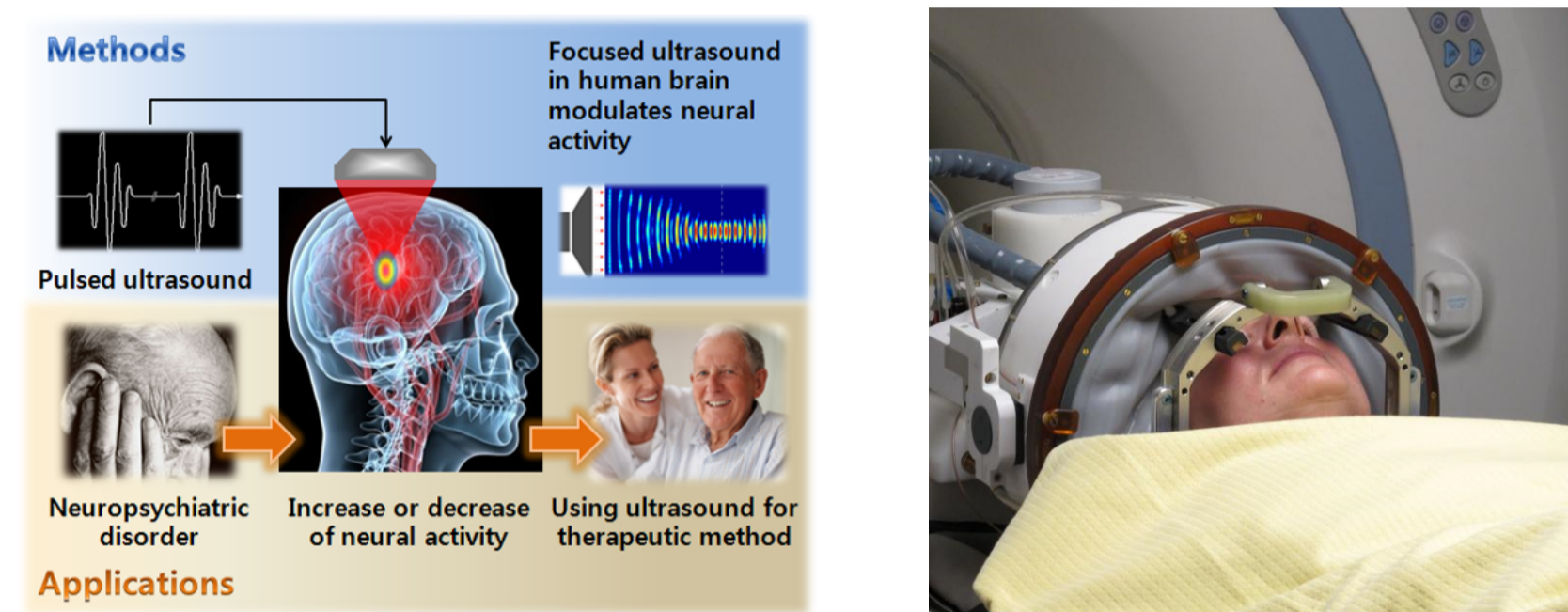


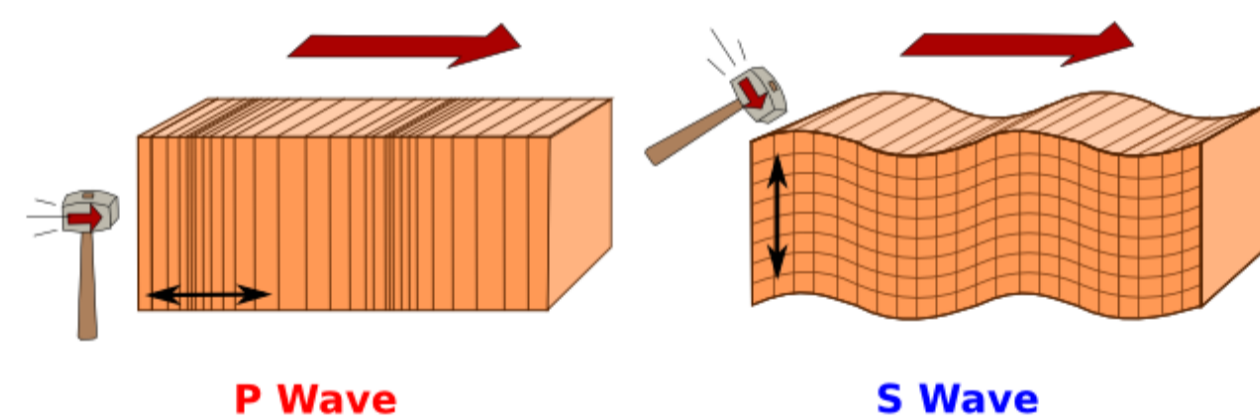
1 Overview

Simulation of elastic wave propagation has many applications in ultrasonics. Non-invasive, focal neurostimulation with ultrasound is a potentially powerful neuroscientific tool that requires effective transcranial focusing of ultrasound to be developed. Time-reversal focusing based on numerical simulations of transcranial ultrasound propagation can account for the effect of the skull but relies on accurate simulations.



