BRNO UNIVERSITY OF TECHNOLOGY

FACULTY OF INFORMATION TECHNOLOGY
DEPARTMENT OF INTELLIGENT SYSTEMS

ALGORITHMIC AND MATHEMATICAL PRINCIPLES OF AUTOMATIC NUMBER PLATE RECOGNITION SYSTEMS

B.SC. THESIS

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BRNO 2007

The author is indebted to the supervisor of this thesis, ***doc. Ing. František Zbořil, CSc.*** *for his great help.*

THIS WORK IS A PART OF THE RESEARCH PLAN "SECURITY-ORIENTED RESEARCH IN INFORMATION TECHNOLOGY, MSM 0021630528" AT BRNO UNIVERSITY OF TECHNOLOGY

Abstract

This work deals with problematic from field of artificial intelligence, machine vision and neural networks in construction of an automatic number plate recognition system (ANPR). This problematic includes mathematical principles and algorithms, which ensure a process of number plate detection, processes of proper characters segmentation, normalization and recognition. Work comparatively deals with methods achieving invariance of systems towards image skew, translations and various light conditions during the capture. Work also contains an implementation of a demonstration model, which is able to proceed these functions over a set of snapshots.

Key Words:

Machine vision, artificial intelligence, neural networks, optical character recognition, ANPR

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Chapter 1

Introduction

1.1 ANPR systems as a practical application of artificial intelligence

Massive integration of information technologies into all aspects of modern life caused demand for processing vehicles as conceptual resources in information systems. Because a standalone information system without any data has no sense, there was also a need to transform information about vehicles between the reality and information systems. This can be achieved by a human agent, or by special intelligent equipment which is be able to recognize vehicles by their number plates in a real environment and reflect it into conceptual resources. Because of this, various recognition techniques have been developed and number plate recognition systems are today used in various traffic and security applications, such as parking, access and border control, or tracking of stolen cars.

In parking, number plates are used to calculate duration of the parking. When a vehicle enters an input gate, number plate is automatically recognized and stored in database. When a vehicle later exits the parking area through an output gate, number plate is recognized again and paired with the first-one stored in the database. The difference in time is used to calculate the parking fee. Automatic number plate recognition systems can be used in access control. For example, this technology is used in many companies to grant access only to vehicles of authorized personnel.

In some countries, ANPR systems installed on country borders automatically detect and monitor border crossings. Each vehicle can be registered in a central database and compared to a black list of stolen vehicles. In traffic control, vehicles can be directed to different lanes for a better congestion control in busy urban communications during the rush hours.

1.2 Mathematical aspects of number plate recognition systems

In most cases, vehicles are identified by their number plates, which are easily readable for humans, but not for machines. For machine, a number plate is only a grey picture defined as a two-dimensional function $f(x, y)$, where x and y are spatial coordinates, and f is a light intensity at that point. Because of this, it is necessary to design robust mathematical machinery, which will be able to extract semantics from spatial domain of the captured image. These functions are implemented in so-called "ANPR systems", where the acronym "ANPR" stands for an "Automatic Number Plate Recognition". ANPR system means transformation of data between the real environment and information systems.

The design of ANPR systems is a field of research in artificial intelligence, machine vision, pattern recognition and neural networks. Because of this, the main goal of this thesis is to study algorithmic and mathematical principles of automatic number plate recognition systems.

Chapter two deals with problematic of number plate area detection. This problematic includes algorithms, which are able to detect a rectangular area of the number plate in original image. Humans define the number plate in a natural language as a "*small plastic or metal plate attached to a vehicle for official identification purposes*", but machines do not understand this definition. Because of this, there is a need to find an alternative definition of the number plate based on descriptors, which will be comprehensible for machines. This is a fundamental problem of machine vision and of this chapter.

Chapter three describes principles of the character segmentation. In most cases, characters are segmented using the horizontal projection of a pre-processed number plate, but sometimes these principles can fail, especially if detected number plates are too warped or skewed. Then, more sophisticated segmentation algorithms must be used.

Chapter four deals with various methods normalization and detection of characters. At first, character dimensions and brightness must be normalized to ensure invariance towards a size and light conditions. Then, a feature extraction algorithm must be applied on a character to filter irrelevant data. It is necessary to extract features, which will be invariant towards character deformations, used font style etc.

Chapter five studies pattern classifiers and neural networks and deals with their usage in recognition of characters. Characters can be classified and recognized by the simple nearest neighbor algorithm (1NN) applied to a vector of extracted features, or there is also possibility to use one of the more sophisticated classification methods, such as feed-forward or Hopfield neural networks. This chapter also presents additional heuristic analyses, which are used for elimination of non-character elements from the plate.

Sometimes, the recognition process may fail and the detected plate can contain errors. Some of these errors can be detected by a syntactical analysis of the recognized plate. If we have a regular expression, or a rule how to evaluate a country-specific license plate, we can reconstruct defective plates using this rule. For example, a number zero "0" can be automatically repaired to a character "O" on positions, where numbers are not allowed. Chapter six deals with this problematic.

1.3 Physical aspects of number plate recognition systems

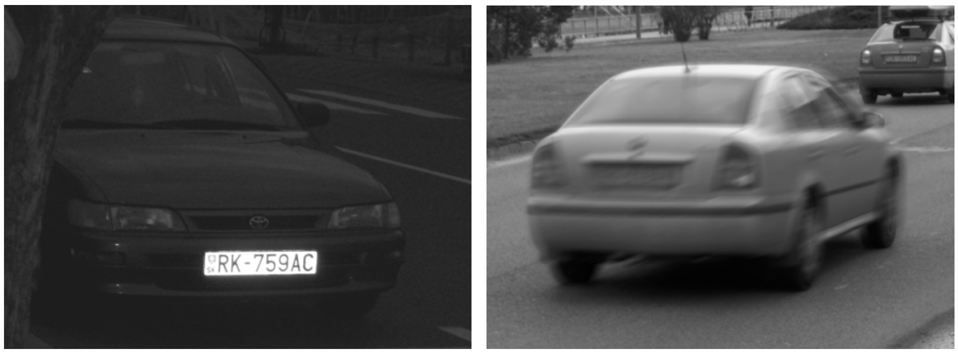
Automatic number plate recognition system is a special set of hardware and software components that proceeds an input graphical signal like static pictures or video sequences, and recognizes license plate characters from it. A hardware part of the ANPR system typically consists of a camera, image processor, camera trigger, communication and storage unit.

The hardware trigger physically controls a sensor directly installed in a lane. Whenever the sensor detects a vehicle in a proper distance of camera, it activates a recognition mechanism. Alternative to this solution is a software detection of an incoming vehicle, or continual processing of the sampled video signal. Software detection, or continual video processing may consume more system resources, but it does not need additional hardware equipment, like the hardware trigger.

Image processor recognizes static snapshots captured by the camera, and returns a text representation of the detected license plate. ANPR units can have own dedicated image processors (all-in-one solution), or they can send captured data to a central processing unit for further processing (generic ANPR). The image processor is running on special recognition software, which is a key part of whole ANPR system.

Because one of the fields of application is a usage on road lanes, it is necessary to use a special camera with the extremely short shutter. Otherwise, quality of captured snapshots will be degraded by an undesired motion blur effect caused by a movement of the vehicle. For example, usage of the standard camera with shutter of *1/100 sec* to capture a vehicle with speed of *80 km/h* will cause a motion skew in amount of *0.22 m*. This skew means the significant degradation of recognition abilities.

There is also a need to ensure system invariance towards the light conditions. Normal camera should not be used for capturing snapshots in darkness or night, because it operates in a visible light spectrum. Automatic number plate recognition systems are often based on cameras operating in an infrared band of the light spectrum. Usage of the infrared camera in combination with an infrared illumination is better to achieve this goal. Under the illumination, plates that are made from reflexive material are much more highlighted than rest of the image. This fact makes detection of license plates much easier.



A | B

Figure 1.1: (a) Illumination makes detection of reflexive image plates easier. (b) Long camera shutter and a movement of the vehicle can cause an undesired motion blur effect.

1.4 Notations and mathematical symbols

Logic symbols

$p \oplus q$	Exclusive logical disjunction (p xor q)
$p \wedge q$	Logical conjunction (p and q)
$p \vee q$	Logical disjunction (p or q)
$\neg p$	Exclusion (not p)

Mathematical definition of image

$f(x,y)$	x and y are spatial coordinates of an image, and f is an intensity of light at that point. This function is always discrete on digital computers. $x \in \mathbb{N}_0 \wedge y \in \mathbb{N}_0$, where $\mathbb{N}_0$ denotes the set of natural numbers including zero.
$f(p)$	The intensity of light at point p. $f(p) = f(x,y)$, where $p = [x,y]$

Pixel neighborhoods

$p_1 \ddot{\mathrm{N}}_4 p_2$	Pixel p_1 is in a four-pixel neighborhood of pixel p_2 (and vice versa)
$p_1 \ddot{\mathrm{N}}_8 p_2$	Pixel p_1 is in an eight-pixel neighborhood of pixel p_2 (and vice versa)

Convolutions

$a(x) * b(x)$	Discrete convolution of signals $a(x)$ and $b(x)$
$a(x) \tilde{*} b(x)$	Discrete periodical convolution of signals $a(x)$ and $b(x)$

Vectors and sets

$\mathbf{m}[x,y]$	The element in xth column and yth row of matrix $\mathbf{m}$.
$\max A$	The maximum value contained in the set A. The scope of elements can be specified by additional conditions
$\min A$	The minimum value contained in the set A
$\text{mean}\, A$	The mean value of the elements contained in the set A
$\text{median}\, A$	The median value of the elements contained in the set A
$\|A\|$	The cardinality of the set A. (Number of elements contained in the set)
$\mathbf{x}$	Vectors or any other ordered sequences of numbers are printed bold.
x_i	The elements of vectors are denoted as x_i, where i is a sequence number (starting with zero), such as $i \in 0 \ldots n-1$, where $n = \|\mathbf{x}\|$ is a cardinality of the vector (number of elements)
$\mathbf{x}[a]$	The element a of the vector $\mathbf{x}$. For example, the vector $\mathbf{x}$ can contain elements a, b, c, d, such as $\mathbf{x} = (a,b,c,d)$
$\mathbf{x}^{(i)}$	If there is more than one vector denoted as $\mathbf{x}$, they are distinguished by their indexes i. The upper index (i) <u>does not</u> mean the ith element of vector.

Intervals

$a < x < b$	x lies in the interval between a and b. This notation is used when x is the spatial coordinate in image (discrete as well as continuous)
$x \in a \ldots b$	This notation has the same meaning as the above one, but it is used when x is a discrete sequence number.

Quantificators

$\exists x$	There exists at least one x
$\exists! x$	There exists exactly one x
$\exists^n x$	There exists exactly n x
$\neg\exists x$	There does not exist x
$\forall x$	For every x

Rounding

$\lfloor x \rfloor$	Number x rounded down to the nearest integer
$\lceil x \rceil$	Number x rounded up to the nearest integer

Chapter 2

Principles of number plate area detection

The first step in a process of automatic number plate recognition is a detection of a number plate area. This problematic includes algorithms that are able to detect a rectangular area of the number plate in an original image. Humans define a number plate in a natural language as a "*small plastic or metal plate attached to a vehicle for official identification purposes*", but machines do not understand this definition as well as they do not understand what "vehicle", "road", or whatever else is. Because of this, there is a need to find an alternative definition of a number plate based on descriptors that will be comprehensible for machines.

Let us define the number plate as a "*rectangular area with increased occurrence of horizontal and vertical edges*". The high density of horizontal and vertical edges on a small area is in many cases caused by contrast characters of a number plate, but not in every case. This process can sometimes detect a wrong area that does not correspond to a number plate. Because of this, we often detect several candidates for the plate by this algorithm, and then we choose the best one by a further heuristic analysis.

Let an input snapshot be defined by a function $f(x,y)$, where x and y are spatial coordinates, and f is an intensity of light at that point. This function is always discrete on digital computers, such as $x \in \mathbb{N}_0 \wedge y \in \mathbb{N}_0$, where $\mathbb{N}_0$ denotes the set of natural numbers including zero. We define operations such as edge detection or rank filtering as mathematical transformations of function f.

The detection of a number plate area consists of a series of convolve operations. Modified snapshot is then projected into axes x and y. These projections are used to determine an area of a number plate.

2.1 Edge detection and rank filtering

We can use a periodical convolution of the function f with specific types of matrices $\mathbf{m}$ to detect various types of edges in an image:

$$f'(x,y) = f(x,y) \tilde{*} \mathbf{m}[x,y] = \sum_{i=0}^{w-1} \sum_{j=0}^{h-1} f(x,y) \cdot \mathbf{m}\left[\operatorname{mod}_w(x-i), \operatorname{mod}_h(y-j)\right]$$

where w and h are dimensions of the image represented by the function f

Note: The expression $\mathbf{m}[x,y]$ *represents the element in x*th *column and y*th *row of matrix* $\mathbf{m}$.

2.1.1 Convolution matrices

Each image operation (or filter) is defined by a convolution matrix. The convolution matrix defines how the specific pixel is affected by neighboring pixels in the process of convolution.

Individual cells in the matrix represent the neighbors related to the pixel situated in the centre of the matrix. The pixel represented by the cell y in the destination image (fig. 2.1) is affected by the pixels $x_0 \dots x_8$ according to the formula:

$$y = x_0 \times m_0 + x_1 \times m_1 + x_2 \times m_2 + x_3 \times m_3 + x_4 \times m_4 + x_5 \times m_5 + x_6 \times m_6 + x_7 \times m_7 + x_8 \times m_8$$

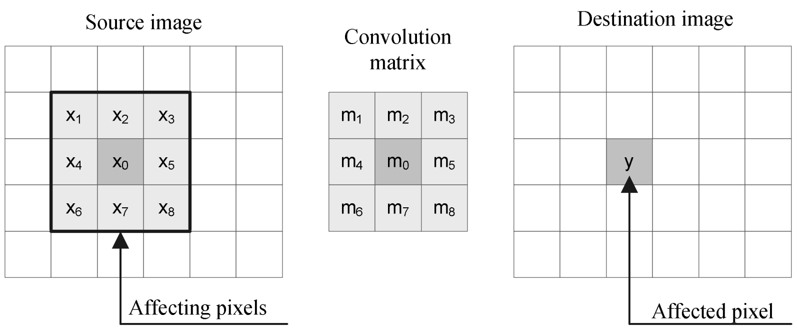


Figure 2.1: The pixel is affected by its neighbors according to the convolution matrix.

Horizontal and vertical edge detection

To detect horizontal and vertical edges, we convolve source image with matrices $\mathbf{m}_{he}$ and $\mathbf{m}_{ve}$. The convolution matrices are usually much smaller than the actual image. Also, we can use bigger matrices to detect rougher edges.

$$\mathbf{m}_{he} = \begin{pmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \; ; \; \mathbf{m}_{ve} = \begin{pmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

Sobel edge detector

The Sobel edge detector uses a pair of 3x3 convolution matrices. The first is dedicated for evaluation of vertical edges, and the second for evaluation of horizontal edges.

$$\mathbf{G}_x = \begin{pmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{pmatrix} \; ; \; \mathbf{G}_y = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix}$$

The magnitude of the affected pixel is then calculated using the formula $|\mathbf{G}| = \sqrt{\mathbf{G}_x^2 + \mathbf{G}_y^2}$. In praxis, it is faster to calculate only an approximate magnitude as $|\mathbf{G}| = |\mathbf{G}_x| + |\mathbf{G}_y|$.

Horizontal and vertical rank filtering

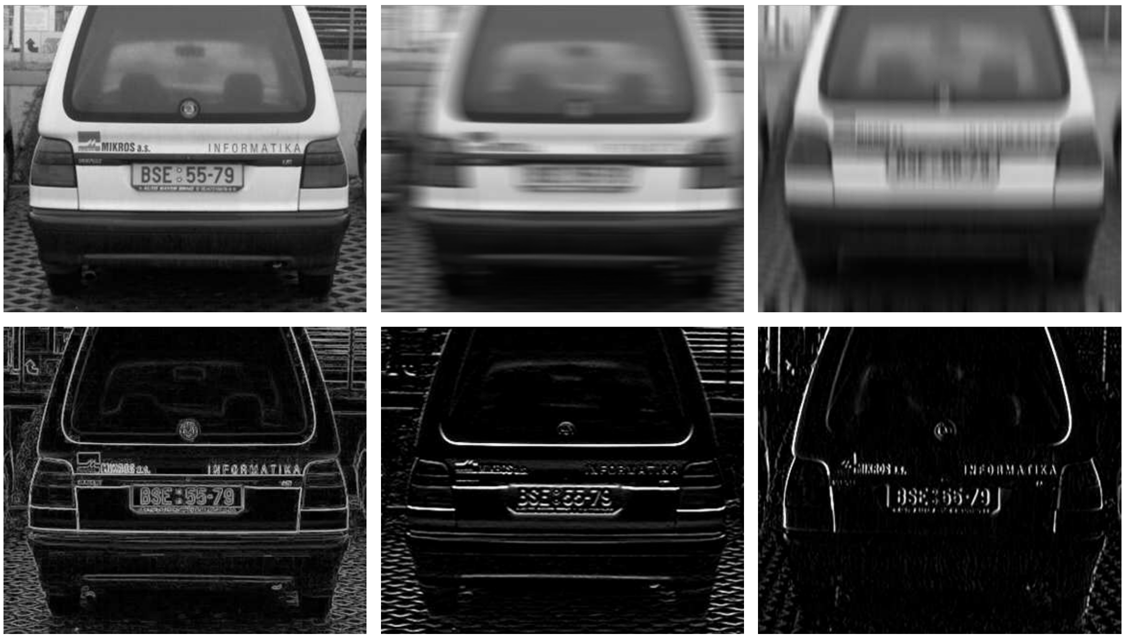
Horizontally and vertically oriented rank filters are often used to detect clusters of high density of bright edges in the area of the number plate. The width of the horizontally oriented rank filter matrix is much larger than the height of the matrix ($w \gg h$), and vice versa for the vertical rank filter ($w \ll h$).

To preserve the global intensity of an image, it is necessary to each pixel be replaced with an average pixel intensity in the area covered by the rank filter matrix. In general, the convolution matrix should meet the following condition:

$$\sum_{i=0}^{w-1}\sum_{j=0}^{h-1}\mathbf{m}_{hr}\left[i,j\right]=1.0$$

where w and h are dimensions of the matrix.

The following pictures show the results of application of the rank and edge detection filters.



A	B	C
D	E	F

Figure 2.2: (a) Original image (b) Horizontal rank filter (c) Vertical rank filter (d) Sobel edge detection (e) Horizontal edge detection (f) Vertical edge detection

2.2 Horizontal and vertical image projection

After the series of convolution operations, we can detect an area of the number plate according to a statistics of the snapshot. There are various methods of statistical analysis. One of them is a horizontal and vertical projection of an image into the axes x and y.

