Single-step Calculation of the Acoustic Field from Arbitrary Continuous-wave Sources

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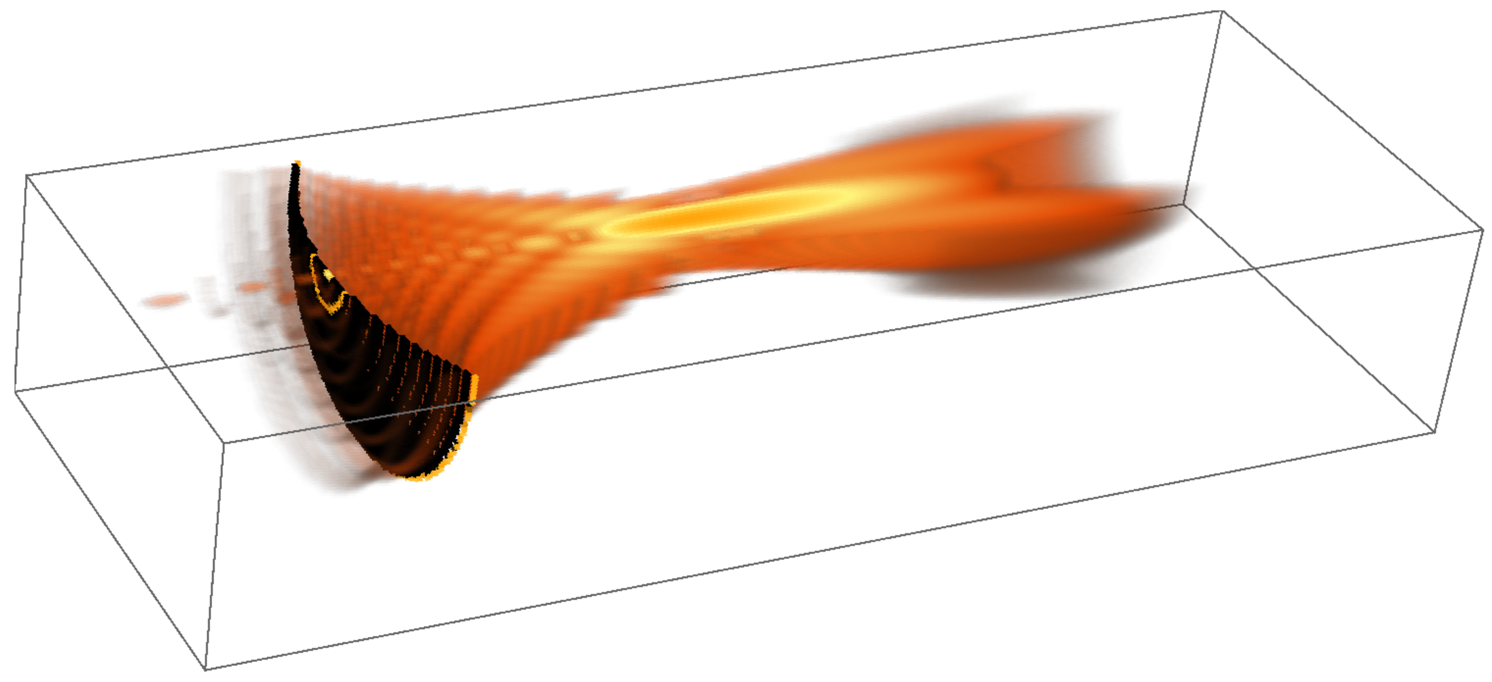
Overview

Existing semi-analytical methods of calculating acoustic field induced by phased array of transducers have several drawbacks – either the computation time is dependent on the number of grid points the pressure is evaluated for, or they impose a limitation on the direction the acoustic waves can propagate.

In case of a single driving frequency, it is possible to calculate pressure field in the whole domain in a single step. This approach is possible by expressing free-space Green's function for the wave equation in the spatial frequency domain, *k*-space.



