Jumping Finite Automata

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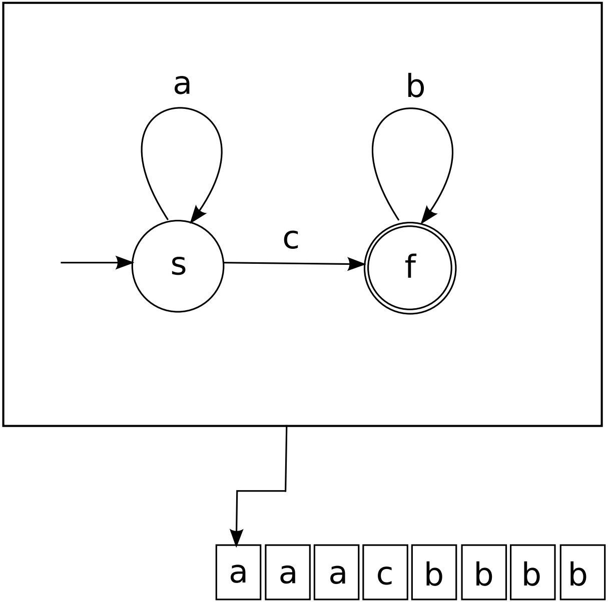
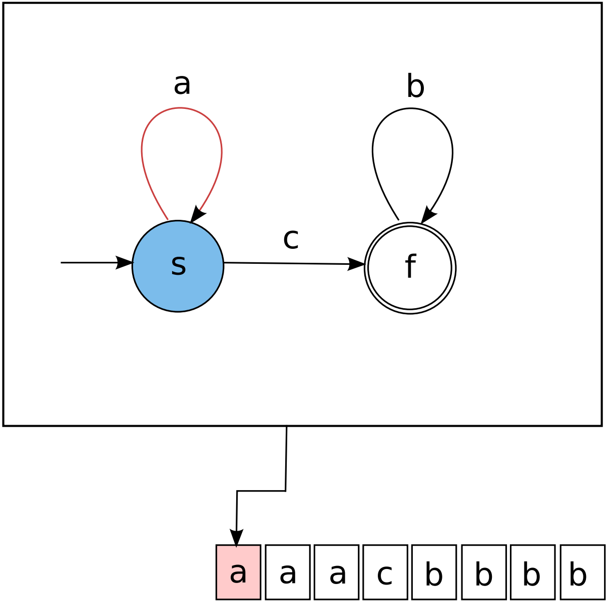
Finite Automata

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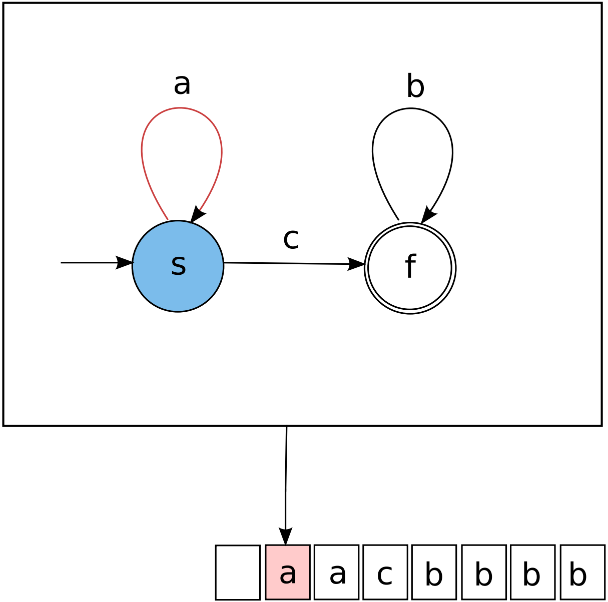
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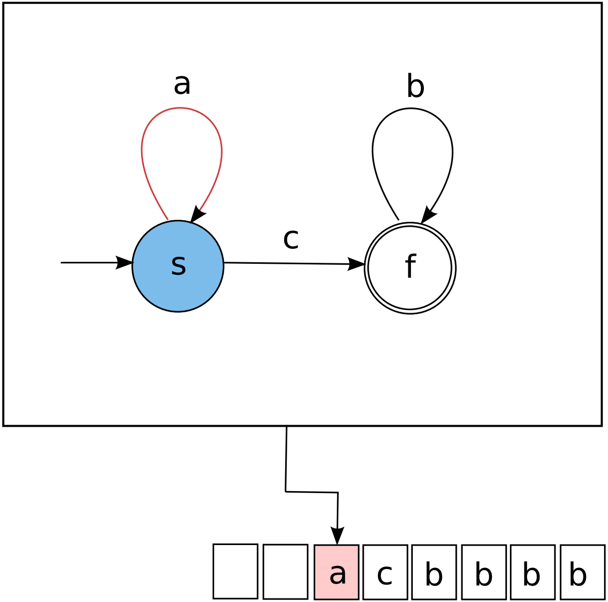
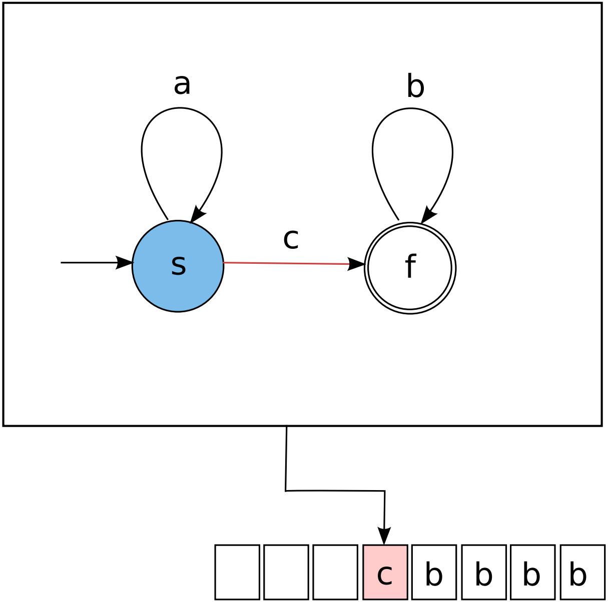
Finite Automata