The vertical projection of the image is a graph, which represents an overall magnitude of the image according to the axis y (see figure 2.3). If we compute the vertical projection of the image after the application of the vertical edge detection filter, the magnitude of certain point represents the occurrence of vertical edges at that point. Then, the vertical projection of so transformed image can be used for a vertical localization of the number plate. The horizontal projection represents an overall magnitude of the image mapped to the axis x.

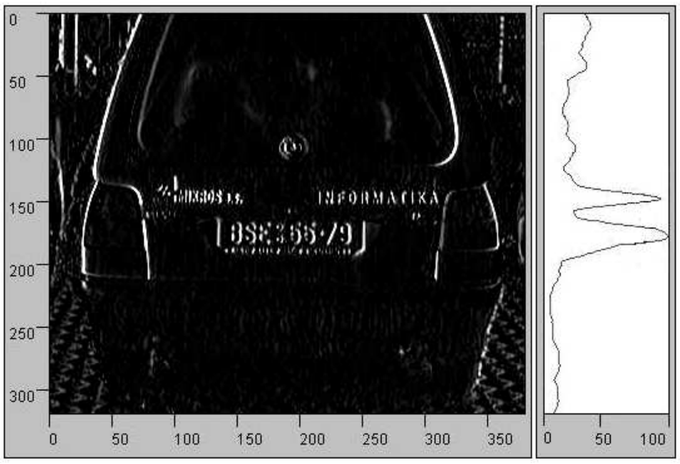


Figure 2.3: Vertical projection of image to a y axis

Let an input image be defined by a discrete function $f(x,y)$. Then, a vertical projection p_y of the function f at a point y is a summary of all pixel magnitudes in the yth row of the input image. Similarly, a horizontal projection at a point x of that function is a summary of all magnitudes in the xth column.

We can mathematically define the horizontal and vertical projection as:

$$p_x(x)=\sum_{j=0}^{h-1} f(x,j) \quad ; \quad p_y(y)=\sum_{i=0}^{w-1} f(i,y)$$

where w and h are dimensions of the image.

2.3 Double-phase statistical image analysis

The statistical image analysis consists of two phases. The first phase covers the detection of a wider area of the number plate. This area is then deskewed, and processed in the second phase of analysis. The output of double-phase analysis is an exact area of the number plate. These two phases are based on the same principle, but there are differences in coefficients, which are used to determine boundaries of clipped areas.

The detection of the number plate area consists of a “band clipping” and a “plate clipping”. The band clipping is an operation, which is used to detect and clip the vertical area of the number plate (so-called band) by analysis of the vertical projection of the snapshot. The plate clipping is a consequent operation, which is used to detect and clip the plate from the band (not from the whole snapshot) by a horizontal analysis of such band.

Snapshot

Assume the snapshot is represented by a function $f(x,y)$, where $x_0 \le x \le x_1$ and $y_0 \le y \le y_1$. The $[x_0, y_0]$ represents the upper left corner of the snapshot, and $[x_1, y_1]$ represents the bottom right corner. If w and h are dimensions of the snapshot, then $x_0 = 0$, $y_0 = 0$, $x_1 = w-1$ and $y_1 = h-1$.

Band

The band b in the snapshot f is an arbitrary rectangle $b = (x_{b0}, y_{b0}, x_{b1}, y_{b1})$, such as:

$$(x_{b0} = x_{\min}) \wedge (x_{b1} = x_{\max}) \wedge (y_{\min} \le y_{b0} < y_{b1} \wedge y_{\max})$$

Plate

Similarly, the plate p in the band b is an arbitrary rectangle $p = (x_{p0}, y_{p0}, x_{p1}, y_{p1})$, such as:

$$(x_{b0} \le x_{p0} \le x_{p1} \le x_{b1}) \wedge (y_{p0} = y_{b0}) \wedge (y_{p0} = y_{b0})$$

The band can be also defined as a <u>vertical selection</u> of the snapshot, and the plate as <u>a horizontal selection</u> of the band. The figure 2.4 schematically demonstrates this concept:

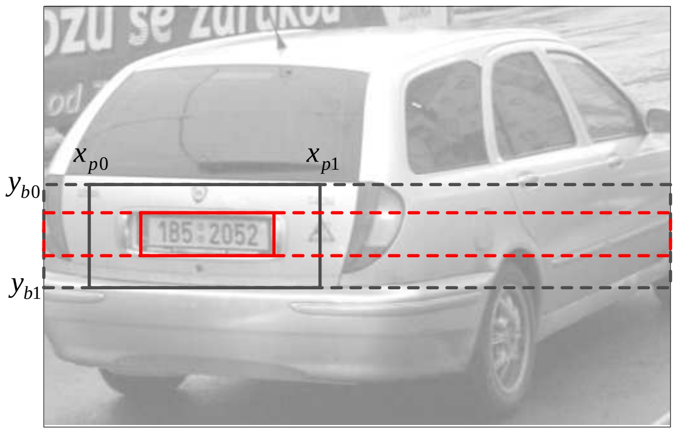


Figure 2.4: The double-phase plate clipping. Black color represents the first phase of plate clipping, and red color represents the second one. Bands are represented by dashed lines, and plates by solid lines.

2.3.1 Vertical detection – band clipping

The first and second phase of band clipping is based on the same principle. The band clipping is a vertical selection of the snapshot according to the analysis of a graph of vertical projection. If h is the height of the analyzed image, the corresponding vertical projection $p_y^r(y)$ contains h values, such as $y \in \langle 0; h-1 \rangle$.

The graph of projection may be sometimes too “ragged” for analysis due to a big statistical dispersion of values $p_y^r(y)$. There are two approaches how to solve this problem. We can blur the source snapshot (costly solution), or we can decrease the statistical dispersion of the ragged projection p_y^r by convolving its projection with a rank vector:

$$p_y(y) = p_y^r(y) \tilde{*} \mathbf{m}_{hr}[y]$$

where $\mathbf{m}_{hr}$ is the rank vector (analogous to the horizontal rank matrix in section 2.1.1). The width of the vector $\mathbf{m}_{hr}$ is nine in default configuration.

After convolution with the rank vector, the vertical projection of the snapshot in figure 2.3 can look like this:

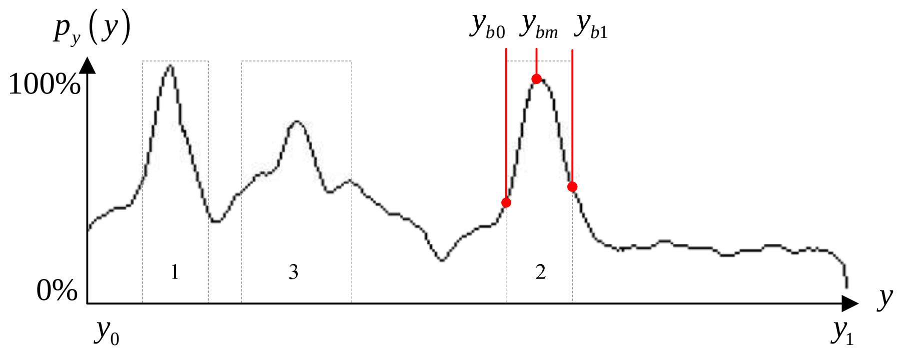


Figure 2.5: The vertical projection of the snapshot 2.3 after convolution with a rank vector. The figure contains three detected candidates. Each highlighted area corresponds to one detected band.

The fundamental problem of analysis is to compute peaks in the graph of vertical projection. The peaks correspond to the bands with possible candidates for number plates. The maximum value of $p_y(y)$ corresponding to the axle of band can be computed as:

$$y_{bm} = \arg \max_{y_0 \le y \le y_1} \{ p_y(y) \}$$

The y_{b0} and y_{b1} are coordinates of band, which can be detected as:

$$y_{b0} = \max_{y_0 \le y \le y_{bm}} \{ y \mid p_y(y) \le c_y \cdot p_y(y_{bm}) \}$$
$$y_{b1} = \min_{y_{bm} \le y \le y_1} \{ y \mid p_y(y) \le c_y \cdot p_y(y_{bm}) \}$$

c_y is a constant, which is used to determine the foot of peak y_{bm}. In praxis, the constant is calibrated to $c_1 = 0.55$ for the first phase of detection, and $c_2 = 0.42$ for the second phase.

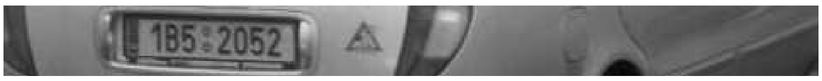


Figure 2.6: The band detected by the analysis of vertical projection

This principle is applied iteratively to detect several possible bands. The y_{b0} and y_{b1} coordinates are computed in each step of iterative process. After the detection, values of projection p_y in interval $\langle y_{b0}, y_{b1} \rangle$ are zeroized. This idea is illustrated by the following pseudo-code:

```
let $\mathbf{L}$ to be a list of detected candidates
for $i$:=0 to number_of_bands_to_be_detected do
begin
    detect $y_{b0}$ and $y_{b1}$ by analysis of projection $p_y$
    save $y_{b0}$ and $y_{b1}$ to a list $\mathbf{L}$
    zeroize interval $\langle y_{b0}, y_{b1} \rangle$
end
```

The list $\mathbf{L}$ of coordinates y_{b0} and y_{b1} will be sorted according to value of peak (y_{bm}). The band clipping is followed by an operation, which detects plates in a band.

2.3.2 Horizontal detection – plate clipping

In contrast with the band clipping, there is a difference between the first and second phase of plate clipping.

First phase

There is a strong analogy in a principle between the band and plate clipping. The plate clipping is based on a horizontal projection of band. At first, the band must be processed by a vertical detection filter. If w is a width of the band (or a width of the analyzed image), the corresponding horizontal projection $p_x^r(x)$ contains w values:

$$p_x(x) = \sum_{j=y_{b0}}^{y_{b1}} f(x, j)$$

Please notice that $p_x(x)$ is a projection of the band, not of the whole image. This can be achieved by a summation in interval $\langle y_{b0}, y_{b1} \rangle$, which represents the vertical boundaries of the band. Since the horizontal projection $p_x^r(x)$ may have a big statistical dispersion, we decrease it by convolving with a rank vector ($p_x(x) = p_x^r(x) \tilde{\ast} \mathbf{m}_{vr}[x]$). The width of the rank vector is usually equal to a half of an estimated width of the number plate.

Then, the maximum value corresponding to the plate can be computed as:

$$x_{bm} = \arg \max_{x_0 \le y \le x_1} \{ p_x(x) \}$$

The x_{b0} and x_{b1} are coordinates of the plate, which can be then detected as:

$$x_{b0} = \max_{x_0 \le x \le x_{bm}} \{ x \mid p_x(x) \le c_x \cdot p_x(x_{bm}) \}$$
$$x_{b1} = \min_{x_{bm} \le x \le x_1} \{ x \mid p_x(x) \le c_x \cdot p_x(x_{bm}) \}$$

where c_x is a constant, which is used to determine the foot of peak x_{bm}. The constant is calibrated to $c_x = 0.86$ for the first phase of detection.

Second phase

In the second phase of detection, the horizontal position of a number plate is detected in another way. Due to the skew correction between the first and second phase of analysis, the wider plate area must be duplicated into a new bitmap. Let $f_n(x, y)$ be a corresponding function of such bitmap. This picture has a new coordinate system, such as $[0,0]$ represents the upper left corner and $[w-1, h-1]$ the bottom right, where w and h are dimensions of the area. The wider area of the number plate after deskewing is illustrated in figure 2.8.

In contrast with the first phase of detection, the source plate has not been processed by the vertical detection filter. If we assume that plate is white with black borders, we can detect that borders as black-to-white and white-to-black transitions in the plate. The horizontal projection $p_x(x)$ of the image is illustrated in the figure 2.7.a. To detect the black-to-white and white-to-black transitions, there is a need to compute a derivative $p_x'(x)$ of the projection $p_x(x)$. Since the projection is not continuous, the derivation step cannot be an infinitely small number

($h \neq \lim\limits_{x \to 0} x$). If we derive a discrete function, the derivation step h must be an integral number (for example $h = 4$). Let the derivative of $p_x(x)$ be defined as:

$$p_x'(x) = \frac{p_x(x) - p_x(x-h)}{h}$$

Where $h = 4$.

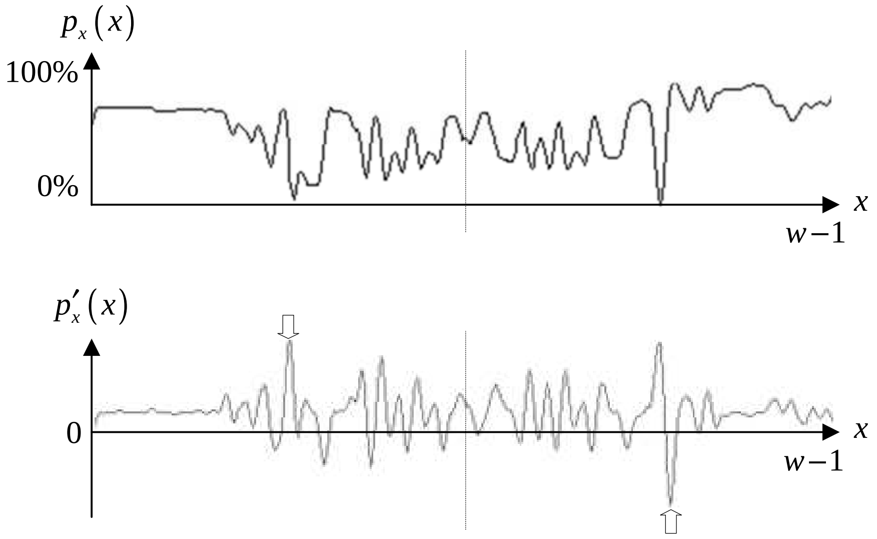


Figure 2.7: (a) The horizontal projection $p_x(x)$ of the plate in figure 2.8. (b) The derivative of $p_x(x)$. Arrows denote the "BW" and "WB" transitions, which are used to determine the boundaries of the plate.

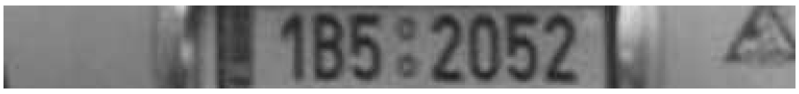


Figure 2.8: The wider area of the number plate after deskewing.

The left and right boundary of the plate can be determined by an analysis of the projection $p_x'(x)$. The left corner x_{p0} is represented by the black-to-white transition (positive peak in figure 2.7.b), and right corner x_{p1} by the white-to-black transition (negative peak in figure 2.7.b):

$$x_{p0} = \min_{0 \le x < \frac{w}{2}} \left\{ x \middle| p_x'(x) \ge c_d \cdot \max_{0 \le x < w} \left\{ p_x'(x) \right\} \right\}$$

$$x_{p1} = \max_{\frac{w}{2} \le x < w} \left\{ x \middle| p_x'(x) \le c_d \cdot \min_{0 \le x < w} \left\{ p_x'(x) \right\} \right\}$$

where c_d is a constant used to determine the most left negative and the most right positive peak. The left and right corners must lie on the opposite halves of the detected plate according to the constraints $0 \le x < \frac{w}{2}$ for x_{p0}, and $\frac{w}{2} \le x < w$ for x_{p1}.

In this phase of the recognition process, it is not possible to select a best candidate for a number plate. This can be done by a heuristic analysis of characters after the segmentation.

2.4 Heuristic analysis and priority selection of number plate candidates

In general, the captured snapshot can contain several number plate candidates. Because of this, the detection algorithm always clips several bands, and several plates from each band. There is a predefined value of maximum number of candidates, which are detected by analysis of projections. By default, this value is equals to nine.

There are several heuristics, which are used to determine the cost of selected candidates according to their properties. These heuristics have been chosen ad hoc during the practical experimentations. The recognition logic sorts candidates according to their cost from the most suitable to the least suitable. Then, the most suitable candidate is examined by a deeper heuristic analysis. The deeper analysis definitely accepts, or rejects the candidate. As there is a need to analyze individual characters, this type of analysis consumes big amount of processor time.

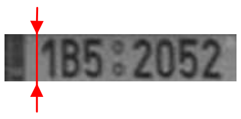
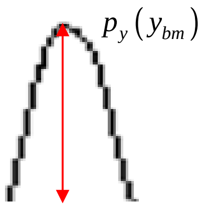
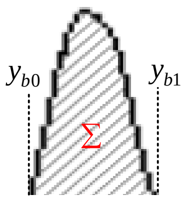
The basic concept of analysis can be illustrated by the following steps:

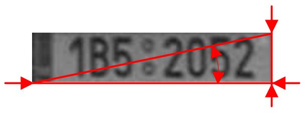
1. Detect possible number plate candidates.
2. Sort them according to their cost (determined by a basic heuristics).
3. Cut the first plate from the list with the best cost.
4. Segment and analyze it by a deeper analysis (time consuming).
5. If the deeper analysis refuses the plate, return to the step 3.

2.4.1 Priority selection and basic heuristic analysis of bands

The basic analysis is used to evaluate the cost of candidates, and to sort them according to this cost. There are several independent heuristics, which can be used to evaluate the cost α_i. The heuristics can be used separately, or they can be combined together to compute an overall cost of candidate by a weighted sum:

$$\alpha = 0.15 \cdot \alpha_1 + 0.25 \cdot \alpha_2 + 0.4 \cdot \alpha_3 + 0.4 \cdot \alpha_4$$

Heuristics	Illustration	Description
$\alpha_1 = \lvert y_{b0} - y_{b1} \rvert$		The height of band in pixels. Bands with a lower height will be preferred.
$\alpha_2 = \dfrac{1}{p_y(y_{bm})}$		The " $p_y(y_{bm})$ " is a maximum value of peak of vertical projection of snapshot, which corresponds to the processed band. Bands with a higher amount of vertical edges will be preferred.
$\alpha_3 = \dfrac{1}{\sum_{y=y_{b0}}^{y_{b1}} p_y(y)}$		This heuristics is similar to the previous one, but it considers not only the value of the greatest peak, but a value of area under the graph between points y_{b0} and y_{b1}. These points define a vertical position of the evaluated band.

$\alpha_4 = \left\lvert \frac{\lvert x_{p0} - x_{p1} \rvert}{\lvert y_{b0} - y_{b1} \rvert} - 5 \right\rvert$		The proportions of the one-row number plates are similar in the most countries. If we assume that width/height ratio of the plate is about five, we can compare the measured ratio with the estimated one to evaluate the cost of the number plate.