Calculating Acoustic Field in a Single Step

Use of a conventional mass, single-frequency source term of the form

$$S(\mathbf{x},t) = \frac{\partial}{\partial t} A(\mathbf{x}) e^{i(\omega_0 t + \phi(\mathbf{x}))}$$

allows the Green's function time convolution integrals to be solved analytically, and remaining spatial integral expressed in a form of Fourier transform, yielding

$$p(\mathbf{x},t) = c_0^2 \mathscr{F}^{-1}\left\{ I(\mathbf{k},t) \mathscr{F}\left\{ A(\mathbf{x}) e^{i\phi(\mathbf{x})} \right\} \right\},$$

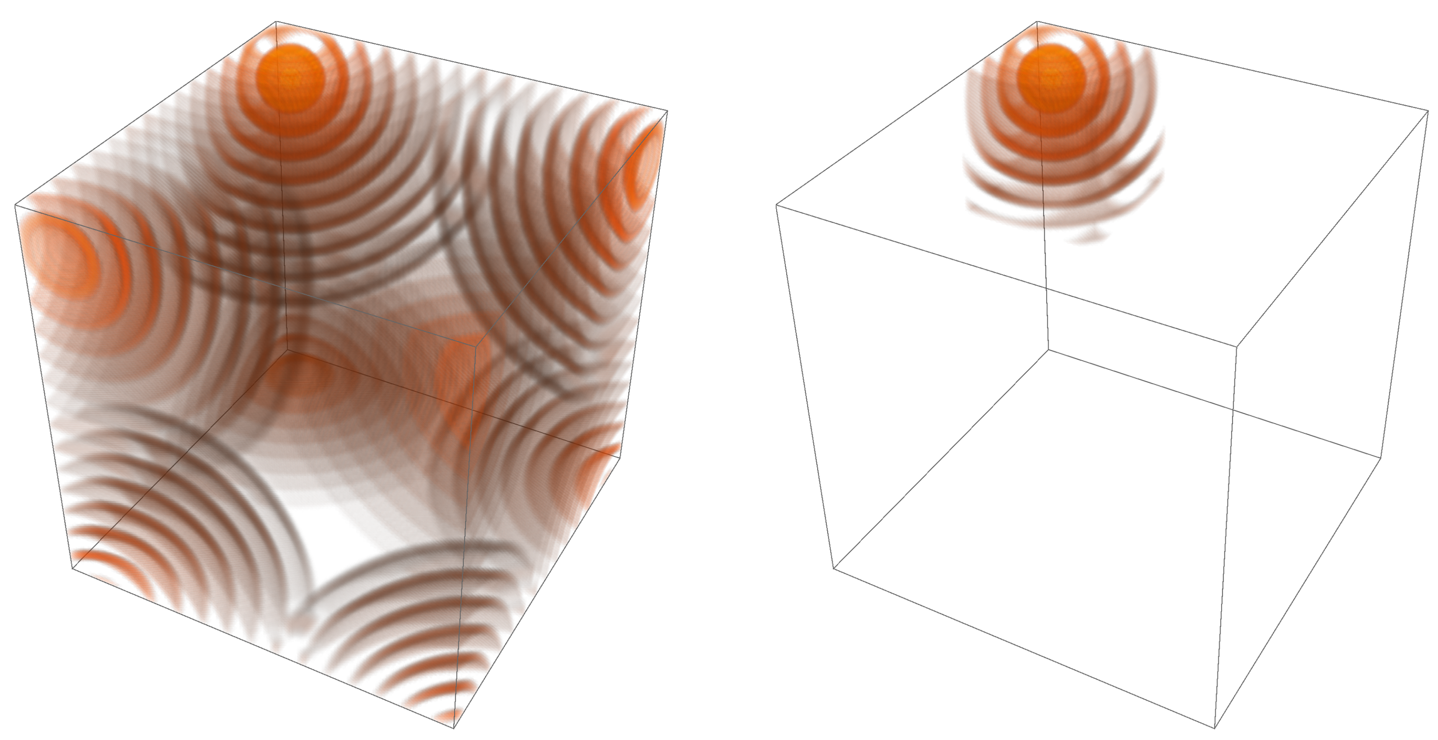
$$I(\mathbf{k},t) = \frac{i\omega_0 c_0 k \left(e^{i\omega_0 t} - \cos(c_0 k t) + \omega_0^2 \sin(c_0 k t) \right)}{(c_0 k)^3 - c_0 \omega_0^2 k} + \frac{\sin(c_0 k t)}{c_0 k}.$$