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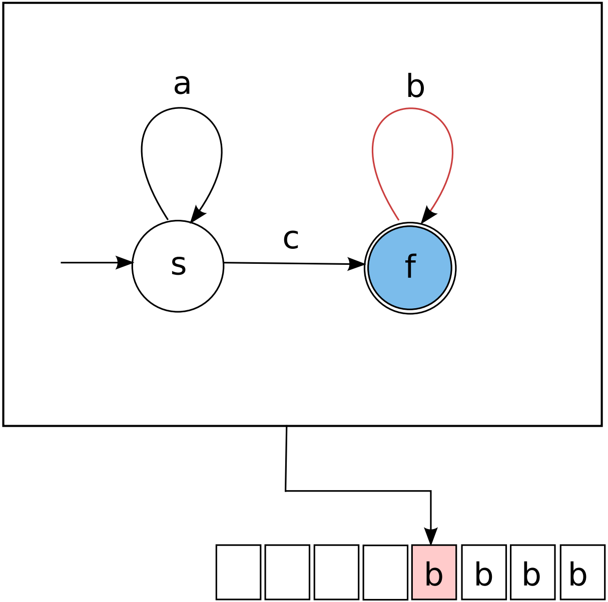
Finite Automata



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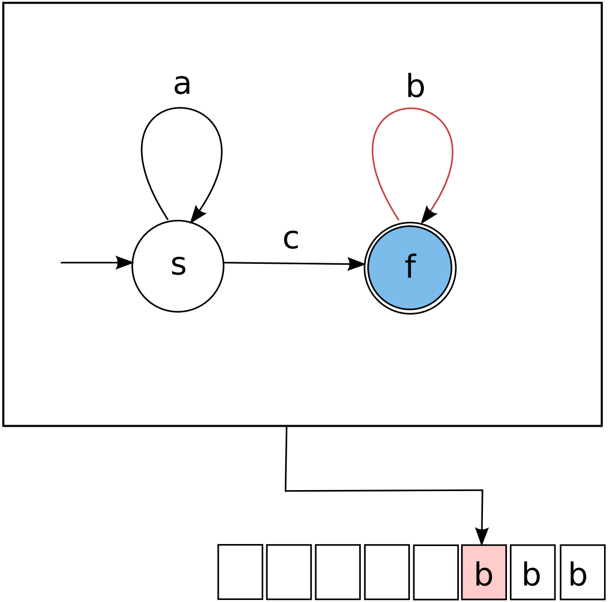

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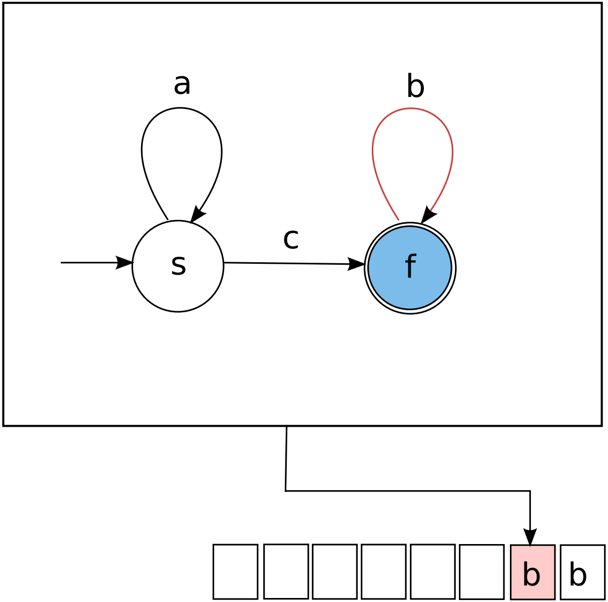

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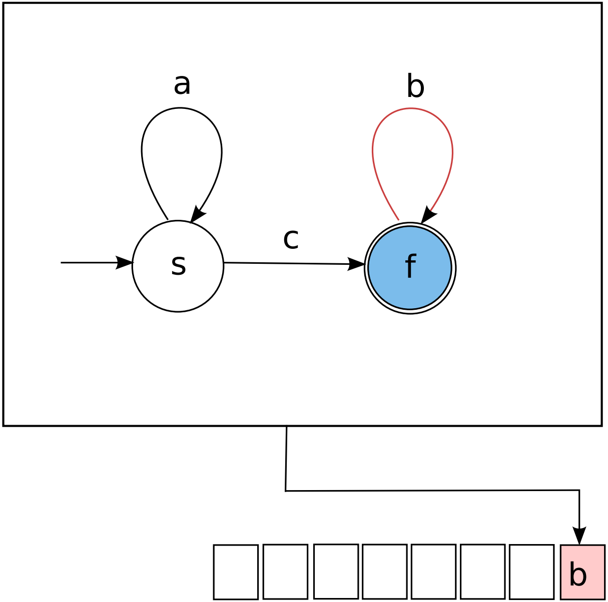