2.4.2 Deeper analysis

The deeper analysis determines the validity of a candidate for the number plate. Number plate candidates must be segmented into the individual characters to extract substantial features. The list of candidates is iteratively processed until the first valid number plate is found. The candidate is considered as a valid number plate, if it meets the requirements for validity.

Assume that plate p is segmented into several characters $p_0 \dots p_{n-1}$, where n is a number of characters. Let w_i be a width of ith character (see figure 2.9.a). Since all segmented characters have roughly uniform width, we can use a standard deviation of these values as a heuristics:

$$\beta_1 = \sqrt{\frac{1}{n} \sum_{i=0}^{n-1} \left(w_i - \bar{w} \right)^2}$$

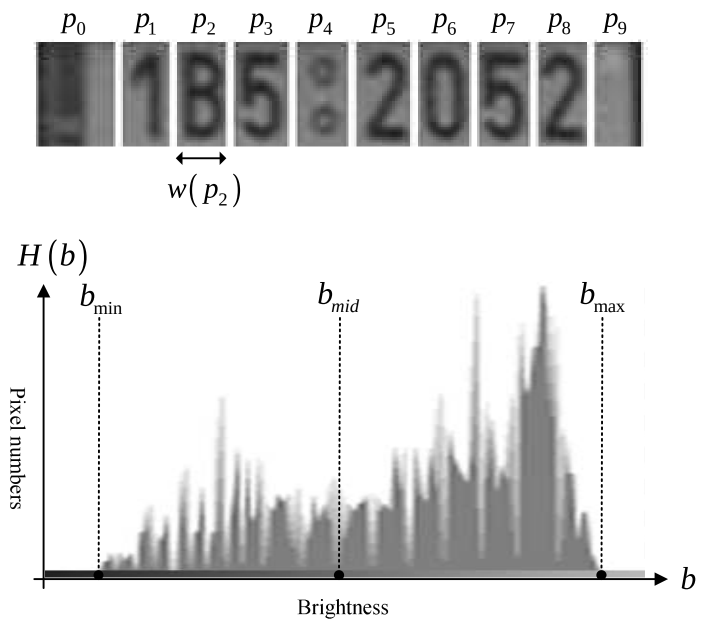
where $\bar{w}$ is an arithmetic average of character widths $\bar{w} = \frac{1}{n} \sum_{i=0}^{n-1} w_i$.

If we assume that the number plate consists of dark characters on a light background, we can use a brightness histogram to determine if the candidate meets this condition. Because some country-specific plates are negative, we can use the histogram to deal with this type of plates (see figure 2.9.b).

Let $H(b)$ be a brightness histogram, where b is a certain brightness value. Let $b_{\min}$ and $b_{\max}$ be a value of a darkest and lightest point. Then, $H(b)$ is a count of pixels, whose values are equal to b. The plate is negative when the heuristics β_2 is negative:

$$\beta_2 = \left(\sum_{b=b_{mid}}^{b_{\max}} H(b) \right) - \left(\sum_{b=b_{\min}}^{b_{mid}} H(b) \right)$$

where b_{mid} is a middle point in the histogram, such as $b_{mid} = \frac{b_{\max} - b_{\min}}{2}$.



A
B

Figure 2.9: (a) The number plate must be segmented into individual characters for deeper heuristic analysis. (b) Brightness histogram of the number plate is used to determine the positivity of the number plate.

2.5 Deskewing mechanism

The captured rectangular plate can be rotated and skewed in many ways due to the positioning of vehicle towards the camera. Since the skew significantly degrades the recognition abilities, it is important to implement additional mechanisms, which are able to detect and correct skewed plates.

The fundamental problem of this mechanism is to determine an angle, under which the plate is skewed. Then, deskewing of so evaluated plate can be realized by a trivial affine transformation.

It is important to understand the difference between the “sheared” and “rotated” rectangular plate. The number plate is an object in three-dimensional space, which is projected into the two-dimensional snapshot during the capture. The positioning of the object can sometimes cause the skew of angles and proportions.

If the vertical line of plate v_p is not identical to the vertical line of camera objective v_c, the plate may be sheared. If the vertical lines v_p and v_c are identical, but the axis a_p of plate is not parallel to the axis of camera a_c, the plate may be rotated. (see figure 2.10)

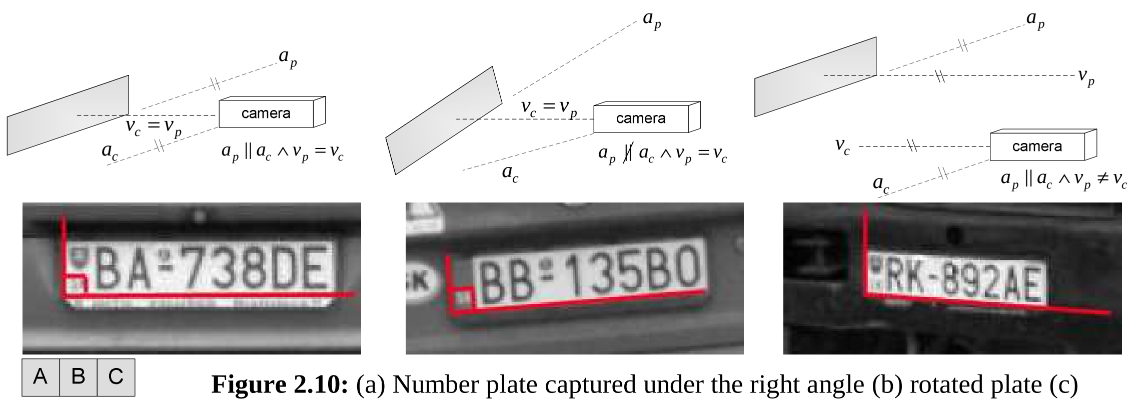

a_p
$v_c = v_p$
camera
a_c
$a_p \parallel a_c \wedge v_p = v_c$
a_p
$v_c = v_p$
camera
a_c
$a_p \nparallel a_c \wedge v_p = v_c$
a_p
v_p
v_c
camera
a_c
$a_p \parallel a_c \wedge v_p \neq v_c$
BA 738DE
BB 135BO
RK-892AE
A
B
C

Figure 2.10: (a) Number plate captured under the right angle (b) rotated plate (c) Sheared plate

2.5.1 Detection of skew

Hough transform is a special operation, which is used to extract features of a specific shape within a picture. The classical Hough transform is used for the detection of lines. The Hough transform is widely used for miscellaneous purposes in the problematic of machine vision, but I have used it to detect the skew of captured plate, and also to compute an angle of skew.

It is important to know, that Hough transform does not distinguish between the concepts such as "rotation" and "shear". The Hough transform can be used only to compute an approximate angle of image in a two-dimensional domain.

The mathematical representation of line in the orthogonal coordinate system is an equation $y = a \cdot x + b$, where a is a slope and b is a y-axis section of so defined line. Then, the line is a set of all points $[x, y]$, for which this equation is valid. We know that the line contains an infinite number of points as well as there are an infinite number of different lines, which can cross a certain point. The relation between these two assertions is a basic idea of the Hough transform.

The equation $y = a \cdot x + b$ can be also written as $b = -x \cdot a + y$, where x and y are parameters. Then, the equation defines a set of all lines (a, b), which can cross the point $[x, y]$. For each point in the "XY" coordinate system, there is a line in an "AB" coordinate system (so called "Hough space")

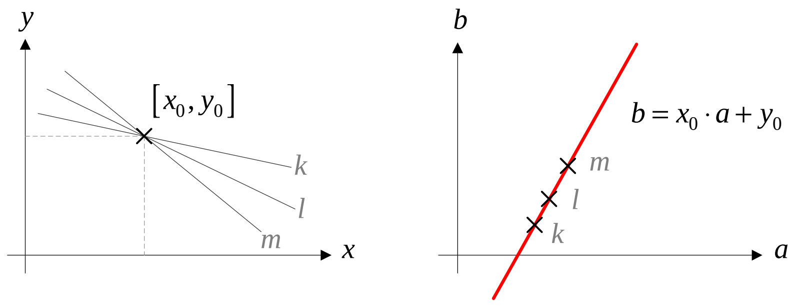


Figure 2.11: The "XY" and "AB" ("Hough space") coordinate systems. Each point $[x_0, y_0]$ in the "XY" coordinate system corresponds to one line in the Hough space (red color). The are several points (marked as k, l, m) in the Hough space, that correspond to the lines in the "XY" coordinate system, which can cross the point. $[x_0, y_0]$.

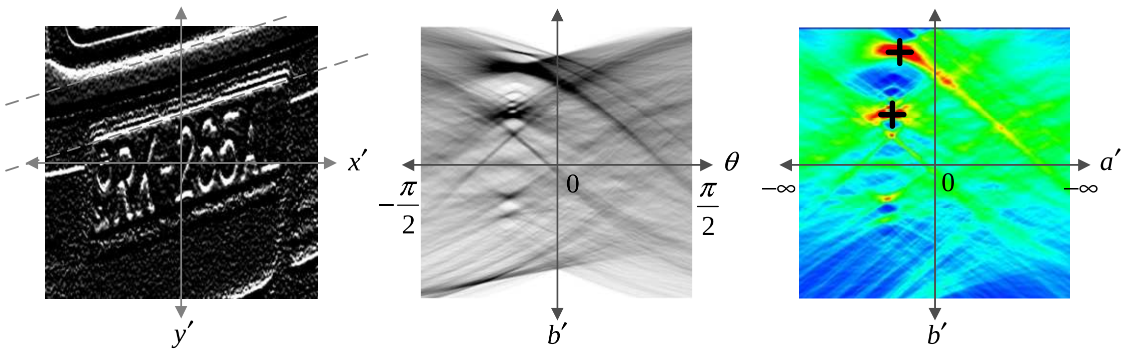
Let $f(x,y)$ be a continuous function. For each point $[a,b]$ in Hough space, there is a line in the "XY" coordinate system. We compute a magnitude of point $[a,b]$ as a summary of all points in the "XY" space, which lie on the line $a \cdot x + b$.

Assume that $f(x,y)$ is a discrete function, which represents the snapshot with definite dimensions $(w \times h)$. To compute the Hough transform of the function like this, it is necessary to normalize it into a unified coordinate system in the following way:

$$x' = \frac{2 \cdot x}{w} - 1 \; ; \; y' = \frac{2 \cdot y}{h} - 1$$

Although the space defined by a unified coordinate system is always discrete (floating point) on digital computers, we will assume that it is continuous. Generally, we can define the Hough transform $h'(a',b')$ of a continuous function $f'(x',y')$ in the unified coordinate system as:

$$h'(a',b') = \int_{-1}^{1} f'(x', a' \cdot x' + b')dx'$$



A | B | C

Figure 2.12: (a) Number plate in the unified " XY " coordinate system after application of the horizontal edge detection filter (b) Hough transform of the number plate in the " θB " coordinate system (c) Colored Hough transform in the " AB " coordinate system.

We use the Hough transform of certain image to evaluate its skew angle. You can see the colored Hough transform on the figure 2.12.c. The pixels with a relatively high value are marked by a red color. Each such pixel corresponds to a long white line in the figure 13.a. If we assume that the angle of such lines determines the overall angle, we can find the longest line as:

$$(a'_m, b'_m) = \arg \max_{\substack{0 \le a' \le 1 \\ 0 \le b' \le 1}} \{ h'(a',b') \}$$

To compute the angle of such a line, there is a need to transform it back to the original coordinate system:

$$[a_m, b_m] = \left[w \cdot \frac{a'_m - 1}{2}, h \cdot \frac{b'_m - 1}{2} \right]$$

where w and h are dimensions of the evaluated image. Then, the overall angle θ of image can be computed as:

$$\theta = \arctan(a_m)$$

The more sophisticated solution is to determine the angle from a horizontal projection of the Hough transform h'. This approach is much better because it covers all parallel lines together, not only the longest one:

$$\hat{\theta} = \arctan\left(w \cdot \frac{\hat{a}'_m - 1}{2} \right) \quad ; \quad \hat{a}'_m = \arg \max_{-1 \le a' \le 1} \left\{ p_{a'}\left(a'\right) \right\}$$

where $p_{a'}\left(a'\right)$ is a horizontal projection of the Hough space, such as:

$$p_{a'}\left(a'\right) = \int_{-1}^{1} f'\left(a', b'\right) db'$$

2.5.2 Correction of skew

The second step of a deskewing mechanism is a geometric operation over an image $f\left(x, y\right)$. As the skew detection based on Hough transform does not distinguish between the shear and rotation, it is important to choose the proper deskewing operation. In praxis, plates are sheared in more cases than rotated. To correct the plate sheared by the angle θ, we use the affine transformation to shear it by the negative angle $-\theta$.

For this transformation, we define a transformation matrix $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} 1 & S_y & 0 \\ S_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\tan\left(\theta\right) & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where S_x and S_y are shear factors. The S_x is always zero, because we shear the plate only in a direction of the Y-axis.

Let $\mathbf{P}$ be a vector representing the certain point, such as $\mathbf{P} = \left[x, y, 1\right]$ where x and y are coordinates of that point. The new coordinates $\mathbf{P}_s = \left[x_s, y_s, 1\right]$ of that point after the shearing can be computed as:

$$\mathbf{P}_s = \mathbf{P} \cdot \mathbf{A}$$

where $\mathbf{A}$ is a corresponding transformation matrix.

Let the deskewed number plate be defined by a function f_s. The function f_s can be computed in the following way:

$$f_s\left(x, y\right) = f\left(\left[x, y, 1\right] \cdot \mathbf{A} \cdot \left[1, 0, 0\right]^T, \left[x, y, 1\right] \cdot \mathbf{A} \cdot \left[0, 1, 0\right]^T \right)$$

After the substitution of the transformation matrix $\mathbf{A}$:

$$f_s\left(x, y\right) = f\left(\left[x, y, 1\right] \cdot \begin{bmatrix} 1 & -\tan\left(\theta\right) & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \; , \; \left[x, y, 1\right] \cdot \begin{bmatrix} 1 & -\tan\left(\theta\right) & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right)$$

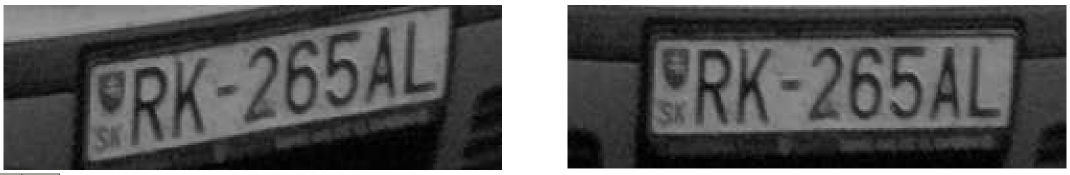


Figure 2.13: (a) Original number plate. (b) Number plate after deskewing.

Chapter 3

Principles of plate segmentation

The next step after the detection of the number plate area is a segmentation of the plate. The segmentation is one of the most important processes in the automatic number plate recognition, because all further steps rely on it. If the segmentation fails, a character can be improperly divided into two pieces, or two characters can be improperly merged together.

We can use a horizontal projection of a number plate for the segmentation, or one of the more sophisticated methods, such as segmentation using the neural networks. If we assume only one-row plates, the segmentation is a process of finding horizontal boundaries between characters. Section 3.2 deals with this problematic.

The second phase of the segmentation is an enhancement of segments. The segment of a plate contains besides the character also undesirable elements such as dots and stretches as well as redundant space on the sides of character. There is a need to eliminate these elements and extract only the character. Section 3.3 deals with these problems.

3.1 Segmentation of plate using a horizontal projection

Since the segmented plate is deskewed, we can segment it by detecting spaces in its horizontal projection. We often apply the adaptive thresholding filter to enhance an area of the plate before segmentation. The adaptive thresholding is used to separate dark foreground from light background with non-uniform illumination. You can see the number plate area after the thresholding in figure 3.1.a.

After the thresholding, we compute a horizontal projection $p_x(x)$ of the plate $f(x,y)$. We use this projection to determine horizontal boundaries between segmented characters. These boundaries correspond to peaks in the graph of the horizontal projection (figure 3.1.b).

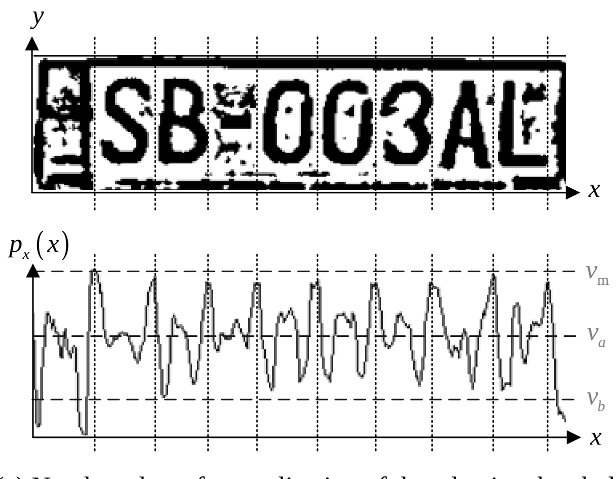


Figure 3.1: (a) Number plate after application of the adaptive thresholding (b) Horizontal projection of plate with detected peaks. Detected peaks are denoted by dotted vertical lines.

The goal of the segmentation algorithm is to find peaks, which correspond to the spaces between characters. At first, there is a need to define several important values in a graph of the horizontal projection $p_x(x)$:

- v_m - The maximum value contained in the horizontal projection $p_x(x)$, such as $v_m = \max\limits_{0 \le x < w} \{ p_x(x) \}$, where w is a width of the plate in pixels.
- v_a - The average value of horizontal projection $p_x(x)$, such as $v_a = \frac{1}{w} \sum\limits_{x=0}^{w-1} p_x(x)$
- v_b - This value is used as a base for evaluation of peak height. The base value is always calculated as $v_b = 2 \cdot v_a - v_m$. The v_a must lie on vertical axis between the values v_b and v_m.

The algorithm of segmentation iteratively finds the maximum peak in the graph of vertical projection. The peak is treated as a space between characters, if it meets some additional conditions, such as height of peak. The algorithm then zeroizes the peak and iteratively repeats this process until no further space is found. This principle can be illustrated by the following steps:

1. Determine the index of the maximum value of horizontal projection:
   $x_m = \arg\max\limits_{0 \le x < w} \{ p_x(x) \}$
2. Detect the left and right foot of the peak as:
   $x_l = \max\limits_{0 \le x \le x_m} \{ x \mid p_x(x) \le c_x \cdot p_x(x_m) \}$
   $x_r = \min\limits_{x_m \le x < w} \{ x \mid p_x(x) \le c_x \cdot p_x(x_m) \}$
3. Zeroize the horizontal projection $p_x(x)$ on interval $\langle x_l, x_r \rangle$
4. <u>If</u> $p_x(x_m) < c_w \cdot v_m$, go to step 7.
5. Divide the plate horizontally in the point x_m.
6. Go to step 1.
7. End.

