One-Sided Random Context Grammars

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Outline

- **Introduction**
- **Motivation**
- **Definitions and Examples**
- **Results**
- **Open Problems**

One-sided random context grammars

- variant of a random context grammar
- $P = P_L \cup P_R$
- $\lfloor A \rightarrow x, U, W \rfloor \in P$

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$$\ldots\ldots \boxed{A} \xrightarrow{\ldots\ldots}$$

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Illustration

$\lfloor A \rightarrow x, \{B, C\}, \{D\} \rfloor \in P_L$

$bBcECbAcD$

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Illustration

$\lfloor A \rightarrow x, \{B, C\}, \{D\} \rfloor \in P_L$

$$\overleftarrow{bBcECb} \boxed{A} cD \Rightarrow bBcECb\, x\, cD$$

Motivation

- A natural generalization of left forbidding grammars and left permitting grammars.
- Theoretical viewpoint:
  - What is the impact of this restriction on the generative power of random context grammars?
  - The achieved results may be useful in the future when solving open problems.
- Practical viewpoint: possible applicability in practice.

Definition

A *one-sided random context grammar* is a quintuple

$$G = (N, T, P_L, P_R, S)$$

where

- N is the alphabet of *nonterminals*;
- T is the alphabet of *terminals*;
- P_L and P_R two are finite sets of *rules* of the form

$$\lfloor A \rightarrow x, U, W \rfloor$$

  where $A \in N$, $x \in (N \cup T)^*$, and $U, W \subseteq N$;
- S is the *starting nonterminal*.

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  where $A \in N$, $x \in (N \cup T)^*$, and $U, W \subseteq N$;
- S is the *starting nonterminal*.

Definition

If $\lfloor A \rightarrow x, U, W \rfloor \in P_L \cup P_R$ implies that $|x| \geq 1$, then G is *propagating*.

Definition

The *direct derivation relation*, denoted by $\Rightarrow$, is defined as

$$uAv \Rightarrow uxv$$

if and only if

$$\lfloor A \rightarrow x, U, W \rfloor \in P_L, U \subseteq \text{alph}(u), \text{ and } W \cap \text{alph}(u) = \emptyset$$

or

$$\lfloor A \rightarrow x, U, W \rfloor \in P_R, U \subseteq \text{alph}(v), \text{ and } W \cap \text{alph}(v) = \emptyset$$

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Definition

The *language of G*, denoted by $L(G)$, is defined as

$$L(G) = \{w \in T^* \mid S \Rightarrow^* w\}$$

where $\Rightarrow^*$ is the reflexive-transitive closure of $\Rightarrow$.

Example

Consider the one-sided random context grammar

$$G = (\{S, A, B, \bar{A}, \bar{B}\}, \{a, b, c\}, P_L, P_R, S)$$

where P_L contains

$$\lfloor S \rightarrow AB, \emptyset, \emptyset \rfloor \qquad \lfloor \bar{B} \rightarrow B, \{A\}, \emptyset \rfloor$$
$$\lfloor B \rightarrow b\bar{B}c, \{\bar{A}\}, \emptyset \rfloor \qquad \lfloor B \rightarrow \varepsilon, \emptyset, \{A, \bar{A}\} \rfloor$$

and P_R contains

$$\lfloor A \rightarrow a\bar{A}, \{B\}, \emptyset \rfloor \qquad \lfloor A \rightarrow \varepsilon, \{B\}, \emptyset \rfloor$$
$$\lfloor \bar{A} \rightarrow A, \{\bar{B}\}, \emptyset \rfloor$$

$$L(G) = \{a^n b^n c^n \mid n \geq 0\}$$

Denotation of Language Families

- $\mathscr{L}_{CF}$ ... the family of context-free languages
- $\mathscr{L}_{CS}$ ... the family of context-sensitive languages
- $\mathscr{L}_{RE}$ ... the family of recursively enumerable languages

- $\mathscr{L}_{RC}$ ... the family of random context languages
- $\mathscr{L}_{RC}^{-\varepsilon}$ ... the family of propagating random context languages

- $\mathscr{L}_{ORC}$ ... the family of one-sided random context languages
- $\mathscr{L}_{ORC}^{-\varepsilon}$ ... the family of propagating one-sided random context languages

Random Context Grammars:

Theorem

$\mathscr{L}_{CF} \subset \mathscr{L}_{RC}^{-\varepsilon} \subset \mathscr{L}_{CS} \subset \mathscr{L}_{RC} = \mathscr{L}_{RE}$

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One-Sided Random Context Grammars:

Theorem

$\mathscr{L}_{ORC}^{-\varepsilon} = \mathscr{L}_{CS}$ and $\mathscr{L}_{ORC} = \mathscr{L}_{RE}$

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Theorem

$\mathscr{L}_{ORC}^{-\varepsilon} = \mathscr{L}_{CS}$ and $\mathscr{L}_{ORC} = \mathscr{L}_{RE}$

Corollary

$\mathscr{L}_{RC}^{-\varepsilon} \subset \mathscr{L}_{ORC}^{-\varepsilon} \subset \mathscr{L}_{RC} = \mathscr{L}_{ORC}$

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If $\lfloor A \rightarrow x, U, W \rfloor \in P_L \cup P_R$ implies that $W = \emptyset$, then G is a *one-sided permitting grammar*.