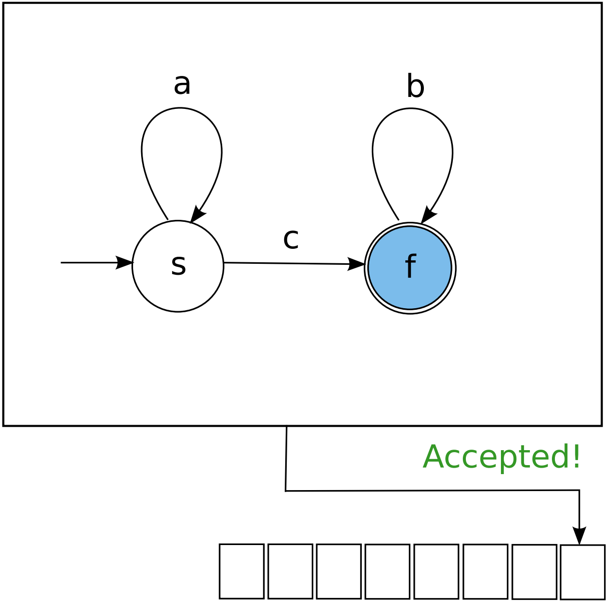
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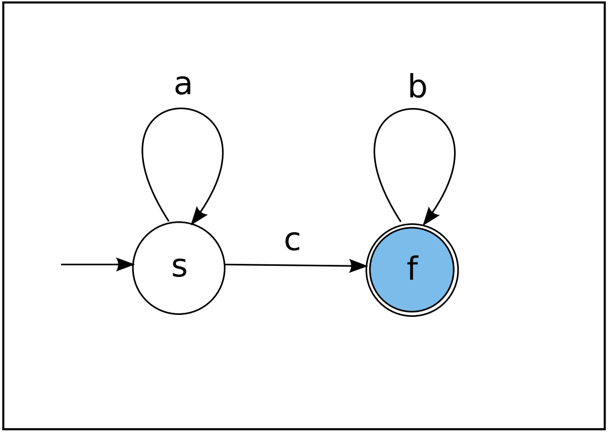
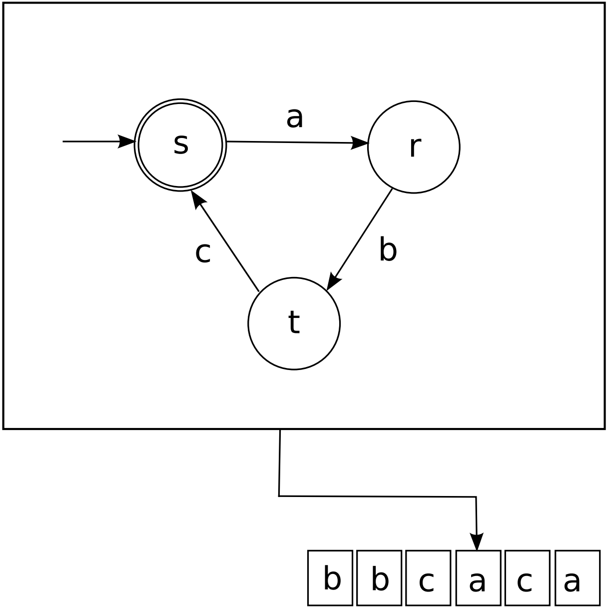
Accepted!