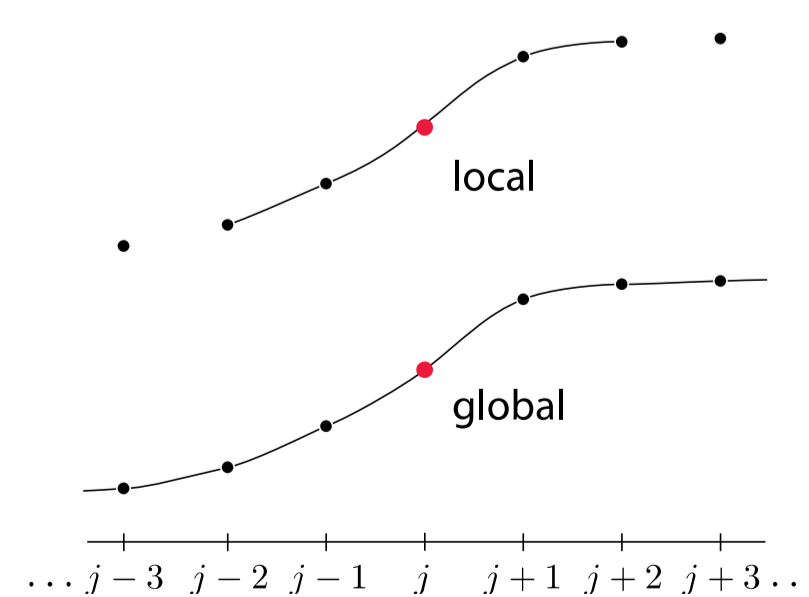
2 Elastic Wave Propagation in Bones and Skull

The governing equations must account for both compressional (P) and shear (S) waves in heterogeneous absorbing media.



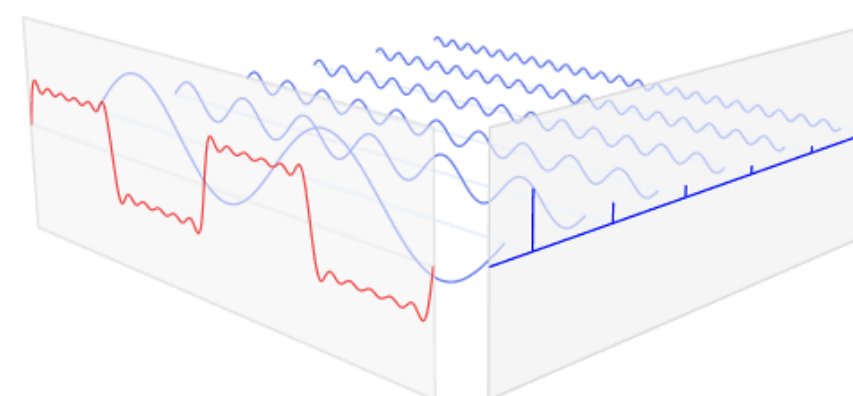
The numerical model is based on an explicit solution of coupled PDEs using the Fourier pseudospectral method. The model uses the Fourier collocation spectral method to compute spatial derivatives, and a leapfrog finite-difference scheme to integrate forward in time.