Two different constants have been used in the algorithm above. The constant c_x is used to determine foots of peak x_m. The optimal value of c_x is <u>0.7</u>.

The constant c_w determines the minimum height of the peak related to the maximum value of the projection (v_m). If the height of the peak is below this minimum, the peak will not be considered as a space between characters. It is important to choose a value of constant c_w carefully. An inadequate small value causes that too many peaks will be treated as spaces, and characters will be improperly divided. A big value of c_w causes that not all regular peaks will be treated as spaces, and characters will be improperly merged together. The optimal value of c_w is <u>0.86</u>. To ensure a proper behavior of the algorithm, constants c_x and c_w should meet the following condition:

$$\forall (x_l, x_m, x_r) \in P : c_w \cdot v_m > p_x(x_l) \wedge p_x(x_r)$$

where P is a set of all detected peaks x_m with corresponding foots x_l and x_r.

3.2 Extraction of characters from horizontal segments

The segment of plate contains besides the character also redundant space and other undesirable elements. We understand under the term "segment" the part of a number plate determined by a horizontal segmentation algorithm. Since the segment has been processed by an adaptive thresholding filter, it contains only black and white pixels. The neighboring pixels are grouped together into larger pieces, and one of them is a character. Our goal is to divide the segment into the several pieces, and keep only one piece representing the regular character. This concept is illustrated in figure 3.2.

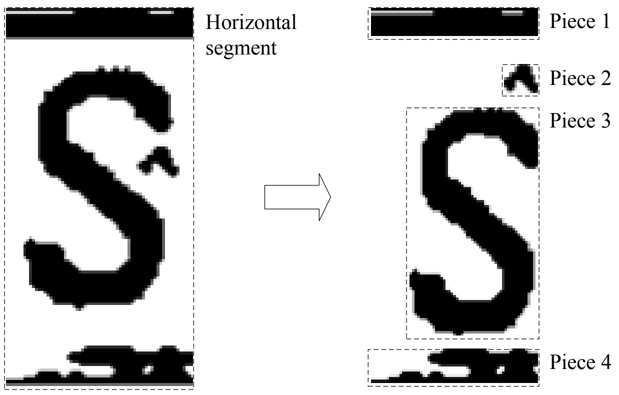


Figure 3.2: Horizontal segment of the number plate contains several groups (pieces) of neighboring pixels.

3.2.1 Piece extraction

Let the segment be defined by a discrete function $f(x,y)$ in the relative coordinate system, such as $[0,0]$ is an upper left corner of the segment, and $[w-1,h-1]$ is a bottom right corner, where w and h are dimensions of the segment. The value of $f(x,y)$ is "1" for the black pixels, and "0" for the white space.

The piece P is a set of all neighboring pixels $[x,y]$, which represents a continuous element. The pixel $[x,y]$ belongs to the piece P if there is at least one pixel $[x',y']$ from the P, such as $[x,y]$ and $[x',y']$ are neighbors:

$$[x,y]\in \mathrm{P} \Leftrightarrow \exists[x',y']\in \mathrm{P}:[x,y]\ddot{\mathrm{N}}_4[x',y']$$

The notation $a\ddot{\mathrm{N}}_4 b$ means a binary relation "a is a neighbor of b in a four-pixel neighborhood":

$$[x,y]\ddot{\mathrm{N}}_4[x',y'] \Leftrightarrow |x-x'|=1 \oplus |y-y'|=1$$

Algorithm

The goal of the piece extraction algorithm is to find and extract pieces from a segment of the plate. This algorithm is based on a similar principle as a commonly known "seed-fill" algorithm.

- Let piece P be a set of (neighboring) pixels $[x, y]$
- Let S be a set of all pieces P from a processed segment defined by the function $f(x, y)$.
- Let X be a set of all black pixels: $X = \{[x, y] \| f(x, y) = 1\}$
- Let A be an auxiliary set of pixels

Principle of the algorithm is illustrated by the following pseudo-code:

```
let set $S = \emptyset$
let set $X = \{[x,y] \| f(x,y) = 1 \wedge [0,0] \leq [x,y] < [w,h]\}$
while set $X$ is not empty do
begin
    let set $\mathrm{P} = \emptyset$
    let set $A = \emptyset$
    pull one pixel from set $X$ and insert it into set $A$
    while set $A$ is not empty do
    begin
        let $[x,y]$ be a certain pixel from $A$
        pull pixel $[x,y]$ from a set $A$
        if $f(x,y) = 1 \wedge [x,y] \notin A \wedge [0,0] \leq [x,y] < [w,h]$ then
        begin
            pull pixel $[x,y]$ from set $A$ and insert it into set $\mathrm{P}$
            insert pixels $[x-1,y]$, $[x+1,y]$, $[x,y-1]$, $[x,y+1]$ into set $A$
        end
    end
    add $\mathrm{P}$ to set $S$
end
```

Note 1: The operation “pull one pixel from a set” is non-deterministic, because a set is an unordered group of elements. In real implementation, a set will be implemented as an ordered list, and the operation “pull one pixel from a set” will be implemented as “pull the first pixel from a list”

Note 2: The mathematical conclusion $[x_{\min}, y_{\min}] < [x, y] < [x_{\max}, y_{\max}]$ means “The pixel $[x, y]$ lies in a rectangle defined by pixels $[x_{\min}, y_{\min}]$ and $[x_{\max}, y_{\max}]$”. More formally:

$$[x, y]\mathrm{R}[x', y'] \Leftrightarrow x\mathrm{R}x' \wedge y\mathrm{R}y'$$

where R is a one of the binary relations: ‘$<$’, ’$>$’, ’$\leq$’, ’$\geq$’ and ’$=$’.

3.2.2 Heuristic analysis of pieces

The piece is a set of pixels in the local coordinate system of the segment. The segment usually contains several pieces. One of them represents the character and others represent redundant elements, which should be eliminated. The goal of the heuristic analysis is to find a piece, which represents character.

Let us place the piece P into an imaginary rectangle (x_0, y_0, x_1, y_1), where $[x_0, y_0]$ is an upper left corner, and $[x_1, y_1]$ is a bottom right corner of the piece:

$$x_0 = \min\left\{x \middle| [x,y] \in \mathrm{P}\right\} \quad y_0 = \min\left\{y \middle| [x,y] \in \mathrm{P}\right\}$$
$$x_1 = \max\left\{x \middle| [x,y] \in \mathrm{P}\right\} \quad y_1 = \max\left\{y \middle| [x,y] \in \mathrm{P}\right\}$$

The dimensions and area of the imaginary rectangle are defined as $w = |x_0 - x_1|$, $h = |y_0 - y_1|$ and $S = w \cdot h$. Cardinality of the set P represents the number of black pixels n_b. The number of white pixels n_w can be then computed as $n_w = S - n_b = w \cdot h - |\mathrm{P}|$. The overall magnitude M of a piece is a ratio between the number of black pixels n_b and the area S of an imaginary rectangle $M = n_b / S$.

In praxis, we use the number of white pixels n_w as a heuristics. Pieces with a higher value of n_w will be preferred.

The piece chosen by the heuristics is then converted to a monochrome bitmap image. Each such image corresponds to one horizontal segment. These images are considered as an output of the segmentation phase of the ANPR process (see figure 3.3)

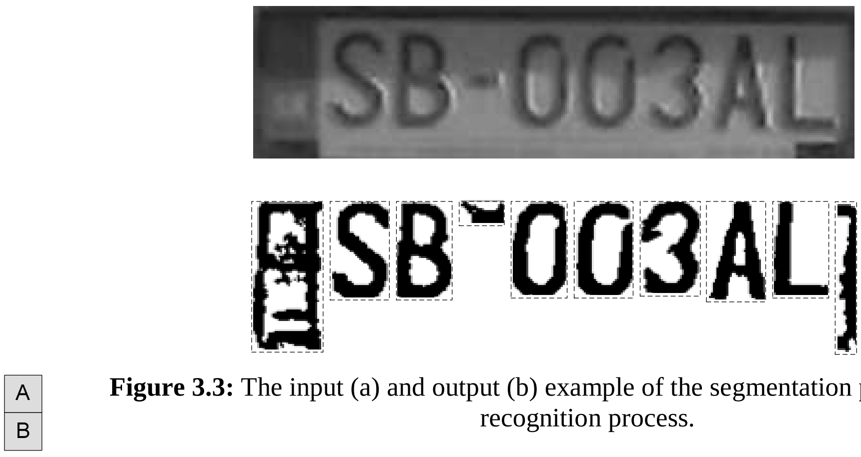


Figure 3.3: The input (a) and output (b) example of the segmentation phase of the ANPR recognition process.

Chapter 4

Feature extraction and normalization of characters

To recognize a character from a bitmap representation, there is a need to extract feature descriptors of such bitmap. As an extraction method significantly affects the quality of whole OCR process, it is very important to extract features, which will be invariant towards the various light conditions, used font type and deformations of characters caused by a skew of the image.

The first step is a normalization of a brightness and contrast of processed image segments. The characters contained in the image segments must be then resized to uniform dimensions (second step). After that, the feature extraction algorithm extracts appropriate descriptors from the normalized characters (third step). This chapter deals with various methods used in the process of normalization.

4.1 Normalization of brightness and contrast

The brightness and contrast characteristics of segmented characters are varying due to different light conditions during the capture. Because of this, it is necessary to normalize them. There are many different ways, but this section describes the three most used: histogram normalization, global and adaptive thresholding.

Through the histogram normalization, the intensities of character segments are re-distributed on the histogram to obtain the normalized statistics.

Techniques of the global and adaptive thresholding are used to obtain monochrome representations of processed character segments. The monochrome (or black & white) representation of image is more appropriate for analysis, because it defines clear boundaries of contained characters.

4.1.1 Histogram normalization

The histogram normalization is a method used to re-distribute intensities on the histogram of the character segments. The areas of lower contrast will gain a higher contrast without affecting the global characteristic of image.

Consider a grayscale image defined by a discrete function $f(x,y)$. Let I be a total number of gray levels in the image (for example $I = 256$). We use a histogram to determine the number of occurrences of each gray level i, $i \in 0 \dots I-1$:

$$H(i) = \left| \left\{ [x,y] \middle| 0 \le x < w \wedge 0 \le y < h \wedge f(x,y) = i \right\} \right|$$

The minimum, maximum and average value contained in the histogram is defined as:

$$H_{\min} = \min_{\substack{0 \le x < w \\ 0 \le y < h}} \left\{ f(x,y) \right\} \quad ; \quad H_{\max} = \max_{\substack{0 \le x < w \\ 0 \le y < h}} \left\{ f(x,y) \right\} \quad ; \quad H_{avg} = \frac{1}{w \cdot h} \sum_{x=0}^{w-1} \sum_{y=0}^{h-1} f(x,y)$$

where the values $H_{\min}$, $H_{\max}$ and H_{avg} are in the following relation:

$$0 \le H_{\min} \le H_{avg} \le H_{\max} \le I-1$$

The goal of the histogram normalization is to obtain an image with normalized statistical characteristics, such as $H_{\min}=0$, $H_{\max}=I-1$, $H_{avg}=\frac{I}{2}$. To meet this goal, we construct a transformation function $g(i)$ as a Lagrange polynomial with interpolation points $[x_1,y_1]=[H_{\min},0]$, $[x_2,y_2]=\left[H_{avg},\frac{I}{2}\right]$ and $[x_3,y_3]=[H_{\max},I-1]$:

$$g(i)=\sum_{j=1}^{3}\left(y_j\prod_{\substack{k=1\\k\neq j}}^{3}\frac{i-x_k}{x_j-x_k}\right)$$

This transformation function can be explicitly written as:

$$g(i)=y_1\cdot\frac{i-x_2}{x_1-x_2}\cdot\frac{i-x_3}{x_1-x_3}+y_2\cdot\frac{i-x_1}{x_2-x_1}\cdot\frac{i-x_3}{x_2-x_3}+y_3\cdot\frac{i-x_1}{x_3-x_1}\cdot\frac{i-x_2}{x_3-x_2}$$

After substitution of concrete points, and concrete number of gray levels $I=256$:

$$g(i)=+128\cdot\frac{i-H_{\min}}{H_{avg}-H_{\min}}\cdot\frac{i-H_{\max}}{H_{avg}-H_{\max}}+255\cdot\frac{i-H_{\min}}{H_{\max}-H_{\min}}\cdot\frac{i-H_{avg}}{H_{\max}-H_{avg}}$$

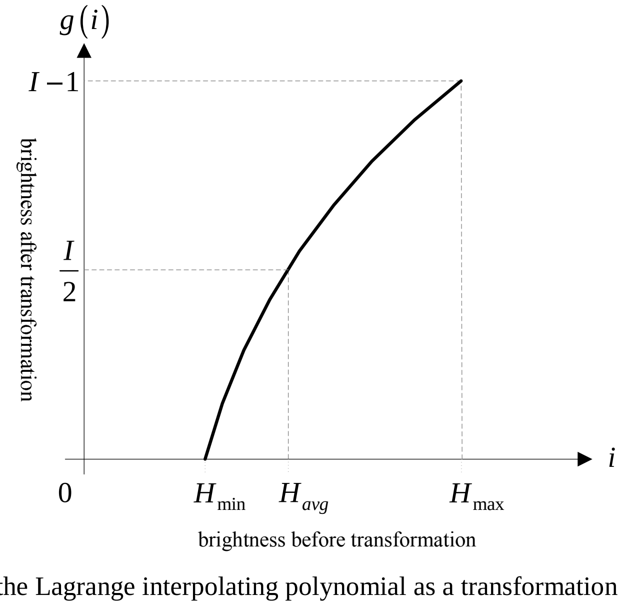


Figure 4.1: We use the Lagrange interpolating polynomial as a transformation function to normalize the brightness and contrast of characters.

The Lagrange interpolating polynomial as a transformation function is a costly solution. It is like harvesting one potato by a tractor. In praxis, there is more useful to construct the transformation using a simple linear function that spreads the interval $\langle H_{\min},H_{\max}\rangle$ into the unified interval $\langle 0,I-1\rangle$:

$$g(i) = \frac{i - H_{\min}}{H_{\max} - H_{\min}}(I - 1)$$

The normalization of image is proceeded by the transformation function in the following way:

$$f_n(x,y) = g\left(f(x,y)\right)$$

4.1.2 Global Thresholding

The global thresholding is an operation, when a continuous gray scale of an image is reduced into monochrome black & white colors according to the global threshold value. Let $\langle 0,1 \rangle$ be a gray scale of such image. If a value of a certain pixel is above the threshold t, the new value of the pixel will be zero. Otherwise, the new value will be one for pixels with values above the threshold t.

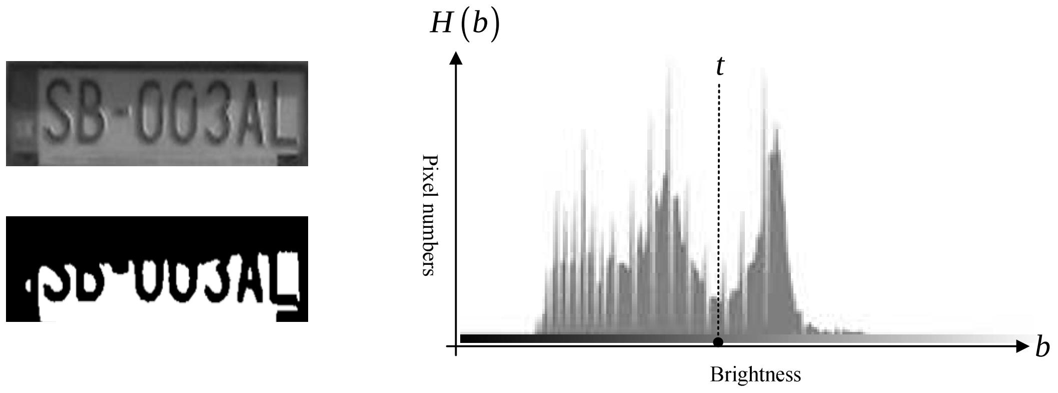
Let v be an original value of the pixel, such as $v \in \langle 0,1 \rangle$. The new value v' is computed as:

$$v' = \begin{cases} 0 & if \quad v \in \langle 0,t) \\ 1 & if \quad v \in \langle t,1 \rangle \end{cases}$$

The threshold value t can be obtained by using a heuristic approach, based on a visual inspection of the histogram. We use the following algorithm to determine the value of t automatically:

1. Select an initial estimate for threshold t (for example $t = 0.5$)
2. The threshold t divides the pixels into the two different sets: $S_a = \left\{ [x,y] \| f(x,y) < t \right\}$, and $S_b = \left\{ [x,y] \| f(x,y) \geq t \right\}$.
3. Compute the average gray level values μ_a and μ_b for the pixels in sets S_a and S_b as: $\mu_a = \frac{1}{|S_a|} \sum_{[x,y] \in S_a} f(x,y)$; $\mu_b = \frac{1}{|S_b|} \sum_{[x,y] \in S_b} f(x,y)$
4. Compute a new threshold value $t = \frac{1}{2}(\mu_a + \mu_b)$
5. Repeat steps 2, 3, 4 until the difference $\vartriangle t$ in successive iterations is smaller than predefined precision t_p

Since the threshold t is global for a whole image, the global thresholding can sometimes fail. Figure 4.2.a shows a partially shadowed number plate. If we compute the threshold t using the algorithm above, all pixels in a shadowed part will be below this threshold and all other pixels will be above this threshold. This causes an undesired result illustrated in figure 4.2.b.



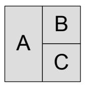


Figure 4.2: (a) The partially shadowed number plate. (b) The number plate after thresholding. (c) The threshold value t determined by an analysis of the histogram.

4.1.3 Adaptive thresholding

The number plate can be sometimes partially shadowed or nonuniformly illuminated. This is most frequent reason why the global thresholding fail. The adaptive thresholding solves several disadvantages of the global thresholding, because it computes threshold value for each pixel separately using its local neighborhood.

Chow and Kaneko approach

There are two approaches to finding the threshold. The first is the *Chow and Kaneko approach*, and the second is a *local thresholding*. The both methods assumes that smaller rectangular regions are more likely to have approximately uniform illumination, more suitable for thresholding. The image is divided into uniform rectangular areas with size of $m \times n$ pixels. The local histogram is computed for each such area and a local threshold is determined. The threshold of concrete point is then computed by interpolating the results of the subimages.