Accepted language: $\{a\}^*\{c\}\{b\}^*$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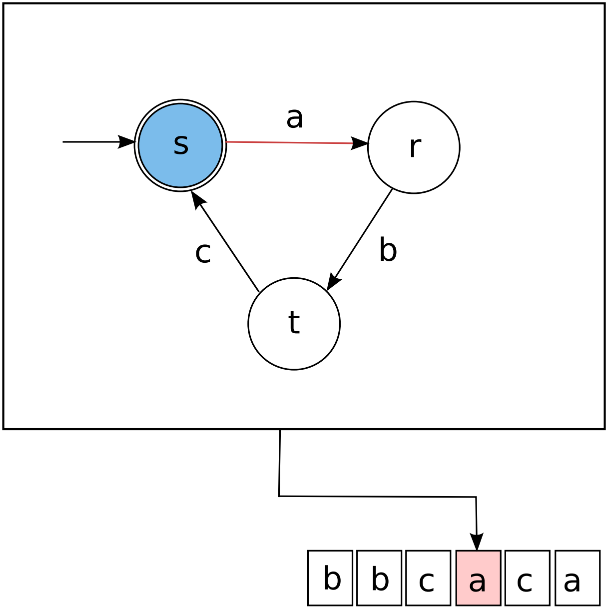
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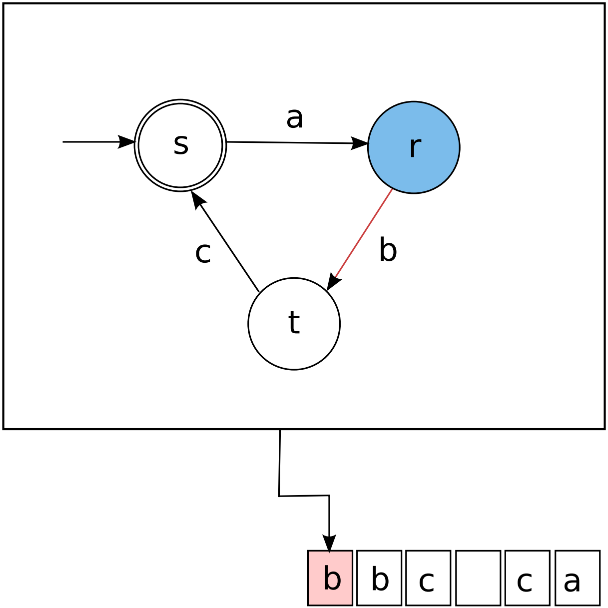
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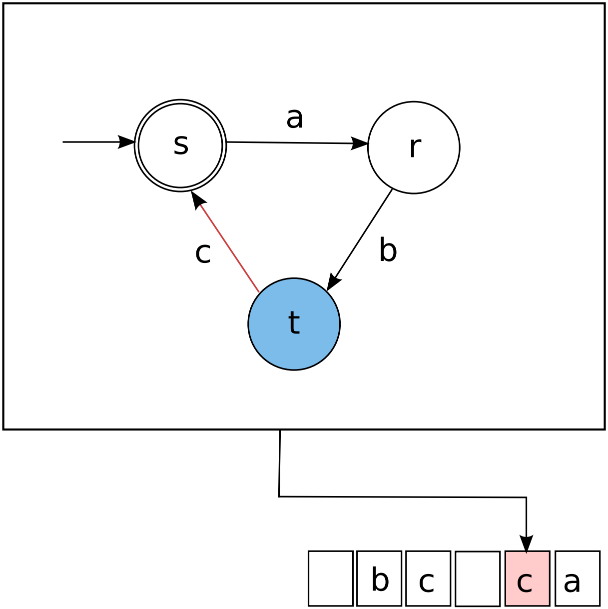
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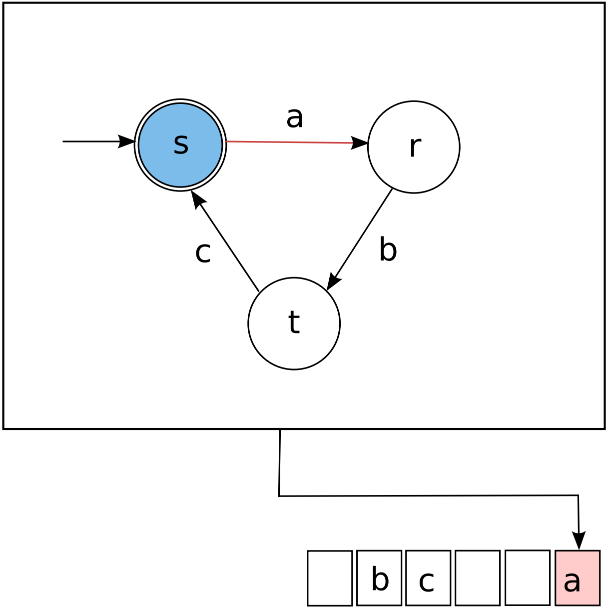
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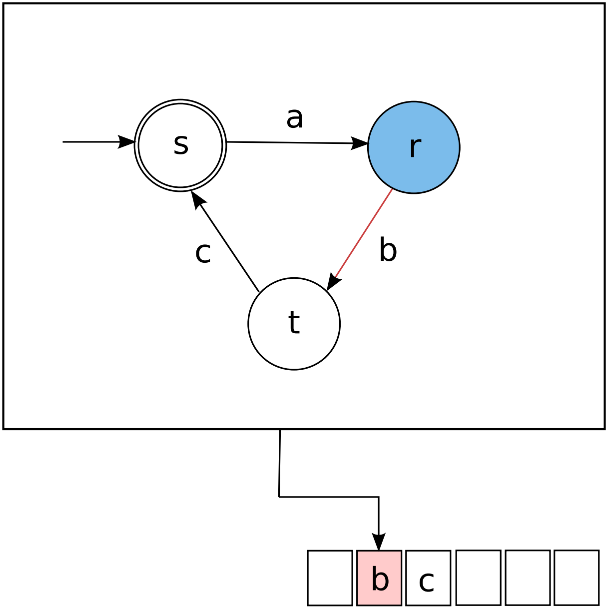
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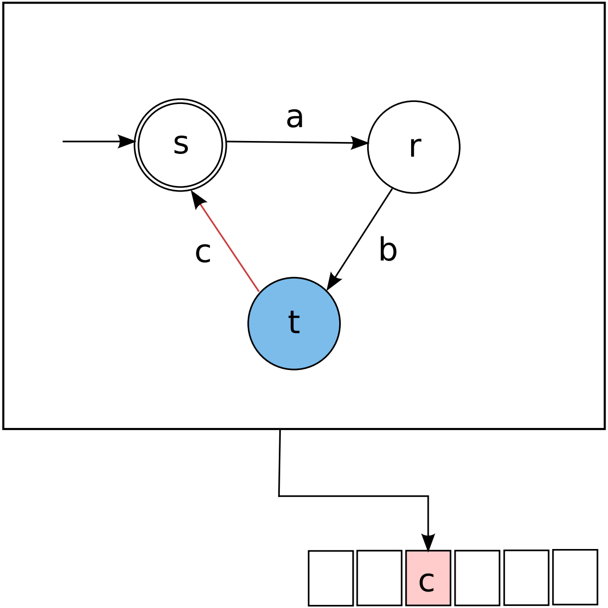
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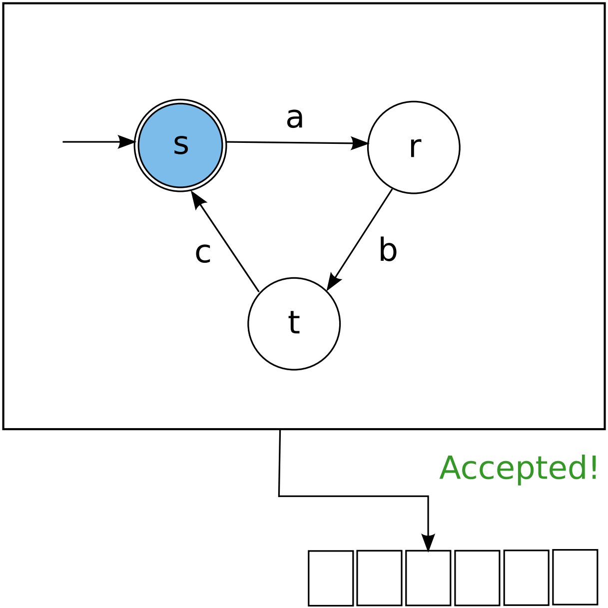
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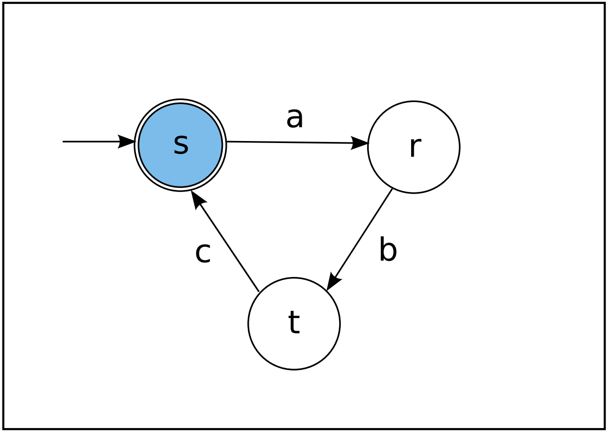

Accepted language: $\{w \in \{a, b, c\}^* : |w|_a = |w|_b = |w|_c\}$

Definition

A *general jumping finite automaton* (*GJFA*) is a quintuple

$$M = (Q, \Sigma, R, s, F)$$

where

- Q is a finite set of *states*;
- Σ is the *input alphabet*;
- R is a finite set of *rules* of the form

$$py \rightarrow q \qquad (p, q \in Q, y \in \Sigma^*)$$

- s is the *start state*;
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Definition

If all rules $py \rightarrow q \in R$ satisfy $|y| \leq 1$, then M is a *jumping finite automaton* (*JFA*).