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- $\mathscr{L}_{OPer}$ . . . the family of one-sided permitting languages
- $\mathscr{L}_{OPer}^{-\varepsilon}$ . . . the family of propagating one-sided permitting languages

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Theorem

$\mathscr{L}_{CF} \subset \mathscr{L}_{OPer}^{-\varepsilon} \subseteq \mathscr{L}_{SC}^{-\varepsilon} \subseteq \mathscr{L}_{CS}$ ○

- $\mathscr{L}_{SC}^{-\varepsilon}$ . . . the family of propagating scattered context languages

Definition

If $\lfloor A \rightarrow x, U, W \rfloor \in P_L \cup P_R$ implies that $U = \emptyset$, then G is a *one-sided forbidding grammar*.

One-Sided Forbidding Grammars

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If $\lfloor A \rightarrow x, U, W \rfloor \in P_L \cup P_R$ implies that $U = \emptyset$, then G is a *one-sided forbidding grammar*.

- $\mathscr{L}_{OFor}$ ... the family of one-sided forbidding languages
- $\mathscr{L}_{OFor}^{-\varepsilon}$ ... the family of propagating one-sided forbidding languages

One-Sided Forbidding Grammars

Definition

If $\lfloor A \rightarrow x, U, W \rfloor \in P_L \cup P_R$ implies that $U = \emptyset$, then G is a *one-sided forbidding grammar*.

- $\mathscr{L}_{\mathrm{OFor}}$ . . . the family of one-sided forbidding languages
- $\mathscr{L}_{\mathrm{OFor}}^{-\varepsilon}$ . . . the family of propagating one-sided forbidding languages

Theorem

$\mathscr{L}_{\mathrm{OFor}}^{-\varepsilon} = \mathscr{L}_{S}^{-\varepsilon}$ *and* $\mathscr{L}_{\mathrm{OFor}} = \mathscr{L}_{S}$

- $\mathscr{L}_{S}$ . . . the family of languages generated by selective substitution grammars
- $\mathscr{L}_{S}^{-\varepsilon}$ . . . the family of languages generated by propagating selective substitution grammars

Definition

If $P_R = \emptyset$, then G is a *left random context grammar*.

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- $\mathscr{L}_{LRC}$ . . . the family of left random context languages
- $\mathscr{L}_{LRC}^{-\varepsilon}$ . . . the family of propagating left random context languages

Definition

If $P_R = \emptyset$, then G is a *left random context grammar*.

- $\mathscr{L}_{LRC}$ ... the family of left random context languages
- $\mathscr{L}_{LRC}^{-\varepsilon}$ ... the family of propagating left random context languages

Open Problem

What is the generative power of left random context grammars?

Definition

A one-sided forbidding grammar G with $P_R = \emptyset$ is a *left forbidding grammar*.

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- $\mathscr{L}_{LFor}$ . . . the family of left forbidding languages
- $\mathscr{L}_{LFor}^{-\varepsilon}$ . . . the family of propagating left forbidding languages

Definition

A one-sided forbidding grammar G with $P_R = \emptyset$ is a *left forbidding grammar*.

- $\mathscr{L}_{LFor}$ . . . the family of left forbidding languages
- $\mathscr{L}_{LFor}^{-\varepsilon}$ . . . the family of propagating left forbidding languages

Theorem

$\mathscr{L}_{LFor} = \mathscr{L}_{LFor}^{-\varepsilon} = \mathscr{L}_{CF}$ ∘

Definition

A one-sided permitting grammar G with $P_R = \emptyset$ is a *left permitting grammar*.

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- $\mathscr{L}_{LPer}$ ... the family of left permitting languages
- $\mathscr{L}_{LPer}^{-\varepsilon}$ ... the family of propagating left permitting languages

Left Permitting Grammars

Definition

A one-sided permitting grammar G with $P_R = \emptyset$ is a *left permitting grammar*.

- $\mathscr{L}_{LPer}$ ... the family of left permitting languages
- $\mathscr{L}_{LPer}^{-\varepsilon}$ ... the family of propagating left permitting languages

Theorem

$\mathscr{L}_{CF} \subset \mathscr{L}_{LPer}^{-\varepsilon} \subseteq \mathscr{L}_{SC}^{-\varepsilon} \subseteq \mathscr{L}_{CS}$

Some Other Interesting Properties

Theorem

Every one-sided random context grammar can be turned into an equivalent one-sided random context grammar satisfying

$$P_L = P_R$$

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Every one-sided random context grammar can be turned into an equivalent one-sided random context grammar satisfying

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Theorem

Every one-sided random context grammar can be turned into an equivalent one-sided random context grammar having at most 10 nonterminals.

Further Properties and Modifications

- other descriptional complexity results
- normal forms
- leftmost derivations
- generalized one-sided forbidding grammars
- LL one-sided random context grammars
- one-sided ET0L systems

Main Open Problems

- What is the generative power of left random context grammars?
- Are the inclusions $\mathscr{L}_{OPer}^{-\varepsilon} \subseteq \mathscr{L}_{SC}^{-\varepsilon}$ and $\mathscr{L}_{LPer}^{-\varepsilon} \subseteq \mathscr{L}_{SC}^{-\varepsilon}$, in fact, proper?
- Can one-sided forbidding grammars generate every recursively enumerable language?

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Discussion