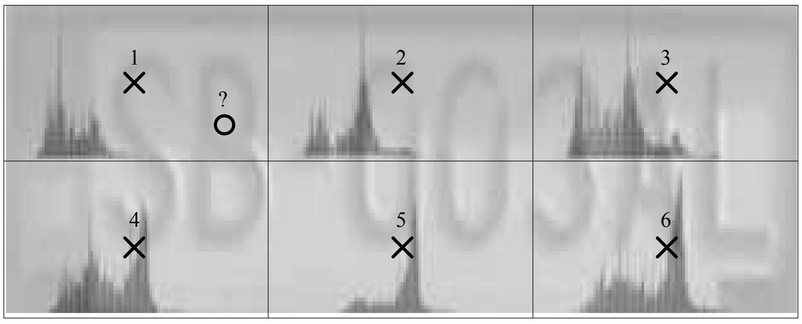


Figure 4.3: The number plate (from figure 4.2) processed by the Chow and Kaneko approach of the adaptive thresholding. The number plate is divided into the several areas, each with own histogram and threshold value. The threshold value of a concrete pixel (denoted by ○) is computed by interpolating the results of the subimages (represented by pixels 1-6).

Local thresholding

The second way of finding the local threshold of pixel is a statistical examination of neighboring pixels. Let $[x, y]$ be a pixel, for which we compute the local threshold t. For simplicity we condider a square neighborhood with width $2 \cdot r + 1$, where $[x-r, y-r]$, $[x-r, y+r]$, $[x+r, y-r]$ and $[x+r, y+r]$ are corners of such square. There are severals approaches of computing the value of threshold:

- Mean of the neighborhood : $t(x,y) = \underset{\substack{x-r \le i \le x+r \\ y-r \le j \le y+r}}{\text{mean}} \{ f(i,j) \}$
- Median of the neighborhood : $t(x,y) = \underset{\substack{x-r \le i \le x+r \\ y-r \le j \le y+r}}{\text{median}} \{ f(i,j) \}$
- Mean of the minimum and maximum value of the heighborhood:
  $t(x,y) = \frac{1}{2} \left(\min_{\substack{x-r \le i \le x+r \\ y-r \le j \le y+r}} \{ f(i,j) \} + \max_{\substack{x-r \le i \le x+r \\ y-r \le j \le y+r}} \{ f(i,j) \} \right)$

The new value $f'(x,y)$ of pixel $[x,y]$ is then computes as:

$$f'(x,y) = \begin{cases} 0 & if \quad f(x,y) \in \langle 0, t(x,y)) \\ 1 & if \quad f(x,y) \in \langle 0, t(x,y) \rangle \end{cases}$$

4.2 Normalization of dimensions and resampling

Before extracting feature descriptors from a bitmap representation of a character, it is necessary to normalize it into unified dimensions. We understand under the term “resampling” the process of changing dimensions of the character. As original dimensions of unnormalized characters are usually higher than the normalized ones, the characters are in most cases downsampled. When we downsample, we reduce information contained in the processed image.

There are several methods of resampling, such as the pixel-resize, bilinear interpolation or the weighted-average resampling. We cannot determine which method is the best in general, because the successfulness of particular method depends on many factors. For example, usage of the weighed-average downsampling in combination with a detection of character edges is not a good solution, because this type of downsampling does not preserve sharp edges (discussed later). Because of this, the problematic of character resampling is closely associated with the problematic of feature extraction.

We will assume that $m \times n$ are dimensions of the original image, and $m' \times n'$ are dimensions of the image after resampling. The horizontal and vertical aspect ratio is defined as $r_x = m'/m$ and $r_y = n'/n$, respectively.

4.2.1 Nearest-neighbor downsampling

The principle of the nearest-neighbor downsamping is a picking the nearest pixel in the original image that corresponds to a processed pixel in the image after resampling. Let $f(x,y)$ be a discrete function defining the original image, such as $0 \le x < m$ and $0 \le y < n$. Then, the function $f'(x',y')$ of the image after resampling is defined as:

$$f'(x',y') = f\left(\left\lfloor \frac{x'}{r_x} \right\rfloor, \left\lfloor \frac{y'}{r_y} \right\rfloor \right)$$

where $0 \le x' < m'$ and $0 \le y' < n'$.

If the aspect ratio is lower than one, then each pixel in the resampled (destination) image corresponds to a group of pixels in the original image, but only one value from the group of source pixels affects the value of the pixel in the resampled image. This fact causes a significant reduction of information contained in original image (see figure 4.5).

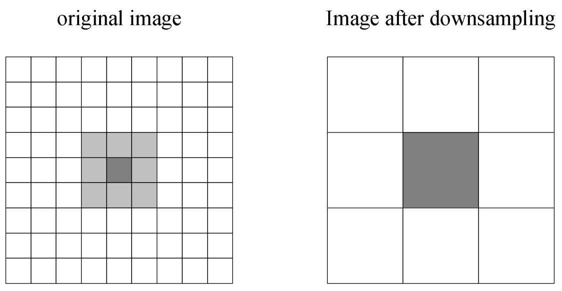


Figure 4.4: One pixel in the resampled image corresponds to a group of pixels in the original image

Although the nearest neighbor downsamping significantly reduces information contained in the original image by ignoring a big amount of pixels, it preserves sharp edges and the strong bipolarity of black and white pixels. Because of this, the nearest neighbor downsamping is suitable in combination with the "edge detection" feature extraction method described in section 4.3.2.

4.2.2 Weighed-average downsampling

In contrast with the nearest-neighbor method, the weighted-average downsamping considers all pixels from a corresponding group of pixels in the original image.

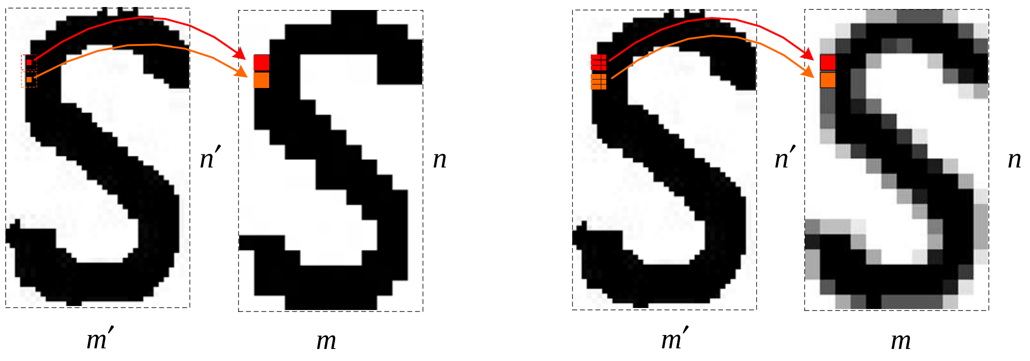
Let r_x and r_y be a horizontal and vertical aspect ratio of the resampled image. The value of the pixel $[x', y']$ in the destination image is computed as a mean of source pixels in the range $[x_{\min}, y_{\min}]$ to $[x_{\max}, y_{\max}]$:

$$f'(x', y') = \frac{1}{(x_{\max} - x_{\min}) \cdot (y_{\max} - y_{\min})} \sum_{i=x_{\min}}^{x_{\max}} \sum_{j=y_{\min}}^{y_{\max}} f(i, j)$$

where:

$$x_{\min} = \left\lfloor \frac{x'}{r_x} \right\rfloor ; \; y_{\min} = \left\lfloor \frac{y'}{r_y} \right\rfloor ; \; x_{\max} = \left\lfloor \frac{x'+1}{r_x} \right\rfloor ; \; y_{\max} = \left\lfloor \frac{y'+1}{r_y} \right\rfloor$$

The weighted-average method of downsampling does not preserve sharp edges of the image (in contrast with the previous method). You can see the visual comparison of these two methods in Figure 4.5.



A B **Figure 4.5:** (a) Nearest-neighbor resampling significantly reduces information contained in the original image, but it preserves sharp edges. (b) Weighted average resampling gives a better visual result, but the edges of the result are not sharp.

4.3 Feature extraction

Information contained in a bitmap representation of an image is not suitable for processing by computers. Because of this, there is need to describe a character in another way. The description of the character should be invariant towards the used font type, or deformations caused by a skew. In addition, all instances of the same character should have a similar description. A description of the character is a vector of numeral values, so-called “descriptors”, or “patterns”:

$$\mathbf{x} = (x_0, \ldots, x_{n-1})$$

Generally, the description of an image region is based on its *internal* and *external* representation. The internal representation of an image is based on its regional properties, such as color or texture. The external representation is chosen when the primary focus is on shape characteristics. The description of normalized characters is based on its external characteristics because we deal only with properties such as character shape. Then, the vector of descriptors includes characteristics such as number of lines, bays, lakes, the amount of horizontal, vertical and diagonal or diagonal edges, and etc. The feature extraction is a process of transformation of data from a bitmap representation into a form of descriptors, which are more suitable for computers.

If we associate similar instances of the same character into the classes, then the descriptors of characters from the same class should be geometrically closed to each other in the vector space. This is a basic assumption for successfulness of the pattern recognition process.

This section deals with various methods of feature extraction, and explains which method is the most suitable for a specific type of character bitmap. For example, the “edge detection” method should not be used in combination with a blurred bitmap.

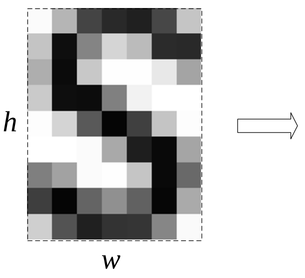
4.3.1 Pixel matrix

The simplest way to extract descriptors from a bitmap image is to assign a brightness of each pixel with a corresponding value in the vector of descriptors. Then, the length of such vector is equal to a square ($w \cdot h$) of the transformed bitmap:

$$x_i = f\left(\left\lfloor \frac{i}{w} \right\rfloor, \mathrm{mod}_w(i)\right)$$

where $i \in 0,\ldots,w \cdot h - 1$.

Bigger bitmaps produce extremely long vector of descriptors, which is not suitable for recognition. Because of this, size of such processed bitmap is very limited. In addition, this method does not consider geometrical closeness of pixels, as well as its neighboring relations. Two slightly biased instances of the same character in many cases produce very different description vectors. Even though, this method is suitable if the character bitmaps are too blurry or too small for edge detection.



$$\mathbf{x} = \begin{pmatrix} 251, 181, 068, 041, 032, 071, 197, \\ 196, 014, 132, 213, 187, 043, 041, \\ 174, 011, 200, 254, 254, 232, 164, \\ 202, 014, 012, 128, 242, 255, 255, \\ 253, 212, 089, 005, 064, 196, 253, \\ 255, 255, 251, 196, 030, 009, 165, \\ 127, 162, 251, 254, 197, 009, 105, \\ 062, 005, 100, 144, 097, 006, 170, \\ 207, 083, 032, 051, 053, 134, 250 \end{pmatrix}$$

Figure 4.6: The “pixel matrix” feature extraction method

4.3.2 Detection of character edges

In contrast with the previous method, the detection of character edges does not consider absolute positioning of each pixel, but only a number of occurrences of individual edge types in a specific region of the character bitmap. Because of this, the resulting vector is invariant towards the intra-regional displacement of the edges, and towards small deformations of characters.

Bitmap regions

Let the bitmap be described by a discrete function $f(x,y)$, where w and h are dimensions, such as $0 \le x < w$ and $0 \le y < h$. We divide it into six equal regions organized to three rows and two columns in the following way:

Let $\left[x_{\min}^{(i)}, y_{\min}^{(i)}\right]$ and $\left[x_{\max}^{(i)}, y_{\max}^{(i)}\right]$ be an upper left and bottom right point of a rectangle, which determinates the region r_i, such as:

- Region r_0 : $x_{\min}^{(0)} = 0$, $y_{\min}^{(0)} = 0$, $x_{\max}^{(0)} = \left\lceil \frac{w}{2} \right\rceil - 1$, $y_{\max}^{(0)} = \left\lceil \frac{h}{3} \right\rceil - 1$
- Region r_1 : $x_{\min}^{(1)} = \left\lceil \frac{w}{2} \right\rceil$, $y_{\min}^{(1)} = 0$, $x_{\max}^{(1)} = w - 1$, $y_{\max}^{(1)} = \left\lceil \frac{h}{3} \right\rceil - 1$
- Region r_2 : $x_{\min}^{(2)} = 0$, $y_{\min}^{(2)} = \left\lceil \frac{h}{3} \right\rceil$, $x_{\max}^{(2)} = \left\lceil \frac{w}{2} \right\rceil - 1$, $y_{\max}^{(2)} = \left\lceil \frac{2 \cdot h}{3} \right\rceil - 1$
- Region r_3 : $x_{\min}^{(3)} = \left\lceil \frac{w}{2} \right\rceil$, $y_{\min}^{(3)} = \left\lceil \frac{h}{3} \right\rceil$, $x_{\max}^{(3)} = w - 1$, $y_{\max}^{(3)} = \left\lceil \frac{2 \cdot h}{3} \right\rceil - 1$

- Region r_4 : $x_{\min}^{(4)} = 0$, $y_{\min}^{(4)} = \left\lceil \frac{2 \cdot h}{3} \right\rceil$, $x_{\max}^{(4)} = \left\lceil \frac{w}{2} \right\rceil$, $y_{\max}^{(4)} = h - 1$
- Region r_5 : $x_{\min}^{(5)} = \left\lceil \frac{w}{2} \right\rceil$, $y_{\min}^{(5)} = \left\lceil \frac{2 \cdot h}{3} \right\rceil$, $x_{\max}^{(5)} = w - 1$, $y_{\max}^{(5)} = h - 1$

There are several ways how to distribute regions in the character bitmap. The regions can be disjunctive as well as they can overlap each other. The figure 4.7 shows the several possible layouts of regions.

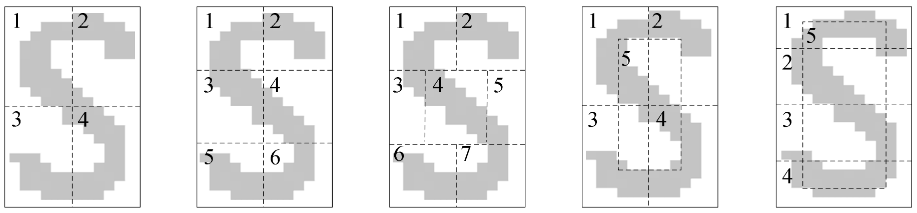


Figure 4.7: Layouts of regions in the character bitmap. The regions can be disjunctive as well as they can overlap each other.

Edge types in region

Let us define an edge of the character as a 2x2 white-to-black transition in a bitmap. According to this definition, the bitmap image can contain fourteen different edge types illustrated in figure 4.8.

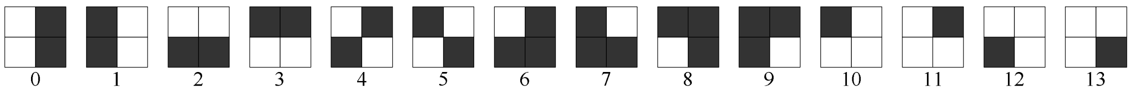


Figure 4.8: The processed bitmap can contain different types of 2x2 edges.

The statistics of occurrence of each edge type causes uselessly long vector of descriptors. Because of this, the "similar" types of edges are considered as the same. The following lists shows how the edges can be grouped together:

1. *0 + 1* (vertical edges)
2. *2 + 3* (horizontal edges)
3. *4 + 6 + 9* ("/"-type diagonal edges)
4. *5 + 7 + 8* ("\"-type diagonal edges)
5. *10* (bottom right corner)
6. *11* (bottom left corner)
7. *12* (top right corner)
8. *13* (top left corner)

For simplicity, assume that edge types are not grouped together. Let η be a number of different edge types, where $\mathbf{h}_i$ is a 2x2 matrix that corresponds to the specific type of edge:

$$\mathbf{h}_0 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \mathbf{h}_1 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \mathbf{h}_2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \mathbf{h}_3 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \mathbf{h}_4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{h}_5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \mathbf{h}_6 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$
$$\mathbf{h}_7 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \mathbf{h}_8 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \mathbf{h}_9 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{h}_{10} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \mathbf{h}_{11} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \mathbf{h}_{12} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \mathbf{h}_{13} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

Let ρ be a number of rectangular regions in the character bitmap, where $x_{\min}^{(i)}$, $y_{\min}^{(i)}$, $x_{\max}^{(i)}$ and $y_{\max}^{(i)}$ are boundaries of the region r_i ($i \in 0 \ldots \rho - 1$). If the statistics consider η different edge types for each of ρ regions, the length of the resulting vector $\mathbf{x}$ is computed as $\eta \cdot \rho$:

$$\mathbf{x} = (x_0, x_1, \ldots, x_{\eta \cdot \rho - 1})$$

Feature extraction algorithm

At first, we have to embed the character bitmap $f(x, y)$ into a bigger bitmap with white padding to ensure a proper behavior of the feature extraction algorithm. Let the padding be one pixel wide. Then, dimensions of the embedding bitmap will be $w + 2$ and $h + 2$. The embedding bitmap $f'(x, y)$ is then defined as:

$$f'(x,y) = \begin{cases} 1 & \textit{if} \quad x = 0 \vee y = 0 \vee x = w+1 \vee y = h+1 \\ f(x-1, y-1) & \textit{if} \quad \neg(x = 0 \vee y = 0 \vee x = w+1 \vee y = h+1) \end{cases}$$

where w and h are dimensions of character bitmap <u>before</u> embedding. Color of the padding is white (value of 1). The coordinates of pixels are shifted one pixel towards the original position.

The structure of vector of output descriptors is illustrated by the pattern below. The notation $\mathbf{h}_j @ r_i$ means "number occurrences of an edge represented by the matrix $\mathbf{h}_j$ in the region r_i".

$$\mathbf{x} = \Big(\underbrace{\mathbf{h}_0 @ r_0, \mathbf{h}_1 @ r_0, \ldots, \mathbf{h}_{\eta-1} @ r_0}_{\text{region } r_0}, \underbrace{\mathbf{h}_0 @ r_1, \mathbf{h}_1 @ r_1, \ldots, \mathbf{h}_{\eta-1} @ r_1}_{\text{region } r_1}, \underbrace{\mathbf{h}_0 @ r_{\rho-1}, \mathbf{h}_1 @ r_{\rho-1}, \ldots, \mathbf{h}_{\eta-1} @ r_{\rho-1}}_{\text{region } r_{\rho-1}} \Big)$$

We compute the position k of the $\mathbf{h}_j @ r_i$ in the vector $\mathbf{x}$ as $k = i \cdot \eta + j$, where η is the number of different edge types (and also the number of corresponding matrices).