Definition

If $x, z, x', z', y \in \Sigma^*$ such that $xz = x'z'$ and $py \rightarrow q \in R$, then M makes a *jump* from $xpyz$ to $x'qz'$, symbolically written as

$$x\underline{p}yz \curvearrowright x'\underline{q}z'$$

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- $\curvearrowright^n$ intuitively, a sequence of n jumps ($n \geq 0$); mathematically, the nth power of $\curvearrowright$
- $\curvearrowright^*$ intuitively, a sequence of jumps (possibly empty); mathematically, the reflexive-transitive closure of $\curvearrowright$

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Definition

The *language accepted by M*, denoted by $L(M)$, is defined as

$$L(M) = \{uv : u, v \in \Sigma^*, u\underline{s}v \curvearrowright^* \underline{f}, f \in F\}$$

Example

The JFA

$$M = (\{s, r, t\}, \{a, b, c\}, R, s, \{s\})$$

with

$$R = \{sa \to r, rb \to t, tc \to s\}$$

accepts

$$L(M) = \{w \in \{a, b, c\}^* : |w|_a = |w|_b = |w|_c\}$$

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For instance:

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Example

The GJFA

$$H = (\{s, f\}, \{a, b\}, R, s, \{f\}),$$

with

$$R = \{sba \to f, fa \to f, fb \to f\}$$

accepts

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Example #2

Example

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For instance:

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Power of GJFAs and JFAs

Theorem

Let K be an arbitrary language. Then, K is accepted by a JFA only if $K = \mathrm{perm}(K)$.

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Proof Idea

When using $pa \rightarrow q$, a can appear on any position. □

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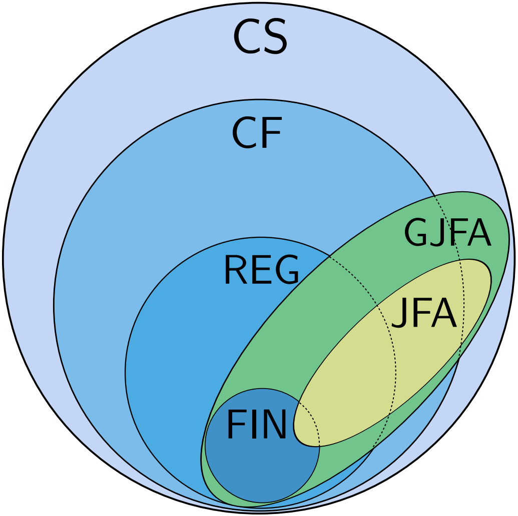
Proof Idea

The language $\{a, b\}^*\{ba\}\{a, b\}^*$ is accepted by the GJFA from Example #2. □

Relations Between Language Families



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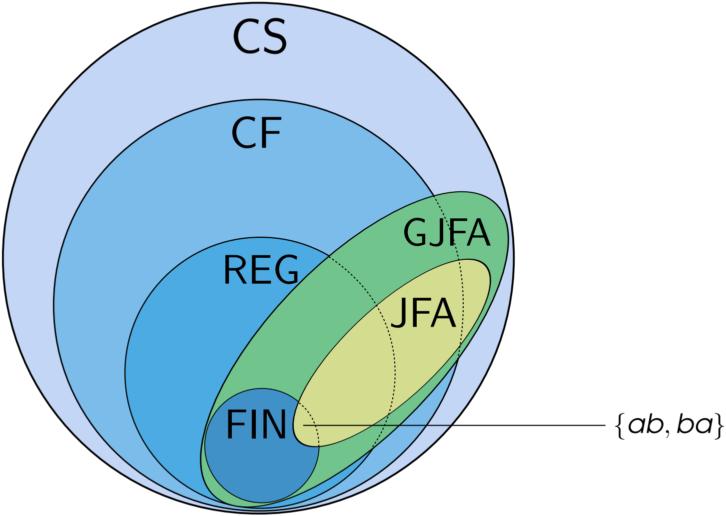

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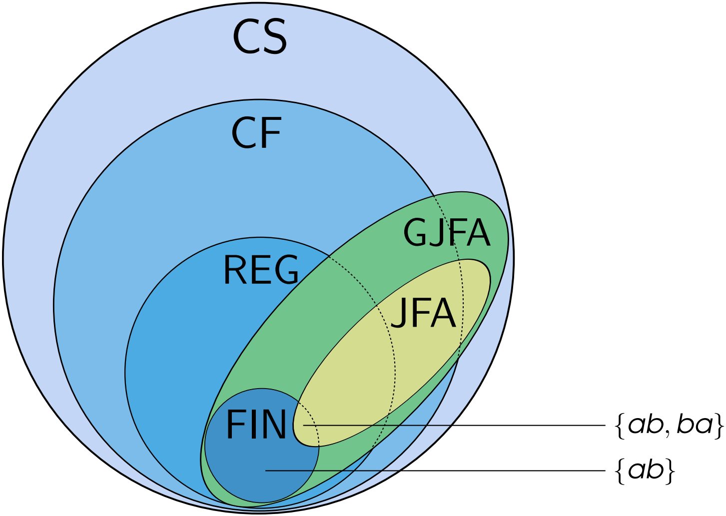