$$\partial_x \sigma_{xyz} = \mathbb{F}_x^{-1} \{ ik_x e^{+ik_x \Delta x/2} \mathbb{F}_x \{ \sigma_{xyz} \} \}$$



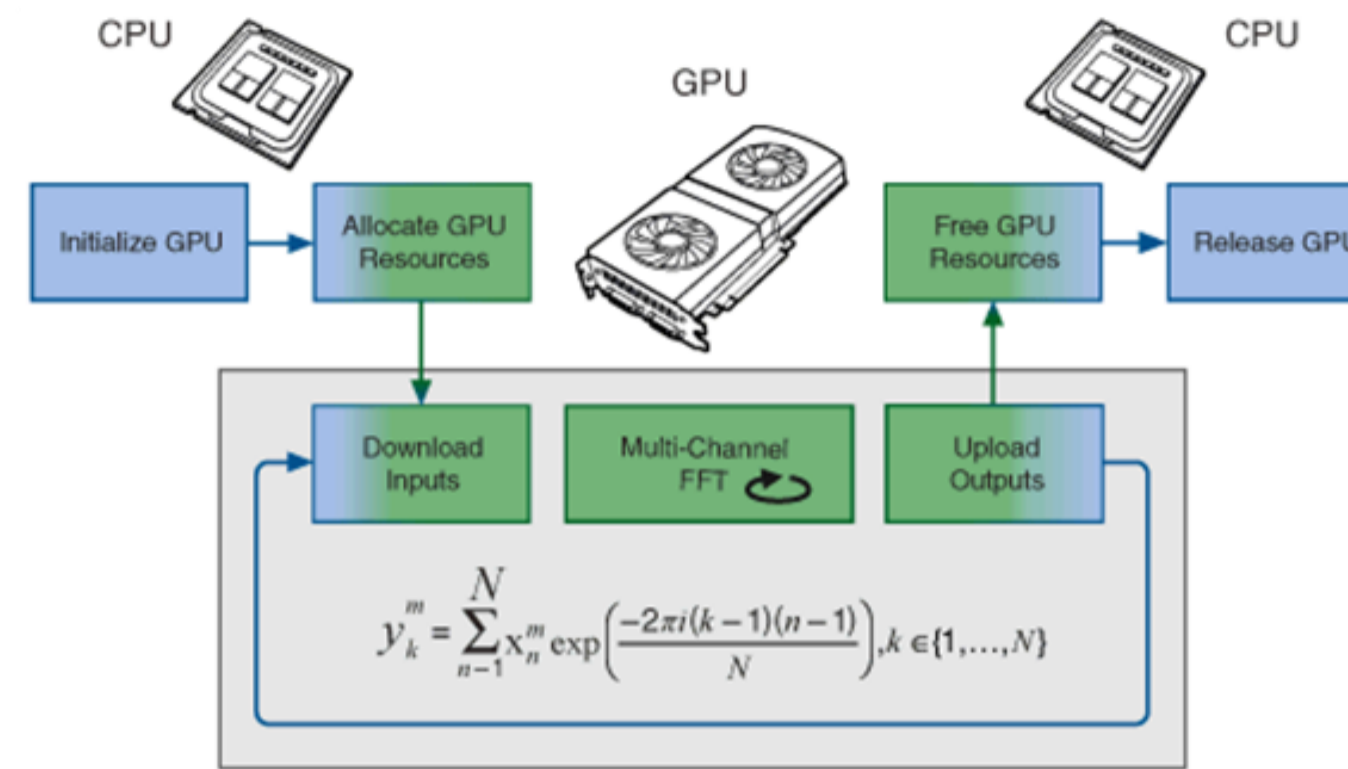
$$\frac{\partial f}{\partial x} \approx \frac{\frac{1}{12}f^{j-2} - \frac{2}{3}f^{j-1} + \frac{2}{3}f^{j+1} - \frac{1}{12}f^{j+2}}{\Delta x}$$

$$\frac{\partial f}{\partial x} \approx \mathbb{F}^{-1} \{ ik_x \mathbb{F} \{ f \} \}$$