Time t needs to be sufficiently large for waves to propagate throughout the domain. As complex numbers are used to represent the sources, pressure amplitude and phase can be recovered from the resulting complex values. Therefore, only a single calculation is required to obtain amplitude, phase, and pressure value at any time in the whole computational domain.

Wave Wrapping

Fourier domain implies periodic boundary conditions. Waves that traverse through a border wrap around and appear on the other side of the domain. Since the calculation is done in a single time step, approaches using an attenuating layer (typically Perfectly Matched Layer) cannot be applied.

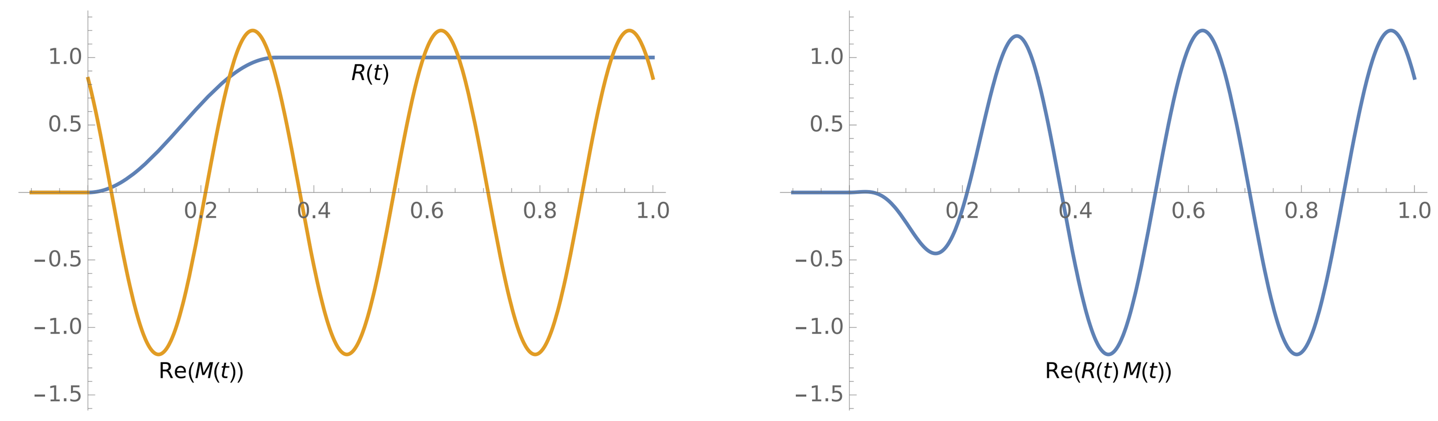
Instead, the computational domain is extended so that the waves cannot reach the domain of interest after traversing the boundary. At first, a minimum propagation time to cover the whole domain is calculated. Using this information, minimum domain expansion is derived.