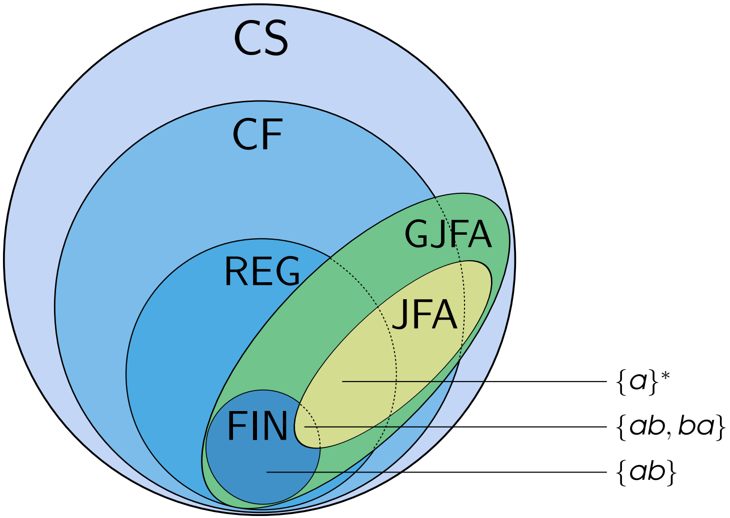
CS
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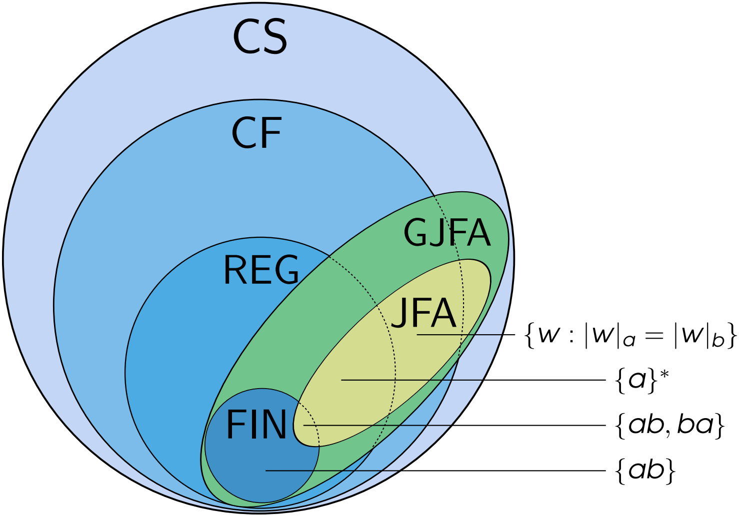
GJFA
JFA
FIN
$\{a\}^*$
$\{ab, ba\}$
$\{ab\}$


CS
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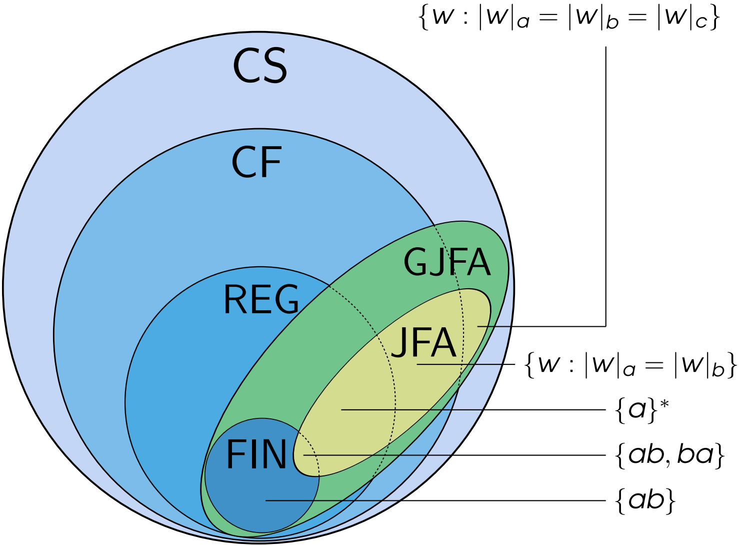
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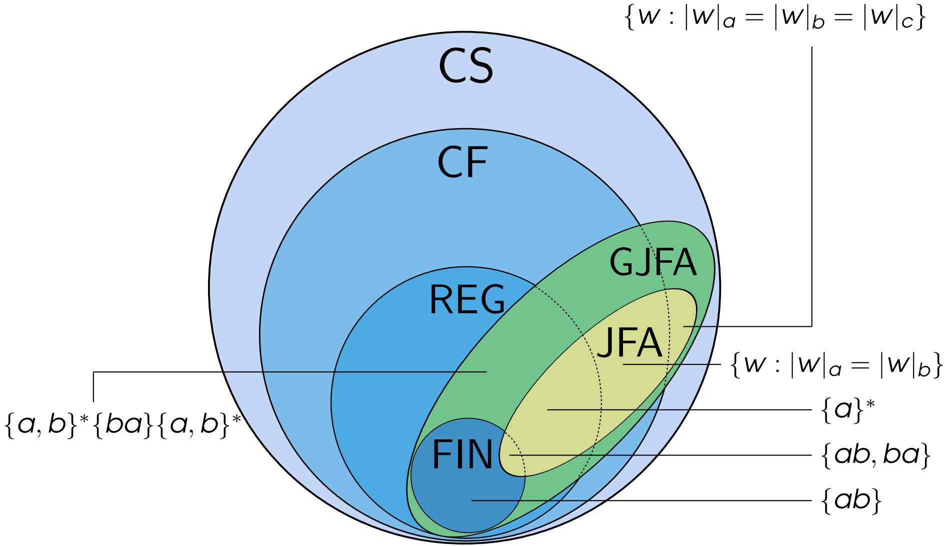
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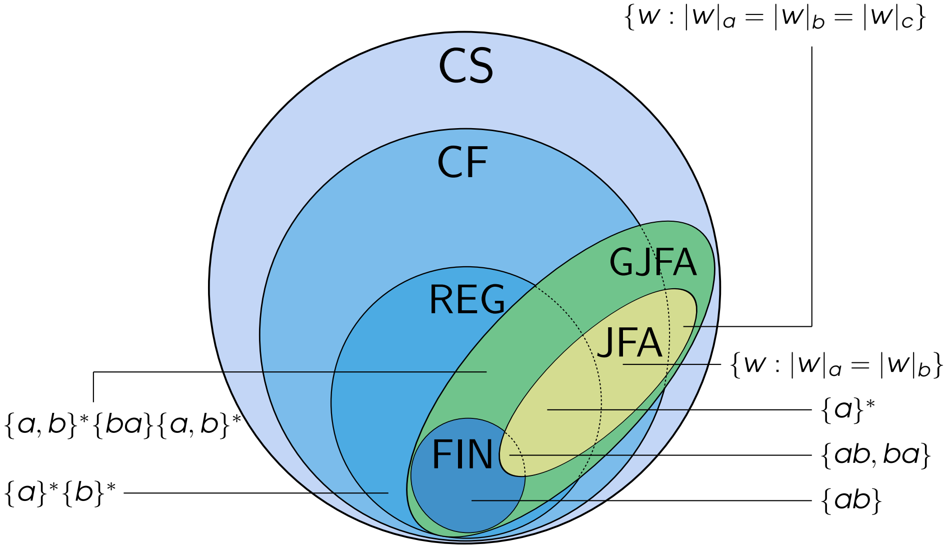