The following algorithm demonstrates the computation of the vector of descriptors $\mathbf{x}$:

```
zeroize vector $\mathbf{x}$
for each region $r_i$, where $i \in 0,\ldots,\rho-1$ do
begin
    for each pixel $[x,y]$ in region $r_i$,where $x_{\min}^{(i)} \le x \le x_{\max}^{(i)}$ and $y_{\min}^{(i)} \le y \le y_{\max}^{(i)}$ do
    begin
        for each matrix $\mathbf{h}_j$, where $j \in 0,\ldots,\eta-1$ do
        begin
            if $\mathbf{h}_j = \begin{bmatrix} f'(x,y) & f'(x+1,y) \\ f'(x,y+1) & f'(x+1,y+1) \end{bmatrix}$ then
            begin
                let $k = i \cdot \eta + j$
                let $\mathbf{x}_k = \mathbf{x}_k + 1$
            end
        end
    end
end
```

4.3.3 Skeletonization and structural analysis

The feature extraction techniques discussed in the previous two chapters are based on the statistical image processing. These methods do not consider structural aspects of analyzed images. The small difference in bitmaps sometimes means a big difference in the structure of contained characters. For example, digits ‘6’ and ‘8’ have very similar bitmaps, but there is a substantial difference in their structures.

The structural analysis is based on higher concepts than the edge detection method. It does not deal with terms such as “pixels” or “edges”, but it considers more complex structures (like junctions, line ends or loops). To analyze these structures, we must involve the thinning algorithm to get a skeleton of the character. This chapter deals with the principle of skeletonization as well as with the principle of structural analysis of skeletonized image.

The concept of skeletonization

The skeletonization is a reduction of the structural shape into a graph. This reduction is accomplished by obtaining a *skeleton* of the region via the skeletonization algorithm. The skeleton of a shape is mathematically defined as a *medial axis transformation.* To define the medial axis transformation and skeletonization algorithm, we must introduce some elementary prerequisite terms.

Let $\ddot{\mathrm{N}}$ be a binary relation between two pixels $\left[x,y\right]$ and $\left[x',y'\right]$, such as $a\ddot{\mathrm{N}}b$ means “a is a neighbor of b”. This relation is defined as:

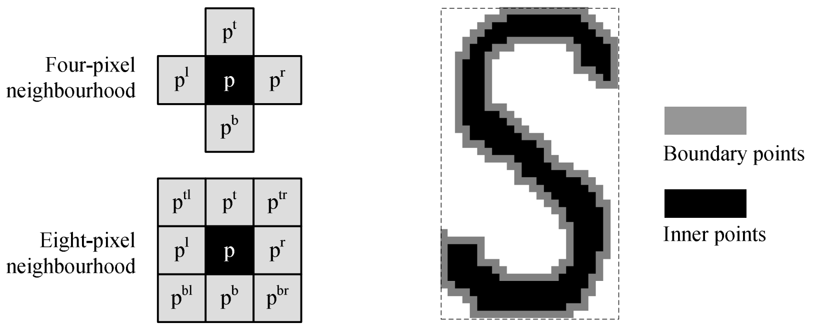
$$\left[x,y\right]\ddot{\mathrm{N}}_8\left[x',y'\right]\Leftrightarrow\left|x-x'\right|=1\vee\left|y-y'\right|=1 \quad \text{for eight-pixel neighbourhood}$$
$$\left[x,y\right]\ddot{\mathrm{N}}_4\left[x',y'\right]\Leftrightarrow\left|x-x'\right|=1\oplus\left|y-y'\right|=1 \quad \text{for four-pixel neighbourhood}$$

The border B of character is a set of boundary pixels. The pixel $\left[x,y\right]$ is a boundary pixel, if it is black and if it has at least one white neighbor in the eight-pixel neighborhood:

$$\left[x,y\right]\in B\Leftrightarrow f\left(x,y\right)=0\wedge\exists\left[x',y'\right]: f\left(x,y\right)=1\wedge\left[x,y\right]\ddot{\mathrm{N}}_8\left[x',y'\right]$$

The inner region I of character is a set of black pixels, which are not boundary pixels:

$$\left[x,y\right]\in I\Leftrightarrow f\left(x,y\right)=0\wedge\left[x,y\right]\notin B$$



A B

Figure 4.9: (a) Illustration of the four-pixel and eight-pixel neighborhood. (b) The set of boundary and inner pixels of character.

The piece P is then a union of all boundary and inner pixels ($\mathrm{P}=B\cup I$). Since there is only one continuous group of black pixels, all black pixels belong to the piece P. The principle and the related terminology of the skeletonization are similar to the piece extraction algorithm discussed in section 3.2.1.

Medial axis transformation

The medial axis transformation of the piece P defined as follows. For each inner pixel $p \in I$, we find the closest boundary pixel $p_b \in B$. If a pixel p has more than one such neighbor, it is said to belong to the medial axis (or skeleton) of the P. The concept of the closest boundary pixel depends on the definition of the Euclidean distance between two pixels in the orthogonal coordinate system. Mathematically, the medial axis (or skeleton) S is a subset of the P defined as:

$$p \in S \Leftrightarrow \exists p_1 \exists p_2 : p_1 \in B \wedge p_2 \in B \wedge d(p, p_1) = d(p, p_2) = \min_{p' \in B} \{ d(p, p') \}$$

The pixel p belongs to the medial axis S if there exists at least two pixels p_1 and p_2, such as Euclidean distance between pixels p and p_1 is equal to the distance between pixels p and p_2, and these pixels are closest boundary pixels to pixel p.

The Euclidean distance between two pixels $p_1 = [x_1, y_1]$ and $p_2 = [x_2, y_2]$ is defined as:

$$d(p_1, p_2) = \sqrt[2]{(x_1 - x_2)^2 \cdot (x_1 - x_2)^2}$$

Skeletonization algorithm

Direct implementation of the mathematical definition of the medial axis transformation is computationally expensive, because it involves calculating the distance from every inner pixel from the set I to every pixel on the boundary B.

The medial axis transformation is intuitively defined by a so-called “fire front” concept. Consider that a fire is lit along the border. All fire fronts will advance into the inner of character at the same speed. The skeleton of a character is then a set of pixels reached by more than one fire front at the same time.

The skeletonization (or thinning) algorithm is based on the “fire front” concept. The thinning is a morphological operation, which preserves end-pixels and does not break connectivity. Assume that pixels of the piece are black (value of zero), and background pixels are white (value of one).

The thinning is an iterative process of two successive steps applied to boundary pixels of a piece. With reference to the eight-pixel neighborhood notation in figure 4.9, the first step flags a boundary pixel p for deletion if each of the following conditions is satisfied:

- At least one of the top, right and bottom neighbor of the pixel p must be white (the pixel p is white just when it does not belong to the piece P).

  $p^t \notin \mathrm{P} \vee p^r \notin \mathrm{P} \vee p^b \notin \mathrm{P}$

- At least one of the left, right and bottom neighbor of pixel p must be white.

  $p^l \notin \mathrm{P} \vee p^r \notin \mathrm{P} \vee p^b \notin \mathrm{P}$

- The pixel p must have at least two, and at most six black neighbors from the piece P. This condition prevents the algorithm from erasing end-points and from breaking the connectivity.

$$2 \leq \left| \left\{ p' \middle| p \ddot{\mathrm{N}}_8 p' \wedge p' \notin \mathrm{P} \right\} \right| \leq 6$$

- The number of white-to-black transitions in the ordered sequence $p^t, p^{tr}, p^r, p^{br}, p^b, p^{bl}, p^l, p^{tl}, p^t$ must be equal to one.

$$v\left(p^t \in \mathrm{P} \wedge p^{tr} \notin \mathrm{P}\right) + v\left(p^{tr} \in \mathrm{P} \wedge p^r \notin \mathrm{P}\right) + v\left(p^r \in \mathrm{P} \wedge p^{br} \notin \mathrm{P}\right) + v\left(p^{br} \in \mathrm{P} \wedge p^b \notin \mathrm{P}\right)$$
$$v\left(p^b \in \mathrm{P} \wedge p^{bl} \notin \mathrm{P}\right) + v\left(p^{bl} \in \mathrm{P} \wedge p^l \notin \mathrm{P}\right) + v\left(p^l \in \mathrm{P} \wedge p^{tl} \notin \mathrm{P}\right) + v\left(p^{tl} \in \mathrm{P} \wedge p^t \notin \mathrm{P}\right) = 1$$

$$v(x) = \begin{cases} 0 & if \quad x \\ 1 & if \quad \neg x \end{cases}$$

The first step flags pixel p for deletion, if its neighborhood meets the conditions above. However, the pixel is not deleted until all other pixels have been processed. If at least one of the conditions is not satisfied, the value of pixel p is not changed.

After step one has been applied to all boundary pixels, the flagged pixels are definitely deleted in the second step. Every iteration of these two steps thins the processed character. This iterative process is applied until no further pixels are marked for deletion. The result of thinning algorithm is a skeleton (or medial axis) of the processed character.

- Let the piece P be a set of all black pixels contained in skeletonized character.
- Let B be a set of all boundary pixels.

The following pseudo-code demonstrates the thinning algorithm more formally. This algorithm proceeds the medial axis transformation over a piece P.

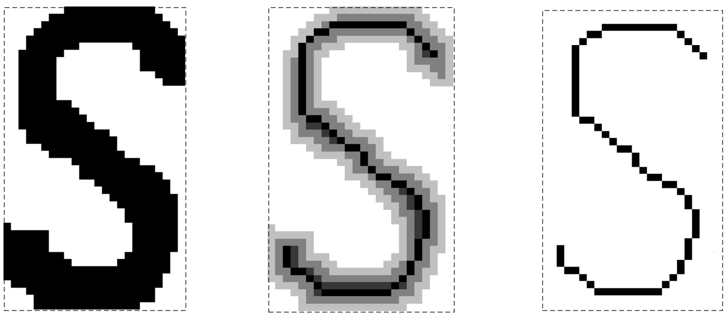
```
do  // iterative thinning process
    let continue = false
    let $B = \emptyset$
    for each pixel $p$ in piece $\mathrm{P}$ do  // create a set of boundary pixels
        if $\exists p' : p' \notin \mathrm{P} \wedge p \ddot{\mathrm{N}}_8 p'$ then  // if the pixel $p$ has at least one white neighbor
            insert pixel $p$ into set $B$  // but keep it also in $\mathrm{P}$

    for each pixel $p$ in set $B$ do  // 1.step of the iteration
    begin
        // if at least one condition is violated, skip this pixel
        if $\neg\left(p^t \notin \mathrm{P} \vee p^r \notin \mathrm{P} \vee p^b \notin \mathrm{P}\right)$ then continue
        if $\neg\left(p^l \notin \mathrm{P} \vee p^r \notin \mathrm{P} \vee p^b \notin \mathrm{P}\right)$ then continue
        if $\neg\left(2 \leq \left| \left\{ p' \middle| p \ddot{\mathrm{N}}_8 p' \wedge p' \notin \mathrm{P} \right\} \right| \leq 6\right)$ then continue
        if
      $v\left(p^t \in \mathrm{P} \wedge p^{tr} \notin \mathrm{P}\right) + v\left(p^{tr} \in \mathrm{P} \wedge p^r \notin \mathrm{P}\right) + v\left(p^r \in \mathrm{P} \wedge p^{br} \notin \mathrm{P}\right) + v\left(p^{br} \in \mathrm{P} \wedge p^b \notin \mathrm{P}\right)$
      $v\left(p^b \in \mathrm{P} \wedge p^{bl} \notin \mathrm{P}\right) + v\left(p^{bl} \in \mathrm{P} \wedge p^l \notin \mathrm{P}\right) + v\left(p^l \in \mathrm{P} \wedge p^{tl} \notin \mathrm{P}\right) + v\left(p^{tl} \in \mathrm{P} \wedge p^t \notin \mathrm{P}\right) \neq 1$
      then
        begin
            continue
        end
        // all tests passed
        flag point $p$ for deletion
        let continue = true
    end

    for each pixel $p$ in set $B$ do // 2.step of the iteration
        if $p$ is flagged then
            pull point $p$ from piece $\mathrm{P}$

while continue = true
```

Note: The pixel p *belongs to the piece* P *when it is black:* $p \in \mathrm{P} \Leftrightarrow f(p) = 0$



A | B | C

Figure 4.10: (a) The character bitmap before skeletonization. (b) The thinning algorithm iteratively deletes boundary pixels. Pixels deleted in the first iteration are marked by a light gray color. Pixels deleted in the second and third iteration are marked by dark gray. (c) The result of the thinning algorithm is a skeleton (or a medial axis).

Structural analysis of skeletonized character

The structural analysis is a feature extraction method that considers more complex structures than pixels. The basic idea is that the substantial difference between two compared characters cannot be evaluated by the statistical analysis. Because of this, the structural analysis extracts features, which describe not pixels or edges, but the more complex structures, such as junctions, line ends and loops.

Junction

The junction is a point, which has at least three black neighbors in the eight-pixel neighborhood. We consider only two types of junctions: the junction of three and four lines. The number of junctions in the skeletonized piece P is mathematically defined as:

$$n_j^3 = \left| \left\{ p \middle| \exists^3 p' : \{p, p'\} \subseteq \mathrm{P} \wedge p \ddot{\mathrm{N}}_8 p' \right\} \right|$$

$$n_j^3 = \left| \left\{ p \middle| \exists^4 p' : \{p, p'\} \subseteq \mathrm{P} \wedge p \ddot{\mathrm{N}}_8 p' \right\} \right|$$

Line end

The line end is a point, which has exactly one neighbor in the eight-pixel neighborhood. The number of line-ends in a skeletonized piece P is defined as:

$$n_e = \left| \left\{ p \middle| \exists! p_1 : \{p, p_1\} \subseteq \mathrm{P} \wedge p \ddot{\mathrm{N}}_8 p_1 \right\} \right|$$

The following algorithm can be used to detect the number of junctions and number line-ends in a skeletonized piece P:

```
let $n_j = 0$
let $n_e = 0$
for each pixel $p$ in piece $\mathrm{P}$ do
begin
    let $\mathrm{neighbors} = 0$
    for each pixel $p'$ in neighborhood $\{p^t, p^{tr}, p^r, p^{br}, p^b, p^{bl}, p^l, p^{tl}\}$ do
        if $p' \in \mathrm{P}$ then
            let $\mathrm{neighbors} = \mathrm{neighbors}+1$

    if $\mathrm{neighbors} = 1$ then
        let $n_e = n_e + 1$
    else if $\mathrm{neighbors} \geq 3$ then
        let $n_j = n_j + 1$
end
```

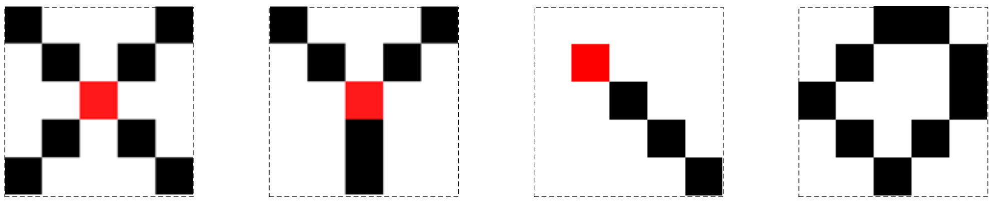


Figure 4.11: (a, b) The junction is a pixel, which as at least three neighbors in eight-pixel neighborhood. (c) The line end is a pixel, which has only one neighbor in eight-pixel neighborhood (d) The loop is a group of pixels, which encloses the continuous white space.

Loops

It is not easy to determine the number of loops n_l in the skeletonized character. The algorithm is based on the following principle. At first, we must negate the bitmap of the skeletonized character. Black pixels will be considered as background and white pixels as foreground. The number of loops in the image is equal to a number of lakes, which are surrounded by these loops. Since the lake is a continuous group of white pixels in the positive image, we apply the piece extraction algorithm on the negative image to determine the number of black pieces. Then, the number of loops is equal to the number of black pieces minus one, because one piece represents the background of the original image (negated to the foreground). Another way is to use a series of morphological erosions.

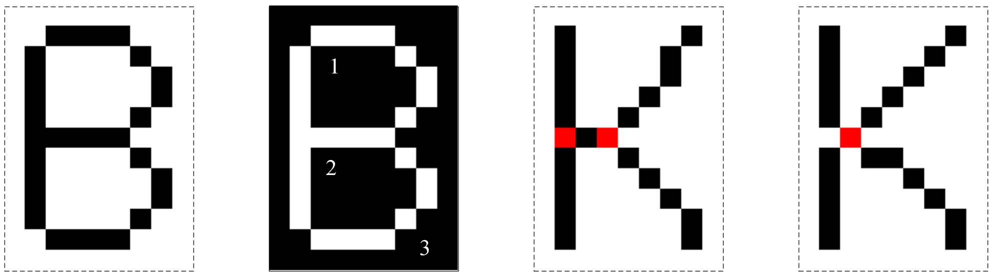


Figure 4.12: (a) We determine the number of lakes in skeleton by applying the piece-extraction algorithm on negative image. The negative image (b) contains three pieces. Since the piece 3 is a background, only two pieces are considered as lakes. (c)(d) The similar skeletons of the same character can differ in the number of junctions

Since we do not know the number of edges of the skeleton, we cannot use the standard cyclomatic equation know from the graph theory. In addition, two similar skeletons of the same character can sometimes differ in a number of junctions (see figure 4.12). Because of this, it is not recommended to use constraints based on the number of junctions.

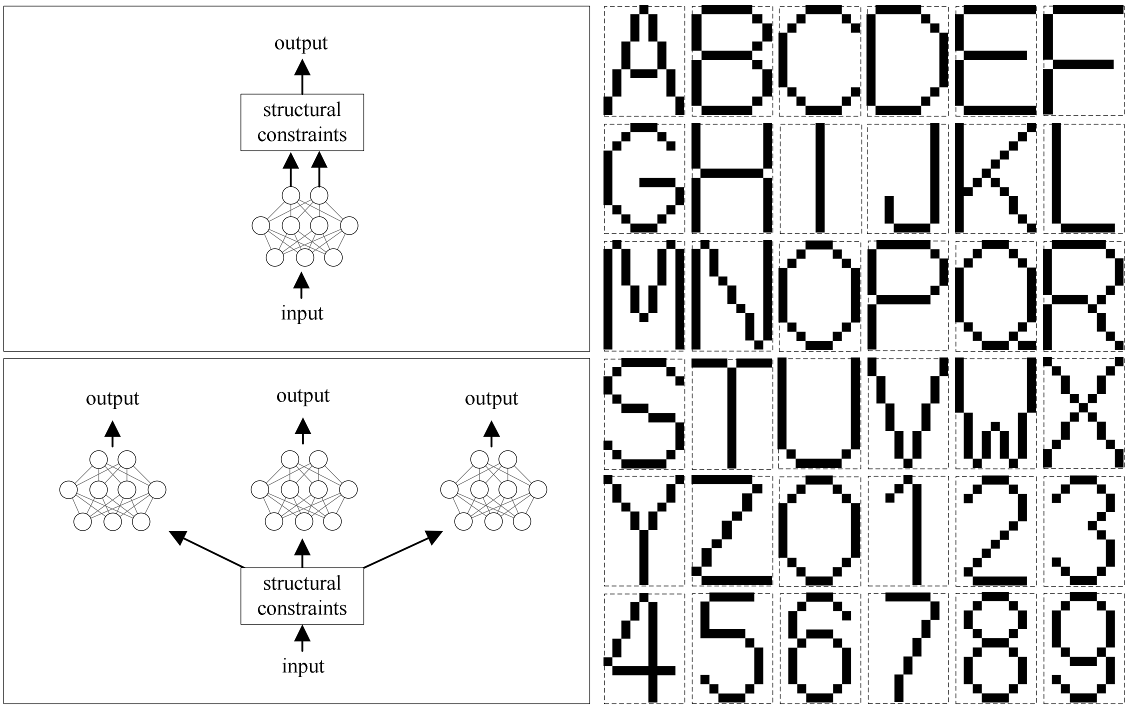
Structural constraints

To improve the recognition process, we can assume structural constraints in the table 4.1. The syntactical analysis can be combined by other methods described in previous chapters, such as edge detection method or pixel matrix.

