n+\frac{1}{2}}\right)\right)$$

Convergence Comparison

The convergence of both integration schemes were investigated using a 1D linear, lossless and homogeneous simulation with increasing timesteps. The outcome of the simulation was then compared with the analytic solution to calculate the relative error.



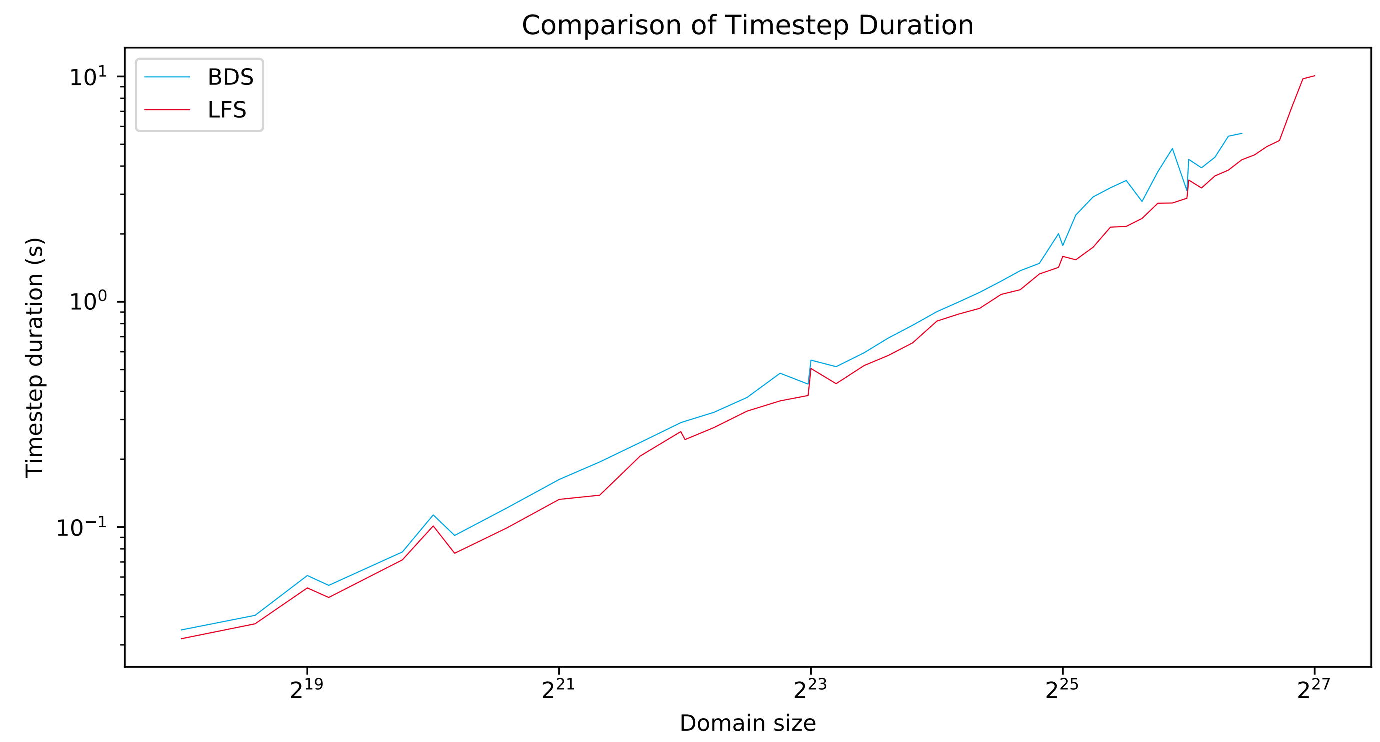
Afterwards, for various error constraint, an appropriate Δt was selected for both schemes and relative speedup was calculated as an reduction in number of timesteps needed to achieve t_{end}.



Speedup	Worst	Best	Average
	1	4.59	3.47

Performance Comparison

For the performance comparison, both code versions ran suite of simulation scenario with varying domain size. The blue line representing the BDS more or less copies behavior of the LFS method with only exception being values around 2^{25} where BDS's higher memory usage causes decrease in cache efficiency earlier on.