3 Native CUDA application

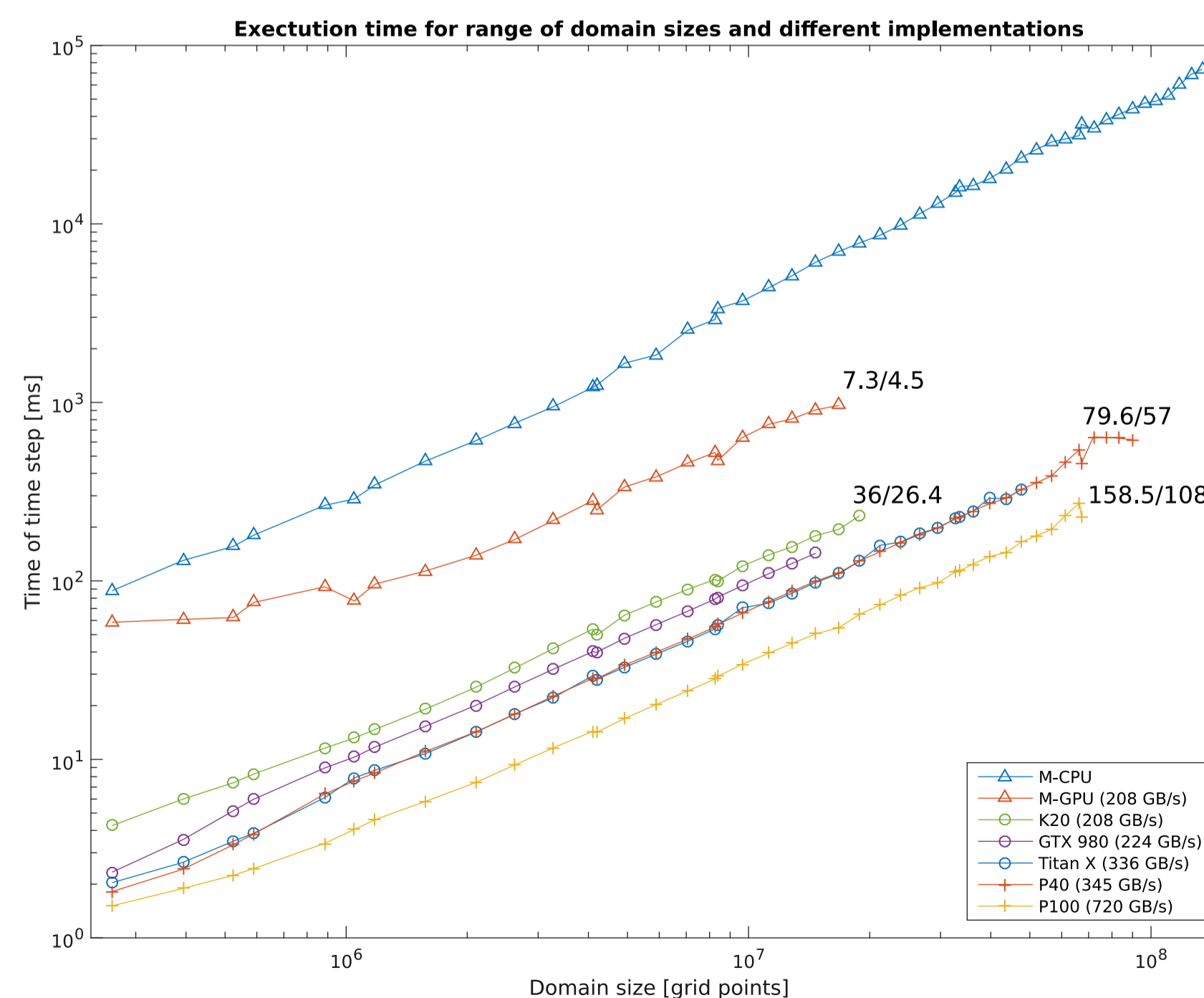
The native CUDA application performs all calculation on the GPU. The processor only loads data from the input HDF5 file, uploads it into GPU memory, samples output quantities and controls the simulation.



The code uses the cuFFT library to compute FFTs and a number of finely tuned CUDA kernels to calculate element-wise matrix operations.

