On (Extended) Szilard Languages

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Outline

- **Preliminaries**
- **Part I: Introduction to (Extended) Szilard Languages**
- **Part II: New Results**
- **Concluding Remarks**

Definition

A *context-free grammar* is a quintuple

$$G = (N, T, \Psi, P, S),$$

where

- N is an alphabet of *nonterminals*;
- T is an alphabet of *terminals* $(N \cap T = \emptyset)$;
- Ψ is a set of *rule labels* $(\mathrm{card}(\Psi) = \mathrm{card}(P))$;
- P is a finite set of *rules* of the form

  $$A \rightarrow x,$$

  where $A \in N$ and $x \in (N \cup T)^*$;
- $S \in N$ is the *starting nonterminal*.

Definition

The relation of a *direct derivation*, denoted by $\Rightarrow$, is defined as follows: if

- $u, v \in (N \cup T)^*$,
- $r\colon A \rightarrow x \in P$,

then

$$uAv \Rightarrow uxv\ [r] \text{ in } G.$$

Definition

The *language generated by* G, denoted by $L(G)$, is defined as

$$L(G) = \left\{ w \in T^* \mid S \Rightarrow^* w\ [\varrho],\ \varrho \in \Psi^* \right\},$$

where $\Rightarrow^*$ is the reflexive and transitive closure of $\Rightarrow$.

Definition

Let $G = (N, T, \Psi, P, S)$ be a context-free grammar.
The *Szilard language* associated to G is defined as

$$Sz(G) = \left\{ \varrho \in \Psi^* \mid S \Rightarrow^* w\ [\varrho],\ w \in L(G) \right\}.$$

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Definition

Let $G = (N, T, \Psi, P, S)$ be a context-free grammar.
The *extended Szilard language* associated to G is defined as

$$ESz(G) = \{w\varrho \in T^*\Psi^* \mid S \Rightarrow^* w\ [\varrho],\ w \in L(G)\}.$$

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- What about *leftmost* Szilard languages?
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- Are there any context-free languages, for which there is no context-free grammar G with context-free $Sz(G)$?

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- Pure grammars, L/P systems, CD/PC grammar systems, etc.
- Applications.
- Recognition and tape complexity ($\mathbf{NC}^1$).

Definition

A *regular-controlled (context-free) grammar* is a pair

$$H = (G, \Xi),$$

where

- $G = (N, T, \Psi, P, S)$ is a context-free grammar;
- $\Xi \subseteq \Psi^*$ is a regular *control language*.

Definition

The *language generated by* H, $L(H)$, is defined as

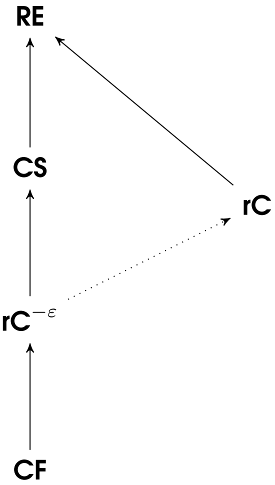
$$L(H) = \{w \in T^* \mid S \Rightarrow^* w\ [\varrho] \text{ with } \varrho \in \Xi\}$$

Example

Let $H = (G, \Xi)$ be a regular-controlled grammar, where P contains the following rules:

$$1\colon S \rightarrow ABC, \quad 5\colon A \rightarrow \varepsilon,$$
$$2\colon A \rightarrow aA, \quad 6\colon B \rightarrow \varepsilon,$$
$$3\colon B \rightarrow bB, \quad 7\colon C \rightarrow \varepsilon,$$
$$4\colon C \rightarrow cC,$$

and $\Xi = \{1\}\{234\}^*\{567\}$.


RE
CS
rC
$rC^{-\varepsilon}$
CF

New Results

Theorem

Let $H = (G, \Xi)$ be a regular-controlled grammar. Then, there is a propagating regular-controlled grammar

$$H' = (G', \Xi'),$$

where $G' = (N', T, \Psi', P', S')$, such that

$$L(H) = \{w \mid w\varrho \in L(H'), S' \Rightarrow^* w\ [\varrho] \text{ in } G'\}.$$

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Theorem

Let K be a recursively enumerable language. Then, there is a propagating regular-controlled grammar with appearance checking

$$H = (G, \Xi, F),$$

where $G = (N, T, \Psi, P, S)$, *such that*

$$K = \{w \mid w\varrho \in L(H), S \Rightarrow^* w\ [\varrho]\ in\ G\}.$$

Concluding Remarks

- Quotients.

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Theorem

Let K be a recursively enumerable language, and let \$ be a symbol such that $\$ \notin \text{alph}(K)$. Then, there is a propagating regular-controlled grammar with appearance checking, H, such that

$$K = L(H) \mathbin{/\!\!/} \{\$\}^+.$$

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- Speed of derivations.
- Coincidental extension.
- Parse trees.

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Discussion