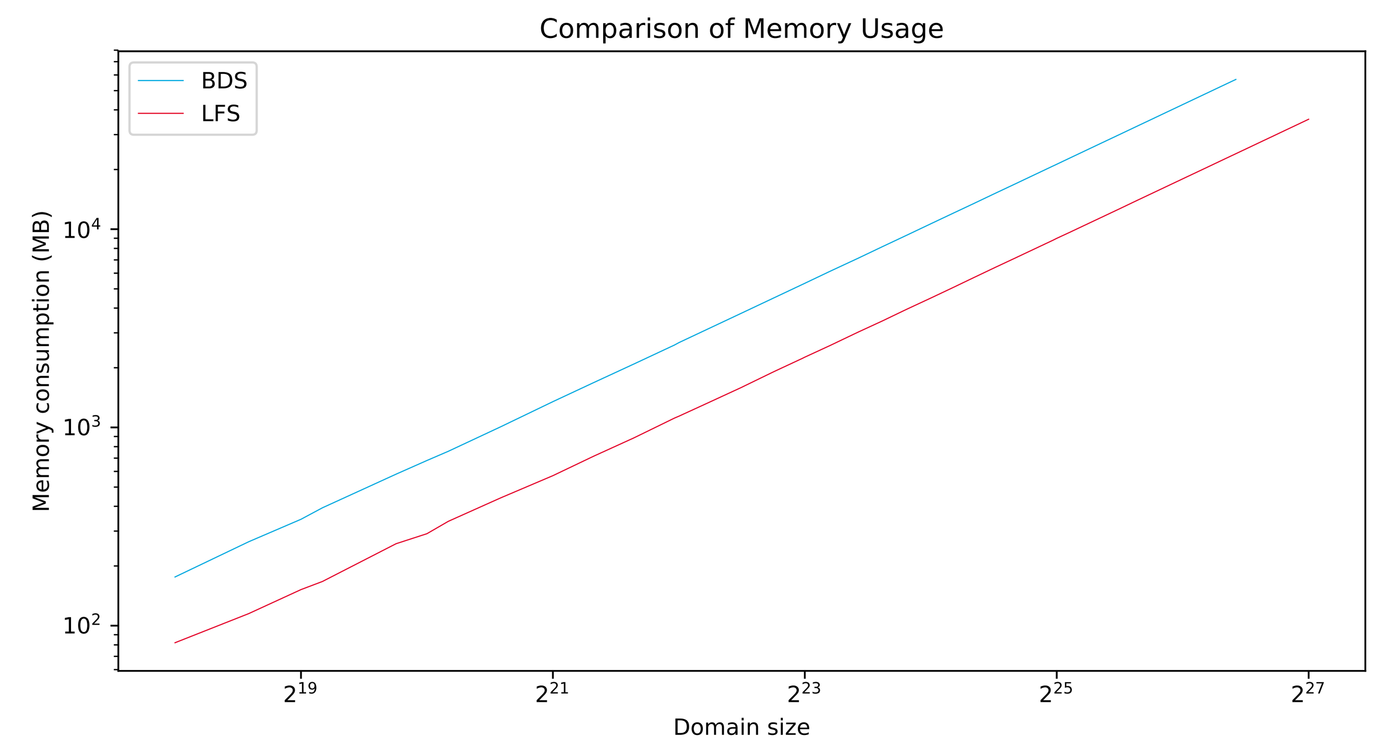


The overhead is calculated as a percentage of additional time required to compute one timestep using BDS taking LFS as a baseline

Overhead	Worst	Best	Average
	74.4%	8%	23.9%

Memory Usage

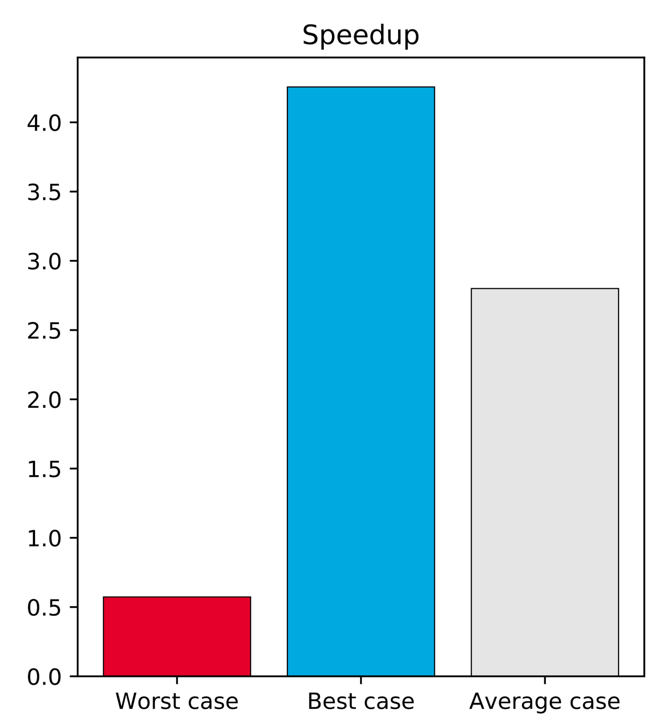
Memory usage was investigated on the same simulation suite. The plot shows that there is almost a constant memory overhead when using BDS which corresponds to the fact that 5 time more memory is required for pressure and particle speed quantities.

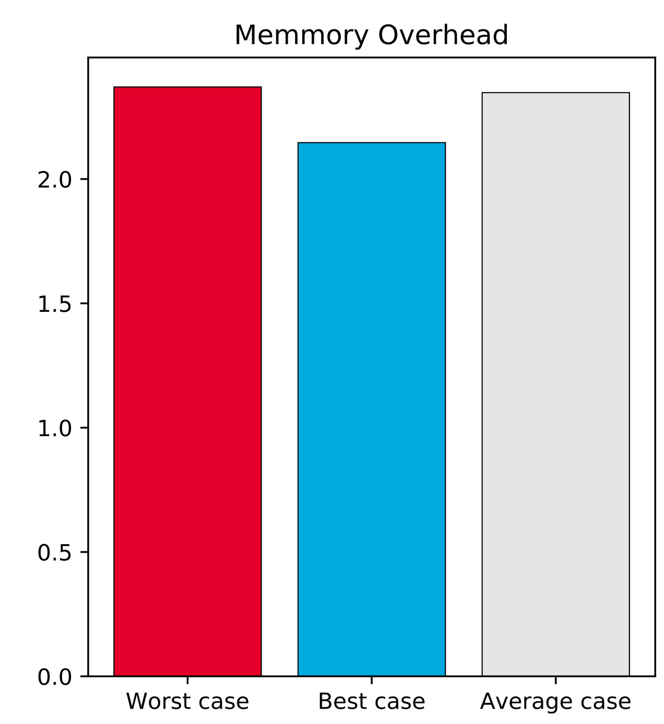


Overhead	Worst	Best	Average
	137%	114.6%	134.7%

Conclusion

According to measured values, the best, the worst and average case values for both speedup and memory overhead was calculated. The BDS can be used to either increase the precision or to speedup the simulation with some error constraint at considerable memory expense. The main class of simulation that can benefit from this method are relatively small, long running simulation which require high accuracy of solution.





k-Wave
A MATLAB toolbox for the time-domain simulation of acoustic wave fields

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