# On Nondeterminism in Programmed Grammars

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# Outline

- **Preliminaries and Introduction**
- **Part I: Degree of Nondeterminism**
- **Part II: Number of Nondeterministic Rules**
- **Part III: Overall Nondeterminism**
- **Concluding Remarks and Open Problems**

# Programmed Grammars – Definition

## Definition

A *programmed grammar* is a quintuple

$$G = (N, T, S, \Psi, P),$$

where

- $N$ is an alphabet of *nonterminals*;
- $T$ is an alphabet of *terminals* ($N \cap T = \emptyset$);
- $S \in N$ is the *starting nonterminal*;
- $\Psi$ is an alphabet of *rule labels*;
- $P$ is a finite set of *rules* of the form

  $$(r\colon A \rightarrow x, \sigma_r),$$

  where $r \in \Psi$, $A \in N$, $x \in (N \cup T)^*$, and $\sigma_r \subseteq \Psi$.

# Programmed Grammars – Definition

### Definition

The relation of a *direct derivation*, symbolically denoted by $\Rightarrow$, is defined over $(N \cup T)^* \times \Psi$ as follows:

$$(u, r) \Rightarrow (v, s)$$

if and only if

$$u = u_1 A u_2,\ v = u_1 x u_2,\ (r\colon A \to x, \sigma_r) \in P, \text{ and } s \in \sigma_r.$$

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The *language generated by G*, $L(G)$, is defined as

$$L(G) = \{w \in T^* \mid (S, r) \Rightarrow^* (w, s), \text{ for some } r, s \in \Psi\}.$$

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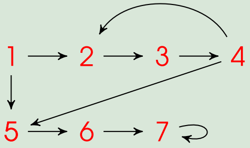
**P** . . . the family of languages generated by programmed grammars

# Programmed Grammars – Example

## Example

$$
\begin{aligned}
&(1\colon S \rightarrow ABC, \{2,5\})\\
&(2\colon A \rightarrow aA, \{3\})\\
&(3\colon B \rightarrow bB, \{4\})\\
&(4\colon C \rightarrow cC, \{2,5\})\\
&(5\colon A \rightarrow a, \{6\})\\
&(6\colon B \rightarrow b, \{7\})\\
&(7\colon C \rightarrow c, \{7\})
\end{aligned}
$$

$$
\begin{aligned}
(S, 1) &\Rightarrow (ABC, 2)\\
&\Rightarrow (aABC, 3)\\
&\Rightarrow (aAbBC, 4)\\
&\Rightarrow (aAbBcC, 5)\\
&\Rightarrow (aabBcC, 6)\\
&\Rightarrow (aabbcC, 7)\\
&\Rightarrow (aabbcc, 7)
\end{aligned}
$$




$$L(G) = \{a^n b^n c^n \mid n \geq 1\}$$

**Definition**

Let $G = (N, T, S, \Psi, P)$ be a programmed grammar. $G$ is of *degree of nondeterminism n*, where $n \geq 1$, if every $(r\colon A \to x, \sigma_r) \in P$ satisfies

$$\mathrm{card}(\sigma_r) \leq n.$$

By $\mathrm{dnd}(G)$, we denote the degree of nondeterminism of $G$.

$\mathbf{DND}(\mathbf{P}, n)$ . . . the family of languages generated by programmed grammars of degree of nondeterminism $n$

**Question**

What happens if we limit the degree of nondeterminism?

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**Theorem**

$\mathbf{DND}(\mathbf{P}, 1) = \mathbf{FIN}$

**FIN** . . . the family of finite languages

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**Theorem**

$\mathbf{DND}(\mathbf{P}, 2) = \mathbf{P}$

# Part II: Number of Nondeterministic Rules

**Question**

What happens if we limit the number of nondeterministic rules?

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${}_n\mathbf{P}$ . . . the family of languages generated by programmed grammars with $n$ nondeterministic rules

**Question**

What happens if we limit the number of nondeterministic rules?

${}_{n}\mathbf{P}$ ... the family of languages generated by programmed grammars with $n$ nondeterministic rules

**Theorem**

${}_{1}\mathbf{P} = \mathbf{P}$

## Definition

Let $G = (N, T, S, \Psi, P)$ be a programmed grammar. For each $(r\colon A \to x, \sigma_r) \in P$, let $\zeta(r)$ be defined as

$$\zeta(r) = \begin{cases} \mathrm{card}(\sigma_r) & \text{if } \mathrm{card}(\sigma_r) \geq 2 \\ 0 & \text{otherwise.} \end{cases}$$

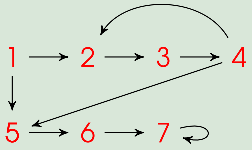
The *overall nondeterminism of G* is denoted by $\mathrm{ond}(G)$ and defined as

$$\mathrm{ond}(G) = \sum_{r \in \Psi} \zeta(r).$$

$\mathbf{OND}(\mathbf{P}, n)$ ... the family of languages generated by programmed grammars with overall nondeterminism $n$

**Example**

$(1\colon S \to ABC, \{2,5\})$
$(2\colon A \to aA, \{3\})$
$(3\colon B \to bB, \{4\})$
$(4\colon C \to cC, \{2,5\})$
$(5\colon A \to a, \{6\})$
$(6\colon B \to b, \{7\})$
$(7\colon C \to c, \{7\})$




**ond($G$)**

ond($G$) $= 4$

# Part III: Overall Nondeterminism

**Question**

What happens if we limit the overall nondeterminism?

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**Theorem**

$\mathbf{OND}(\mathbf{P}, n) \subset \mathbf{OND}(\mathbf{P}, n+1)$

### Open Problems

- Appearance checking?
- Propagating programmed grammars?

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# Discussion