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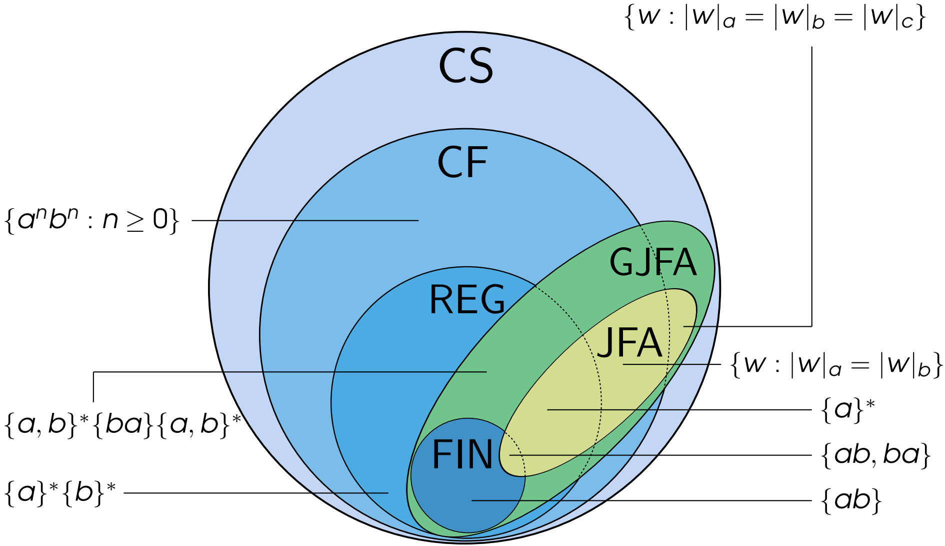
$\{w : |w|_a = |w|_b = |w|_c\}$
CS
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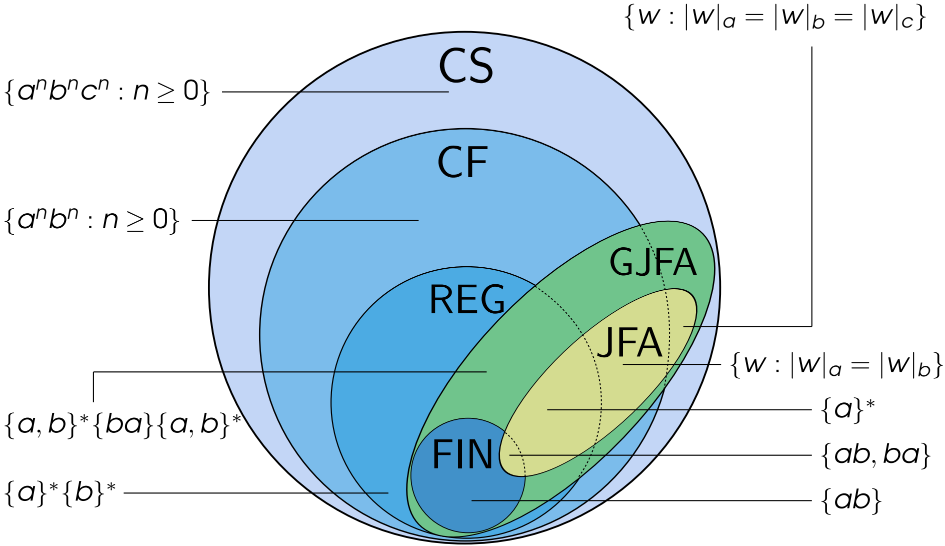
$\{w : |w|_a = |w|_b = |w|_c\}$
CS
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By analogy with finite automata:

- removal of ε-*moves* ($p \to q$ and $qa \to r \Rightarrow pa \to r$)
- making JFAs deterministic

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Corollary

The language of primes

$$\{a^p : p \text{ is a prime number}\}$$

cannot be accepted by any JFA.

Closure Properties

Theorem

JFA *is closed under union.*

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Proof

We have: Two JFAs

- $M_1 = (Q_1, \Sigma_1, R_1, s_1, F_1)$
- $M_2 = (Q_2, \Sigma_2, R_2, s_2, F_2)$ $\quad (Q_1 \cap Q_2 = \emptyset)$

We need: JFA $H = (Q, \Sigma, R, s, F)$ such that $L(H) = L(M_1) \cup L(M_2)$

Construction:

$$\begin{aligned} Q &= Q_1 \cup Q_2 \cup \{s\} & (s \notin Q_1 \cup Q_2) \\ \Sigma &= \Sigma_1 \cup \Sigma_2 \\ R &= R_1 \cup R_2 \cup \{s \rightarrow s_1, s \rightarrow s_2\} \\ F &= F_1 \cup F_2 \end{aligned}$$

□

Theorem

JFA *is* *not* *closed under concatenation.*

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Proof

- Consider $K_1 = \{a\}$ and $K_2 = \{b\}$.
- The JFA $M_1 = (\{s, f\}, \{a\}, \{sa \rightarrow f\}, s, \{f\})$ accepts K_1.
- The JFA $M_2 = (\{s, f\}, \{b\}, \{sb \rightarrow f\}, s, \{f\})$ accepts K_2.
- However, there is no JFA that accepts $K_1K_2 = \{ab\}$. $\square$

	GJFA	JFA	REG
union	+	+	+
intersection	−	+	+
concatenation	−	−	+
intersection with reg. lang.	−	−	+
complement	−	+	+
shuffle	?	+	+
mirror image	?	+	+
Kleene star	?	−	+
Kleene plus	?	−	+
substitution	−	−	+
regular substitution	−	−	+
finite substitution	+	−	+
homomorphism	+	−	+
ε-free homomorphism	+	−	+
inverse homomorphism	+	+	+

Decidability – Summary

	GJFA	JFA
membership	+	+
emptiness	+	+
finiteness	+	+
infiniteness	+	+

Definition

A GJFA $M = (Q, \Sigma, R, s, F)$ is of *degree n*, where $n \geq 0$, if $py \to q \in R$ implies that $|y| \leq n$.