The simplest way is to use one global neural network that returns several candidates and then select the best candidate that meets the structural constraints (figure 4.13.a). More sophisticated solution is to use the structural constraints for adaptive selection of local neural networks (figure 4.13.b).

	Line ends	Loops	Junctions
0	BDO08	CEFGHIJKLMNSTUVWXYZ123457	CDGIJLMNOSUVWZ012357
1	PQ69	ADOPQR09	EFKPQTXY469
2	ACGIJLMNRSUVWZ123457	B8	ABHR8
3	EFTY		
4	HKX		

Table 4.1: Structural constraints of characters.



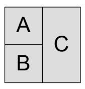


Figure 4.13: (a, b) Structural constraints can be applied before and after the recognition by the neural network. (c) Example of the skeletonized alphabet.

Feature extraction

In case we know the position of structural elements, we can form a vector of descriptors directly from this information. Assume that there are several line-ends, loops, and junctions in the image. The position of loop is defined by its centre. To form the vector, we must convert rectangular coordinates of the element into polar coordinates $[r,\theta]$ (see figure 4.14):

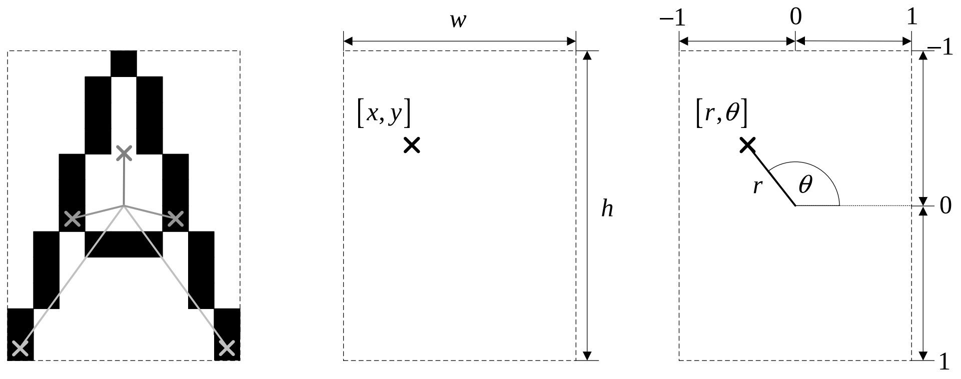
$$r=\sqrt{x'^2+y'^2}\ ;\ \theta=atg\left(\frac{y'}{x'}\right)\ ;\ x'=\frac{2\cdot x-w}{w}\ ;\ y'=\frac{2\cdot y-h}{h}$$

where x' and y' are normalized rectangular coordinates.

The length and the structure of resulting vector vary according to a number and type of structural elements contained in the character. Since the structural constraints divide characters into the several classes, there are several possible types of description vector. Each type of vector corresponds to one class of character.

For example, consider character with two line ends and one junction. This constraint determines the following class of possible characters: (G, I, J, L, M, N, S, U, V, W, Z, 1, 2, 3, 5, 7). We define a vector of descriptors to distinguish between these characters as follows:

$$\mathbf{x}=(\underbrace{r_1,\theta_1}_{\text{line end 1}},\underbrace{r_2,\theta_2}_{\text{line end 2}},\underbrace{r_3,\theta_3}_{\text{junction}})$$



A | B | C

Figure 4.14: (a) The skeleton of the character contains several structural elements, such as junctions, loops and line ends. (b, c) Each element can be positioned in the rectangular or polar coordinate system.

Chapter 5

Recognition of characters

The previous chapter deals with various methods of feature extraction. The goal of these methods is to obtain a vector of descriptors (so-called pattern), which comprehensively describes the character contained in a processed bitmap. The goal of this chapter is to introduce pattern recognition techniques, such as neural networks, which are able to classify the patterns into the appropriate classes.

5.1 General classification problem

The general classification problem is formulated using the mapping between elements in two sets. Let A be a set of all possible combinations of descriptors, and B be a set of all classes. The classification means the projection of group of similar element from the set A into a common class represented by one element in the set B. Thus, one element in the set B corresponds to one class. Usually the group of distinguishable instances of the same character corresponds to the one class, but sometimes one class represents two mutually indistinguishable characters, such as "0" and "O".

Let F be a hypothetic function that assign each element from the set A to an element from the set B:

$$\begin{gathered} F : A \to B \\ \hat{\mathbf{x}} = F(\mathbf{x}) \end{gathered}$$

where $\mathbf{x} \in A$ is a description vector (pattern) which describes the structure of classified character and $\hat{\mathbf{x}} \in B$ is a classifier, which represents the semantics of such character .

The function F is the probably best theoretical classificator, but its construction is impossible since we cannot deal with each combination of descriptors. In praxis, we construct pattern classifier by using only a limited subset of the $A \to B$ mappings. This subset is known as a "training set", such as $A_t \subset A$ and $B_t \subset B$. Our goal is to construct an approximation $\tilde{F}(\mathbf{x}, \mathbf{w})$ of the hypothetic function F, where $\mathbf{w}$ is a parameter that affects the quality of the approximation:

$$\begin{gathered} \tilde{F}(\mathbf{w}) : A_t \to B_t \\ \hat{\mathbf{x}} = \tilde{F}(\mathbf{x}, \mathbf{w}) \end{gathered}$$

where $\mathbf{x} \in A_t \subset A$, $\hat{\mathbf{x}} \in B_t \subset B$. Formally we can say that $\hat{F}(\mathbf{w})$ is a restriction of the projection F over a set $A_t \subset A$. We assume that for each $\mathbf{x}_i \in A_t$ we know the desired value $\hat{\mathbf{x}}_i \in B_t$:

$$\mathbf{x}_0 \to \hat{\mathbf{x}}_0, \mathbf{x}_1 \to \hat{\mathbf{x}}_1, \mathbf{x}_2 \to \hat{\mathbf{x}}_2, \ldots, \mathbf{x}_{n-1} \to \hat{\mathbf{x}}_{n-1}$$

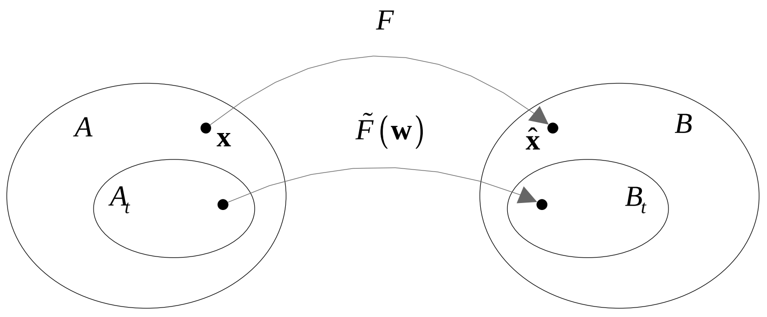


Figure 5.1: The projection between sets A and B. The F is a hypothetic function that maps every possible combination of input pattern $\mathbf{x} \in A$ to a corresponding class $\hat{\mathbf{x}} \in B$. This projection is approximated by a function $\tilde{F}(\mathbf{w})$, which maps input patterns from training set A_t into the corresponding classes from the set B_t

The problem is to find an optimal value (or values) of a parameter $\mathbf{w}$. The $\mathbf{w}$ is typically a vector (or matrix) of syntactical weights in a neural network. According to this parameter, the values of the function $\tilde{F}(\mathbf{x},\mathbf{w})$ should be as closest as possible to the values of $F(\mathbf{x})$ for input patterns $\mathbf{x}$ from the training set A_t. We define an error function to evaluate worthiness of the parameter $\mathbf{w}$:

$$E(\mathbf{w}) = \frac{1}{2}\sum_{i=0}^{m-1}\left(\tilde{F}(\mathbf{x}_i,\mathbf{w}) - \hat{\mathbf{x}}_i\right)^2$$

where m is a number of patterns $\mathbf{x}_0 \ldots \mathbf{x}_{m-1}$ in the training set A_t. Let $\mathbf{w}_+$ to be an optimal value of the parameter $\mathbf{w}$, such as $\mathbf{w}_+ = \arg\min_{\mathbf{w}\in W}\left\{E(\mathbf{w})\right\}$. Then, the approximation $\tilde{F}(\mathbf{x},\mathbf{w}_+)$ of the function $F(\mathbf{x})$ is considered as adapted. The adapted approximation $\tilde{F}(\mathbf{x},\mathbf{w}_+)$ simulates original function $F(\mathbf{x})$ for patterns $\mathbf{x}$ from the training set A_t. In addition, this approximation is able to predict the output classifier $\hat{\mathbf{x}}$ for unknown pattern $\mathbf{x}$ from the "test" set A_x ($A_x = A - A_t$). The function with such prediction ability partially substitutes the hypothetic classificator $F(\mathbf{x})$. Since the function $\tilde{F}(\mathbf{x},\mathbf{w})$ is only a model, we use a feed-forward neural network for its implementation.

5.2 Biological neuron and its mathematical models

For a better understanding of artificial neural network architecture, there is a need to explain the structure and functionality of a biological neuron. The human brain is a neural network of about ten billions interconnected neurons. Each neuron is a cell that uses a biochemical reaction to process and transmit information. The neural cell has a body of size about several micrometers and thousands of input connections called "dendrites". It also has one output connection called "axon", which can be several meters long. The data flow in the biological neural network is represented by electrical signal, which propagates along the axon. When the signal reaches a synaptic connection between the axon and a consecutive dendrite, it relieves molecules of chemical agent (called mediators or neuro-transmitters) into such dendrite. This action causes a local change of polarity of a dendrite transmission membrane. The difference in the polarity of the transmission membrane activates a dendrite-somatic potential wave, which advances in a system of branched dendrites into the body of neuron.

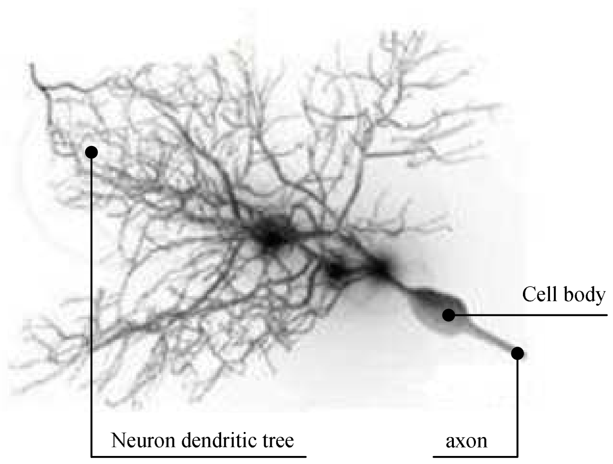


Figure 5.2: The biological neuron

The biological neural network contains two types of synaptic connections. The first is an excitive connection, which amplifies the passing signal. The second (inhibitive) connection suppresses the signal. The behavior of the connection is represented by its "weight". The neural network contains mechanism which is able to alter the weights of connections. Because of this, the system of synaptic weights is a realization of human memory. As the weights are continually altered, the old information is being forgotten little by little.

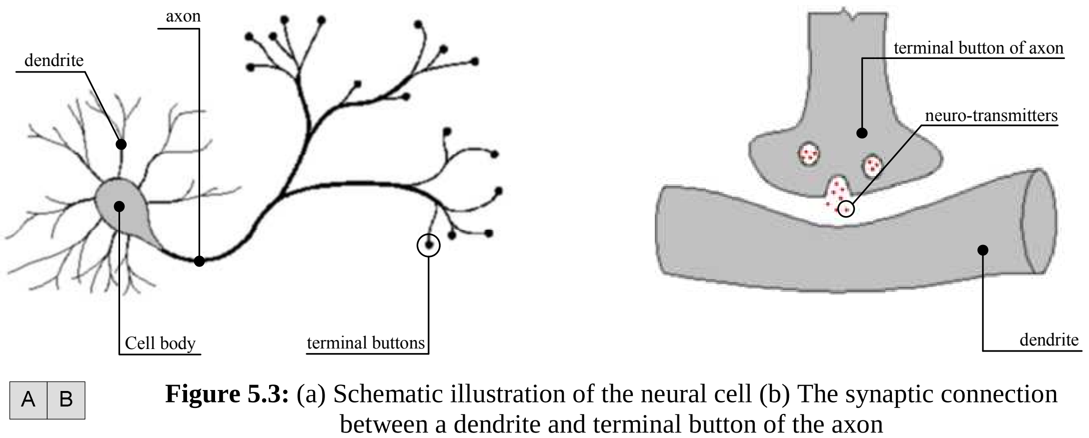


Figure 5.3: (a) Schematic illustration of the neural cell (b) The synaptic connection between a dendrite and terminal button of the axon

Since the problematic of the biological neuron is very difficult, the scientists proposed several mathematical models, such as McCulloch-Pitts binary threshold neuron, or the percepton.

5.2.1 McCulloch-Pitts binary threshold neuron

The McCulloch-Pitts binary threshold neuron was the first model proposed by McCulloch and Pitts in 1943. The neuron has only two possible output values (0 or 1) and only two types of the synaptic weights: the fully excitative and the fully inhibitive. The excitative weight (1) does not affect the input, but the inhibitive one negates it (-1).

The weighted inputs are counted together and processed by a neuron as follows:

$$y = g\left(\sum_{j=0}^{J-1} w_{i,j} \cdot x_j - \vartheta_i\right)$$

$$g(\xi) = \begin{cases} 0 & if \quad \xi < 0 \\ 1 & if \quad \xi \geq 0 \end{cases}$$

This type of neuron can perform logical functions such as AND, OR, or NOT. In addition, McCulloch and Pitts proved that synchronous array of such neurons is able to realize arbitrary computational function, similarly as a Turing machine. Since the biological neurons have not binary response (but continuous), this model of neuron is not suitable for its approximation.

5.2.2 Percepton

Another model of neuron is a percepton. It has been proved that McCulloch-Pitts networks with modified synaptic connections can be trained for the recognition and classification. The training is based on a modification of a neuron weights, according to the reaction of such neuron as follows. If the neuron is not active and it should be, we increase the weights. If the neuron is active and it should not be, we decrease them. This principle was been used in a first model of the neural classifier called ADALINE (adaptive linear neuron). The major problem of such networks is that they are not able to solve linearly nonseparable problems.

This problem has been solved when the scientists Rumelhart, Hilton and Williams proposed the error back-propagation method of learning for multilayered percepton networks. The simple McCulloch-Pitts binary threshold neurons have been replaced by neurons with continuous saturation input/output function.

Percepton has multiple analogous inputs and one analogous output. Let $x_o \ldots x_{j-1}$ be inputs with corresponding weights $w_{i,0} \ldots w_{i,j-1}$. The weighted inputs are counted, thresholded and saturated together in the following way:

$$y = g\left(\sum_{j=0}^{J-1} w_{i,j} \cdot x_j - \vartheta_i\right)$$

$$g(\xi) = \frac{1}{1+e^{-\xi}}$$

where $g(\xi)$ is a sigmoid saturation function (see figure 5.4.b) and ϑ_i is a threshold value. Sometimes, the threshold is implemented as a dedicated input with a constant weight of -1 (see figure 5.4.a). Then, the function of a neuron can be simplified to $y = g(\mathbf{x} \cdot \mathbf{w})$, where $\mathbf{x}$ is a vector of inputs (including the threshold value), and $\mathbf{w}$ is a vector of weights (including the constant weight -1).

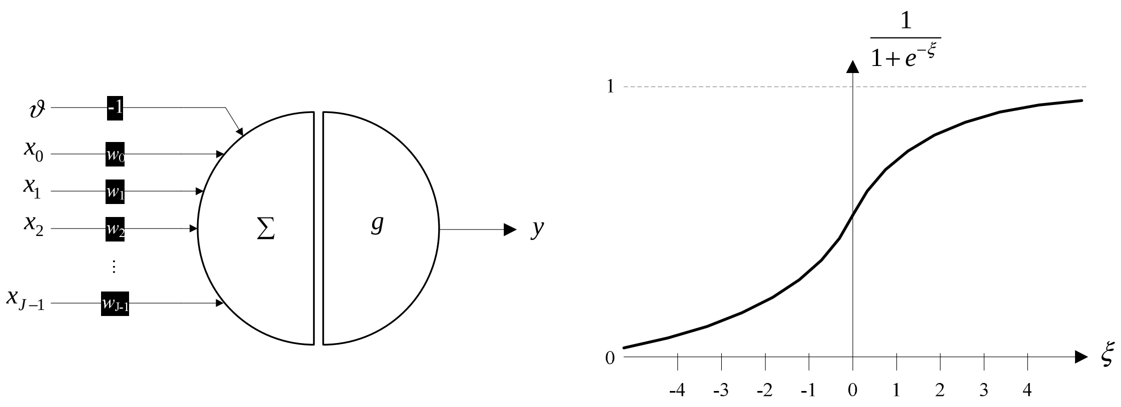


Figure 5.4: (a) The summation Σ and gain (saturation) g function of the percepton with a threshold implemented as a dedicated input. (b) The sigmoid saturation function.

5.3 Feed-forward neural network

Formally, the neural network is defined as an oriented graph $G=(N,E)$, where N is a non-empty set of neurons, and E is a set of oriented connections between neurons. The connection $e(n,n')\in E$ is a binary relation between two neurons n and n'. The set of all neurons N is composed of disjunctive sets N_0, N_1, N_2, where N_i is a set of all neurons from the ith layer.

$$N = N_0 \cup N_1 \cup N_2$$

The jth weight of a ith neuron in a kth layer is denoted as $w_{i,j}^{(k)}$ and the threshold of ith neuron in a kth layer is denoted as $\vartheta_i^{(k)}$. Numbers of neurons for the input (0), hidden (1) and output (2) layer are denoted as m, n, o, such as $m=|N_0|$, $n=|N_1|$ and $o=|N_2|$.