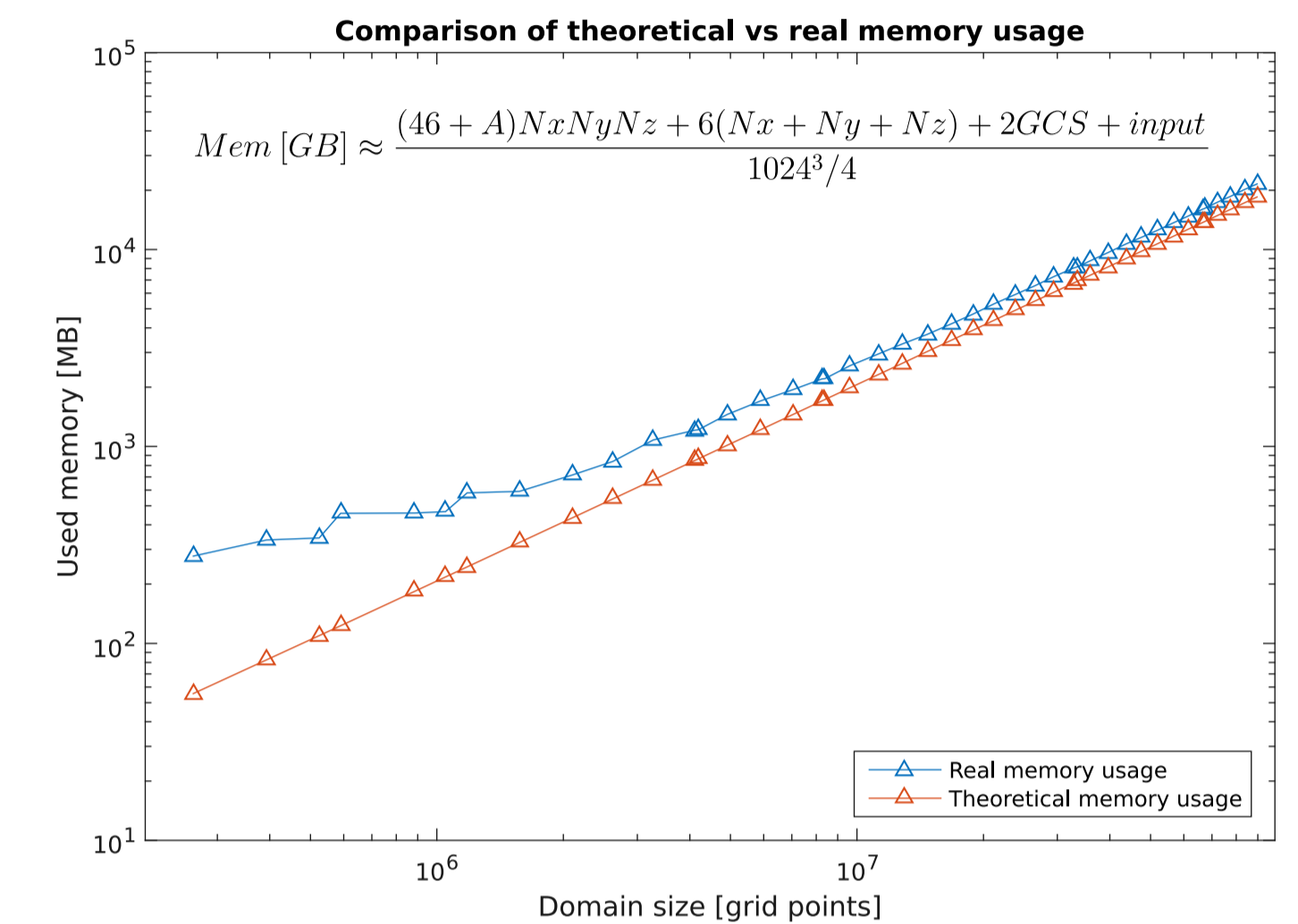
4 Performance Comparison

Performance evaluation was carried out on several GPUs with different domain sizes ranging between 64³ and 512³ grid points. The Matlab implementation was taken as reference. In the plot, the first two curves M-CPU and M-GPU stand for the Matlab code running on CPU and GPU using the Matlab Parallel Computing Toolbox, while the others represent our native CUDA application. The numbers at the curves represent the maximal and average speed-up with respect to the Matlab CPU code.