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Example

The GJFA $M = (\{s, p, f\}, \{a, b, c\}, R, s, \{f\})$ with

$$R = \{sabc \to p, pcc \to f, fa \to f\}$$

is of degree 3.

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$\mathbf{GJFA}_n$ the family of languages accepted by GJFAs of degree n

Theorem

$\mathbf{GJFA}_n \subset \mathbf{GJFA}_{n+1}$ *for all* $n \geq 0$

Left and Right Jumps

Definition

A GJFA makes a *left jump* from $wxpyz$ to $wqxz$ by $py \rightarrow q$:

$$wx\underline{p}yz \;{}_{l}\!\curvearrowright\; w\underline{q}xz$$

where $w, x, y, z \in \Sigma^*$.

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Definition

A GJFA makes a *right jump* from $wpyxz$ to $wxqz$ by $py \rightarrow q$:

$$w \underline{p} y x z \;{}_{r}\!\curvearrowright\; w x \underline{q} z$$

where $w, x, y, z \in \Sigma^*$.

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${}_{l}$**GJFA**	GJFAs using only left jumps
${}_{l}$**JFA**	JFAs using only left jumps
${}_{r}$**GJFA**	GJFAs using only right jumps
${}_{r}$**JFA**	JFAs using only right jumps

Left and Right Jumps – Results

Theorem

$_{r}\mathbf{GJFA} = {}_{r}\mathbf{JFA} = \mathbf{REG}$

Left and Right Jumps – Results

Theorem

$_r\mathbf{GJFA} = {_r\mathbf{JFA}} = \mathbf{REG}$

Proof Idea

- $_r\mathbf{JFA} = \mathbf{REG}$ simulating a finite automaton
- $_r\mathbf{GJFA} = \mathbf{REG}$ simulating a *general finite automaton* □

Left and Right Jumps – Results

Theorem

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Theorem

${}_l\mathbf{JFA} - \mathbf{REG} \neq \emptyset$

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- $_r\mathbf{JFA} = \mathbf{REG}$ simulating a finite automaton
- $_r\mathbf{GJFA} = \mathbf{REG}$ simulating a *general finite automaton* □

Theorem

$_l\mathbf{JFA} - \mathbf{REG} \neq \emptyset$

Proof Idea

$$M = (\{s, p, q\}, \{a, b\}, R, s, \{s\})$$

with

$$R = \{sa \rightarrow p, pb \rightarrow s, sb \rightarrow q, qa \rightarrow s\}$$

accepts

$$_lL(M) = \{w : |w|_a = |w|_b\}$$ □

A Variety of Start Configurations

Definition

Let $M = (Q, \Sigma, R, s, F)$ be a GJFA. Set

$$
\begin{aligned}
{}^{b}L(M) &= \{w \in \Sigma^* : \underline{s}w \curvearrowright^* \underline{f} \text{ with } f \in F\} && \text{(beginning)}\\
{}^{a}L(M) &= \{uv : u, v \in \Sigma^*, u\underline{s}v \curvearrowright^* \underline{f} \text{ with } f \in F\} && \text{(anywhere)}\\
{}^{e}L(M) &= \{w \in \Sigma^* : w\underline{s} \curvearrowright^* \underline{f} \text{ with } f \in F\} && \text{(end)}
\end{aligned}
$$

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{}^{e}L(M) &= \{w \in \Sigma^* : w\underline{s} \curvearrowright^* \underline{f} \text{ with } f \in F\} && \text{(end)}
\end{aligned}
$$

^{b}**GJFA**	GJFAs starting at the beginning
^{a}**GJFA**	GJFAs starting anywhere
^{e}**GJFA**	GJFAs starting at the end
^{b}**JFA**	JFAs starting at the beginning
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^{b}**GJFA**	GJFAs starting at the beginning
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^{b}**JFA**	JFAs starting at the beginning
^{a}**JFA**	JFAs starting anywhere
^{e}**JFA**	JFAs starting at the end

Observations:

- ${}^{a}L(M) = L(M)$
- ^{a}**GJFA** = **GJFA** and ^{a}**JFA** = **JFA**

A Variety of Start Configurations – Results

Theorem

$^{a}\mathbf{JFA} \subset {}^{b}\mathbf{JFA}$

Theorem

${}^{a}\mathbf{JFA} \subset {}^{b}\mathbf{JFA}$

Proof Idea

The JFA

$$M = (\{s, f\}, \{a, b\}, \{sa \to f, fb \to f\}, s, \{f\})$$

satisfies ${}^{b}L(M) = \{a\}\{b\}^*$ $(\{a\}\{b\}^* \notin {}^{a}\mathbf{JFA})$. $\square$

Theorem

$^{a}\mathbf{JFA} \subset {}^{b}\mathbf{JFA}$

Proof Idea

The JFA

$$M = (\{s, f\}, \{a, b\}, \{sa \to f, fb \to f\}, s, \{f\})$$

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Proof Idea

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Theorem

${}^{a}\mathbf{GJFA} \subset {}^{b}\mathbf{GJFA}$

Theorem

${}^{e}\mathbf{GJFA} = {}^{a}\mathbf{GJFA}$ *and* ${}^{e}\mathbf{JFA} = {}^{a}\mathbf{JFA}$

Conclusion and Open Problem Areas

- closure properties of **GJFA** (shuffle, Kleene star, Kleene plus, and mirror image)
- other decision problems of **GJFA** and **JFA**, like equivalence, universality, inclusion, or regularity
- the effect of left jumps to the power of JFAs and GJFAs (we only know that $_l\mathbf{JFA} - \mathbf{REG} \neq \emptyset$)
- strict determinism
- applications: verification of a relation concerning the number of symbol occurrences (genetics)

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Discussion