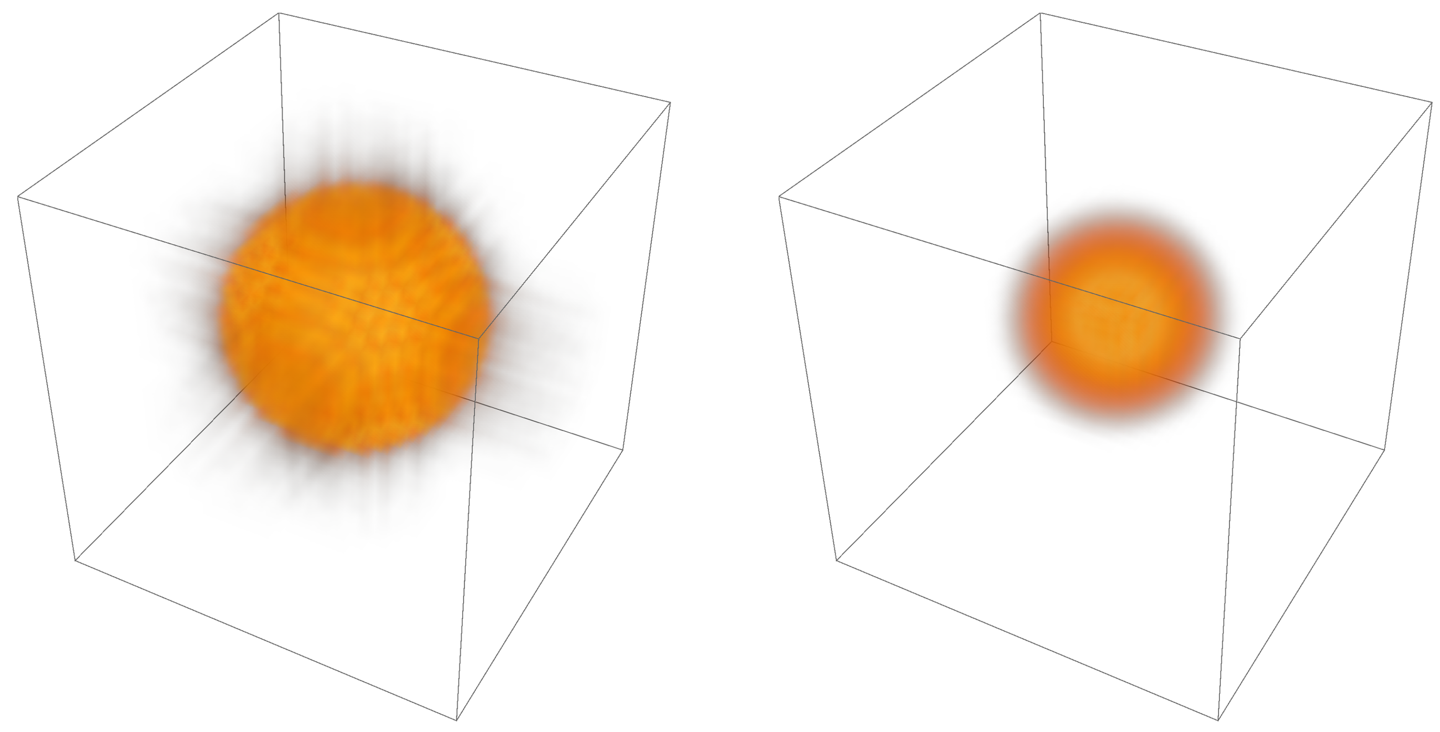
Fast Fourier transform performance is heavily dependent on the domain size. As a part of the expansion, sizes with a low prime factors are preferred over a generic ones, speeding up the computation.

Using a Start-up Ramp Function

The band-limiting imposed by the use of the discrete Fourier transform can lead to Gibbs oscillations in the calculated pressure field, particularly at the front edge of the wave where the pressure changes rapidly from zero. To avoid this, a cosine-based, time varying ramp function is added to the mass source term.