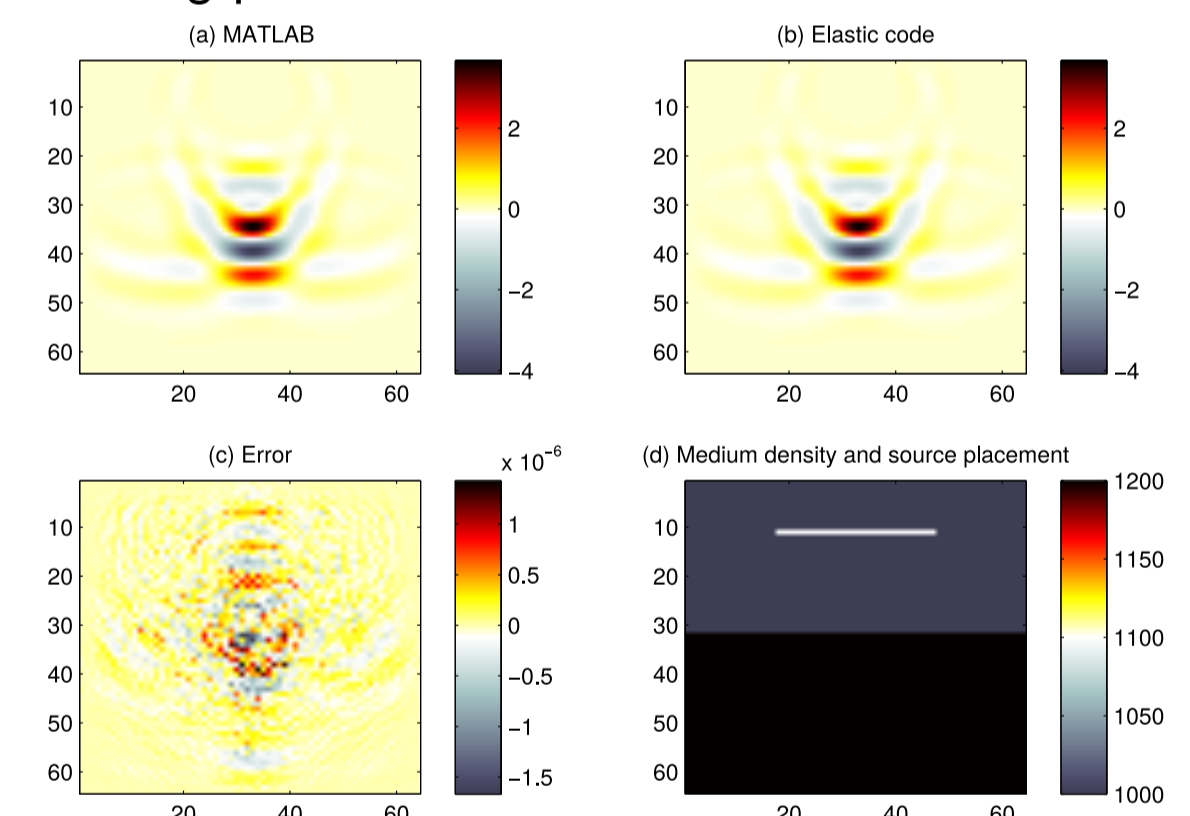
6 Memory Consumption

To be able to predict memory consumption of a certain simulation, a memory consumption model was created. In a figure below, there could be seen close agreement between measured and predicted memory consumption.



7 Accuracy of Implementation

The accuracy of implementation was tested against Matlab code and, as presented in error figure, the maximum relative error is around 1.5 x 10⁻⁶, which is bordering with the accuracy of single precision floating-point numbers.



8 Conclusions

The GPU implementation has both significantly decreased the simulation time and extended the range of computable simulation scenarios. For example, the elastic wave propagation simulation with 448³ grid points and 4,655 time steps can now be completed in 48 minutes, instead of several hours.