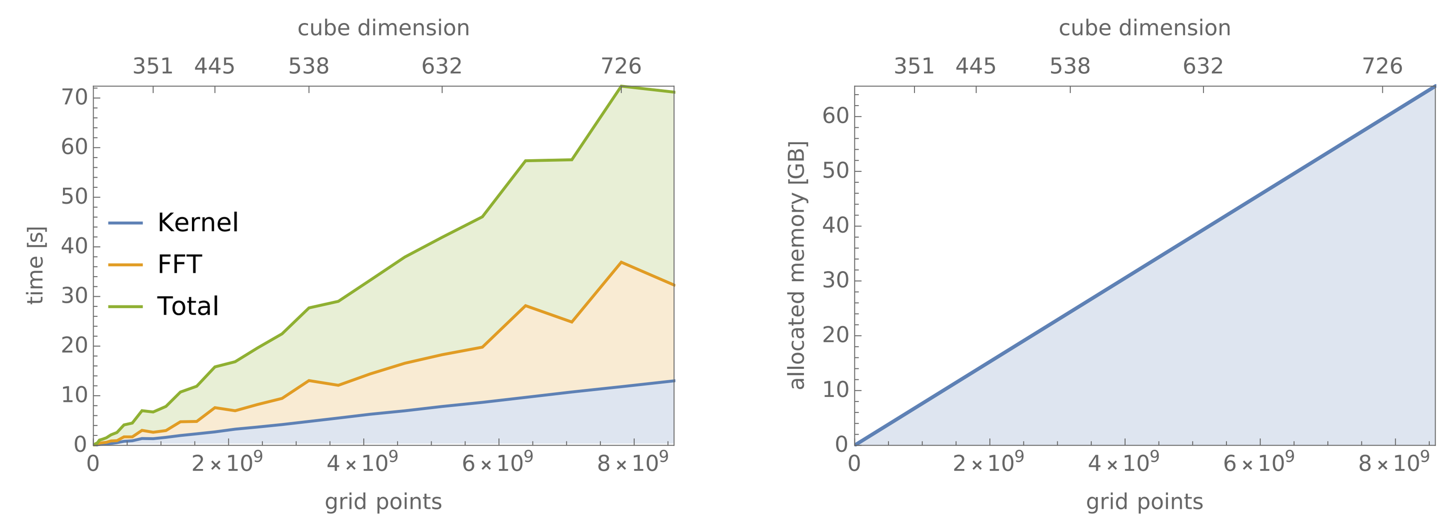
In the volumetric images below an amplitude is plotted in a case without and with a ramp function respectively. Even though the use of a ramp is smoothing out the wave front, the error visible in the image on the left is eliminated. To obtain correct results, propagation time needs to be slightly increased so that the wave front is not a part of the result.



Performance and Memory Usage

Method was implemented using C++ with a help of SIMD 4.0 directives for code vectorization. The Intel MKL library was used as a back-end to perform Fast Fourier Transform (FFT).

Performance was tested on a single node of Salomon supercomputer. Cubic extended domains with their size ranging form 64 to 2048 points were chosen for the testing. The graphs below contain the total amount of grid points and a maximum size of a cube that can be properly represented in the extended domain without the wave wrapping.



Spikes in the measured time can be seen where the extended size has a high prime factor and the performance of FFT suffers. Required memory scales linearly with the number of grid points, requiring only 2 single precision numbers per point.

Conclusions

Method for calculating steady-state pressure from a continuous wave source with an arbitrary distribution of amplitude and phase has been derived and implemented. It does not use time-stepping nor numerical quadrature and its precision is only dependent on spatial discretization and used data type.

Compared to semi-analytical approaches based on the spatial impulse response or Rayleigh integral, this method allows rapid calculation of the pressure field in large 3D domains. This is advantageous for modelling the output from non-planar transducer arrays, including hemispherical arrays used for transcranial ultrasound therapy.

This work was supported by the Engineering and Physical Sciences Research Council, UK, grant numbers EP/L020262/1, EP/M011119/1, and EP/P008860/1. This work was also supported by The Ministry of Education, Youth and Sports of the Czech Republic from the Large Infrastructures for Research, Experimental Development and Innovations project "IT4Innovations National Supercomputing Center – LM2015070", and the National Programme of Sustainability (NPU II) project "IT4Innovations Excellence in Science – LQ1602". This project has received funding from the European Union's Horizon 2020 research and innovation programme H2020 ICT 2016-2017 under grant agreement No 732411 and is an initiative of the Photonics Public Private Partnership.