The number of neurons in the input layer (m) is equal to a length of an input pattern $\mathbf{x}$ in order that each value of the pattern is dedicated to one neuron. Neurons in the input layer do not perform any computation function, but they only distribute values of an input pattern to neurons in the hidden layer. Because of this, the input layer neuron has only one input directly mapped into multiple outputs. Because of this, the threshold value $\vartheta_i^{(0)}$ of the input layer neuron is equal to zero, and the weights of inputs $w_{i,0}^{(0)}$ are equal to one.

The number of neurons in the hidden layer (n) is scalable, but it affects the recognition abilities of a neural network at a whole. Too few neurons in the hidden layer causes that the neural network would not be able to learn new patterns. Too many neurons cause network to be overlearned, so it will not be able to generalize unknown patterns as well.

The information in a feed-forward neural network is propagated from lower layers to upper layers by one-way connections. There are connections only between adjacent layers, thus feed-forward neural network does not contain feedback connections, or connections between arbitrary two layers. In addition, there exist no connections between neurons from the same layer.

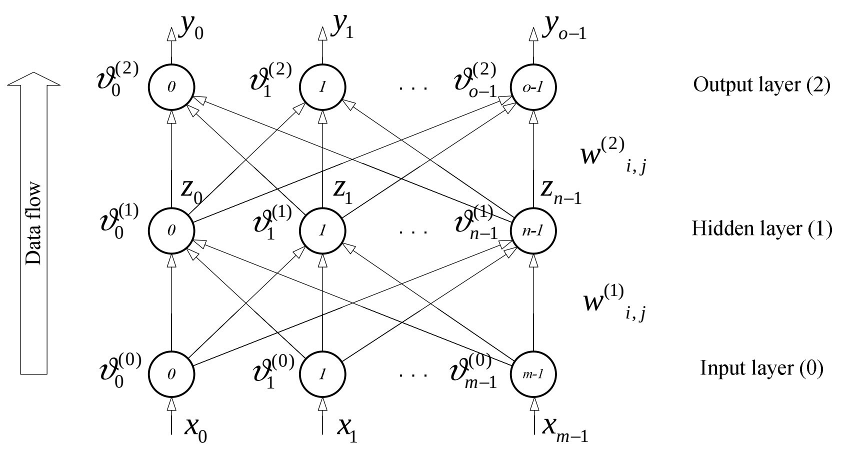


Figure 5.5: Architecture of the three layer feed-forward neural network.

5.4 Adaptation mechanism of feed-forward neural network

There has been proven that a multilayered neural network composed of perceptons with a sigmoid saturation function can solve an arbitrary non-linear problem. Mathematically, for each function $F:\mathbb{R}^m \rightarrow \mathbb{R}^o$ there exists a multilayered feed-forward neural network that is able to realize this function. The proof is based on the Kolmogorov's theorem, which tells that every continuously growing function f defined on interval $\langle 0,1 \rangle^m$ can be written as:

$$f\left(x_0 \dots x_{m-1}\right) = \sum_{i=0}^{2\cdot m} \alpha_i \left(\sum_{j=0}^{m-1} \phi_{i,j}\left(x_j\right) \right)$$

where α_i are properly chosen continuous functions with one parameter.

The problem is how to construct the neural network corresponding to a given non-linear function. At first, we choose a proper topology of the network. The number of neurons in the input and output layer is given by lengths of the input and output patterns, while the number of neurons in the hidden layer is scalable.

An adaptation of the neural network means finding the optimal parameter $\mathbf{w}_+$ of the approximation function $\tilde{F}\left(\mathbf{x},\mathbf{w}\right)$ discussed in section 5.1. Let us define two error functions to evaluate a worthiness of the parameter $\mathbf{w}$:

$$E_t = \frac{1}{2}\sum_{i}^{A_t}\left(\tilde{F}\left(\mathbf{x}_i,\mathbf{w}\right) - \tilde{\mathbf{x}}_i\right)^2 \ ; \ E_x = \frac{1}{2}\sum_{i}^{A_x}\left(\tilde{F}\left(\mathbf{x}_i,\mathbf{w}\right) - \tilde{\mathbf{x}}_i\right)^2$$

where subscript “t” means “train”, and “x” means “test”. The E_t is an error function defined for patterns from the training set, and E_x for patterns from the test set. The response of the neural network to an input pattern $\mathbf{x}$ is given as $y_i = \tilde{F}\left(\mathbf{x}_i,\mathbf{w}\right)$.

The error function E_t goes down as a number of neurons in the hidden layer grows. This relation is valid also between the function E_t and a number of iterative steps of the adaptation process. These relations can be mathematically described as follows:

$$\lim_{n\to\infty} E_t = 0 \;;\; \lim_{k\to\infty} E_t = 0$$

where n is the number of neurons in the input layer and k is the number of iteration steps of the adaptation process.

The error function E_x does not have a limit at zero as n and k goes to infinity. Because of this, there exists an optimal number of neurons and optimal number of iteration steps, in which the function E_x has a minimum (see figure 5.6).

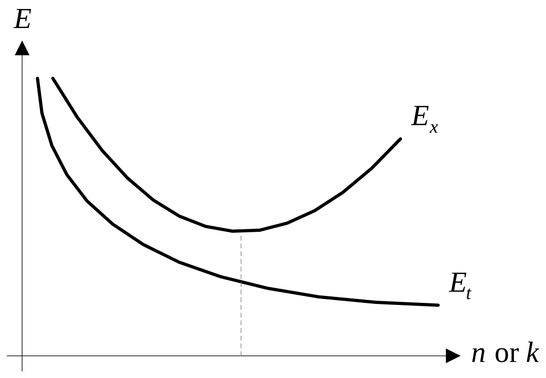


Figure 5.6: Dependency of error functions E_t and E_x on the number of neurons in input layer (n) and the number of iteration steps (k **).**

For simplicity, we will assume only a feed-forward neural network with one layer of hidden neurons defined in section 5.3. All neurons in adjacent layers are connected by oriented connections. There are no feedback connections, or connections between neurons within a single layer.

The activities of hidden and output neurons are defined as:

$$\underbrace{z_i = g\left(\sum_{j=0}^{m-1} w_{i,j}^{(1)} \cdot x_j - \vartheta_i^{(1)}\right)}_{\text{activities of neurons in the hidden layer}} \;;\; \underbrace{y_i = g\left(\sum_{j=0}^{n-1} w_{i,j}^{(2)} \cdot z_j - \vartheta_i^{(2)}\right)}_{\text{activities of neurons in the output layer}}$$

where $g(\xi)$ is a sigmoid saturation function (see figure 5.4.b).

5.4.1 Active phase

Evaluation of the activities of hidden and output neurons is performed in so-called "active phase". The active phase consists of two steps in three-layer neural networks. The first step is an evaluation of activities z_i in the hidden layer, and the second step is an evaluation of activities y_i. Since the evaluation of activities is performed from bottom layers to top ones, the term "feed-forward" refers to this principle. The active phase corresponds to an approximation $\tilde{F}(\mathbf{x},\mathbf{w})$ of function $F(\mathbf{x})$, and it is performed every time when there is a need to classify the input pattern $\mathbf{x}$.

The following pseudo-code demonstrates the active phase of feed-forward neural network. The notation is the same as in figure 5.5.

```
procedure activePhase (input:   $\mathbf{w}$ ,  // vector of thresholds and weights
                                $\mathbf{x}$ ;  // input pattern to be classified
                       output:  $\mathbf{z}$ ,  // vector of activities of neurons in hidden layer
                                $\mathbf{y}$    // vector of activities of neurons in output layer (neural
                                                   network response)
                      )
begin
    // first step: evaluate activities of neurons in the hidden layer
    for each neuron in hidden layer with index $i \in 0,\ldots,n-1$ do
    begin
        let $\xi = \mathbf{w}\left[\vartheta_i^{(1)}\right]$
        for each input with index $j \in 0,\ldots,m-1$ do
            let $\xi = \xi + \mathbf{w}\left[w_{i,j}^{(1)}\right] \cdot x_j$
        let $z_i = g(\xi)$
    end

    // second step: evaluate activities of neurons in the output layer
    for each neuron in output layer with index $i \in 0,\ldots,o-1$ do
    begin
        let $\xi = \mathbf{w}\left[\vartheta_i^{(2)}\right]$
        for each input with index $j \in 0,\ldots,n-1$ do
            let $\xi = \xi + \mathbf{w}\left[w_{i,j}^{(2)}\right] \cdot z_j$
        let $y_i = g(\xi)$
    end
end
```

5.4.2 Partial derivatives and gradient of error function

The goal of the training phase is to find optimal values of thresholds and weights to minimize the error function E_t. Adaptation phase is an iterative process in which a response $\mathbf{y}$ to an input pattern $\mathbf{x}$ is compared with the desired response $\hat{\mathbf{x}}$. The difference between the obtained and desired response is used for a correction of weights. The weights are iteratively altered until the value of the error function E_t is negligible.

Gradient of error function related to a single pattern

We compute a gradient $\mathbf{g}$ of an error function related to a single pattern $\mathbf{x}$ with desired and obtained responses $\mathbf{y}$ and $\hat{\mathbf{x}}$. The gradient $\mathbf{g}$ is computed in direction from upper layers to lower layers as follows:

At first, we compute components of the gradient related to thresholds $\vartheta_i^{(2)}$ in the output layer as

$$\frac{\partial E}{\partial \vartheta_i^{(2)}} = \left(y_i - \hat{x}_i\right) \cdot \left(1 - y_i\right) \cdot y_i$$

Then, we compute components of the gradient related to thresholds $\vartheta_i^{(1)}$ in the hidden layer. These components are computed using the components $\frac{\partial E}{\partial \vartheta_i^{(2)}}$ from the previous step as follows:

$$\frac{\partial E}{\partial \vartheta_i^{(1)}} = z_i \cdot (1 - y_i) \cdot \sum_{j=0}^{o-1} \frac{\partial E}{\partial \vartheta_i^{(2)}} w_{i,j}^{(2)}$$

Similarly, we compute components of the gradient related to weights $w_{i,j}^{(2)}$ and $w_{i,j}^{(1)}$ in the following way:

$$\frac{\partial E}{\partial w_{i,j}^{(2)}} = \frac{\partial E}{\partial \vartheta_i^{(2)}} \cdot z_j \; ; \; \frac{\partial E}{\partial w_{i,j}^{(1)}} = \frac{\partial E}{\partial \vartheta_i^{(1)}} \cdot x_j$$

The gradient $\mathbf{g}$ is a vector of components is given as follows:

$$\mathbf{g} = \left(\frac{\partial E}{\partial \vartheta_0^{(1)}}, \cdots, \frac{\partial E}{\partial \vartheta_{n-1}^{(1)}}, \frac{\partial E}{\partial \vartheta_0^{(2)}}, \cdots, \frac{\partial E}{\partial \vartheta_{o-1}^{(2)}}, \frac{\partial E}{\partial w_{0,0}^{(1)}}, \cdots, \frac{\partial E}{\partial w_{n-1,m-1}^{(1)}}, \frac{\partial E}{\partial w_{0,0}^{(2)}}, \cdots, \frac{\partial E}{\partial w_{o-1,n-1}^{(2)}} \right)$$

Overall gradient

The overall gradient is defined as a summary of gradients related to individual patterns of the training set A_t. Let $\mathbf{g}_{\mathbf{x}/\hat{\mathbf{x}}}$ be a gradient related to a training pair $\mathbf{x}/\hat{\mathbf{x}}$. The overall gradient is computed as $\sum_{\mathbf{x}/\hat{\mathbf{x}}}^{A_t} \mathbf{g}_{\mathbf{x}/\hat{\mathbf{x}}}$.

5.4.3 Adaptation phase

The adaptation phase is an iterative process of finding optimal values of weight and thresholds, for which a value of the error function E_t is in a local minimum. The figure 5.7 schematically illustrates a graph of the function E_t - so called “error landscape”. Generally, the error landscape is $|\mathbf{w}|+1$ dimensional, where $|\mathbf{w}|$ is a cardinality of the vector of thresholds and weights, such as:

$$\mathbf{w} = \left(\vartheta_0^{(1)}, \cdots, \vartheta_{n-1}^{(1)}, \vartheta_0^{(2)}, \cdots, \vartheta_{o-1}^{(2)}, w_{0,0}^{(1)}, \cdots, w_{n-1,m-1}^{(1)}, w_{0,0}^{(2)}, \cdots, w_{o-1,n-1}^{(2)} \right)$$

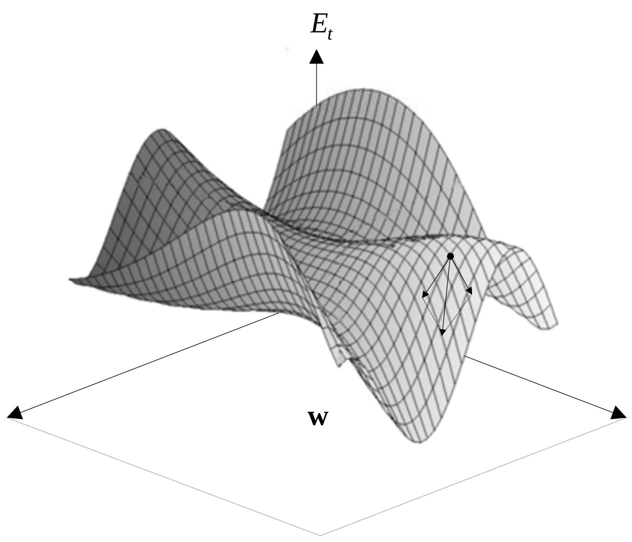


Figure 5.7: The numeric approach of finding the global minimum in the error landscape.

The vector of optimal thresholds and weights $\mathbf{w}_+$ is represented by a global minimum in the error landscape. Since we cannot compute this minimum analytically, we have to use a numeric approach. There are various numeric optimization algorithms, such as Newton's method, or the gradient descent. We use the gradient descent algorithm to find the global minimum in the error landscape. The single step of the iterative algorithm can looks like follows:

$$
\begin{aligned}
{}^{k+1}\vartheta_i^{(l)} &= {}^{k}\vartheta_i^{(l)} - \lambda \frac{\partial E}{\partial\, {}^{k}\vartheta_i^{(l)}} + \mu \cdot \left({}^{k}\vartheta_i^{(l)} - {}^{k-1}\vartheta_i^{(l)} \right) \\
{}^{k+1}w_{i,j}^{(l)} &= {}^{k}w_{i,j}^{(l)} - \lambda \frac{\partial E}{\partial\, {}^{k}w_{i,j}^{(l)}} + \mu \cdot \left({}^{k}w_{i,j}^{(l)} - {}^{k-1}w_{i,j}^{(l)} \right)
\end{aligned}
$$

where ${}^{k}w_{i,j}^{(l)}$ is a weight of the connection between the ith neuron in lth layer and jth neuron in l-1th layer computed in a kth step of iterative process.

The speed of convergence is represented by a parameter λ. Too small value of the parameter λ causes excessively slow convergence. Too big value of the λ breaks the monotony of convergence. The μ is a momentum value, which prevents the algorithm of getting stuck in local minimums.

Note: The notation $\mathbf{g}\left[\frac{\partial E}{\partial \vartheta_i^{(l)}}\right]$ means "the component $\frac{\partial E}{\partial \vartheta_i^{(l)}}$ of the vector (or gradient). The $\frac{\partial E}{\partial \vartheta_i^{(l)}}$ is a partial derivative of error function E by the threshold value $\vartheta_i^{(l)}$. Similarly, the $\frac{\partial E}{\partial w_{i,j}^{(l)}}$ is a partial derivative of function E by the value of weight $w_{i,j}^{(l)}$.

The whole adaptation algorithm of feed-forward neural network can be illustrated by the following pseudo code.

```
procedure adaptation (input:  $A_t$ ,  // training set of patterns $\mathbf{x}/\hat{\mathbf{x}}$
                              $\lambda$ ,  // speed of convergence
                              $\mu$ ,  // momentum value
                              $k_{\max}$ ,  // maximum number of iterations
                              $\varepsilon$ ,  // precision of adaptation process
                     output:  $\mathbf{w}_+$ ,  // vector of optimal weights and thresholds
                     )
begin
   initialize weights and thresholds in $\mathbf{w}$ to random values
   let $\mathbf{w}_{prev} = \mathbf{w}$  // we haven't a previous value of $\mathbf{w}$ at the beginning
   let $k = 0$ , $E = \infty$
   while $k \le k_{\max} \wedge E > \varepsilon$ do
   begin
       // compute overall gradient
       zeroize overall gradient $\mathbf{g}$
       for each pair $\mathbf{x}/\hat{\mathbf{x}}$ of $A_t$ do
       begin
          // compute gradient $\mathbf{g}_{\mathbf{x}/\hat{\mathbf{x}}}$ for training pair $\mathbf{x}/\hat{\mathbf{x}}$
          zeroize gradient $\mathbf{g}_{\mathbf{x}/\hat{\mathbf{x}}}$
          activePhase( $\mathbf{w}$ , $\mathbf{x}$ , $\mathbf{z}$ , $\mathbf{y}$ )// compute activities $\mathbf{z}$ , $\mathbf{y}$
          for each threshold $\vartheta_i^{(2)}$ do $\mathbf{g}_{\mathbf{x}/\hat{\mathbf{x}}}\left[\frac{\partial E}{\partial \vartheta_i^{(2)}}\right] = (y_i - \hat{x}_i)\cdot(1-y_i)\cdot y_i$
          for each threshold $\vartheta_i^{(1)}$ do
          $\mathbf{g}_{\mathbf{x}/\hat{\mathbf{x}}}\left[\frac{\partial E}{\partial \vartheta_i^{(1)}}\right] = z_i\cdot(1-y_i)\cdot\sum_{j=0}^{o-1}\mathbf{g}_{\mathbf{x}/\hat{\mathbf{x}}}\left[\frac{\partial E}{\partial \vartheta_i^{(2)}}\right]\cdot\mathbf{w}\left[w_{i,j}^{(2)}\right]$
          for each weight $w_{i,j}^{(2)}$ do $\mathbf{g}_{\mathbf{x}/\hat{\mathbf{x}}}\left[\frac{\partial E}{\partial w_{i,j}^{(2)}}\right] = \mathbf{g}_{\mathbf{x}/\hat{\mathbf{x}}}\left[\frac{\partial E}{\partial \vartheta_i^{(2)}}\right]\cdot z_j$
          for each weight $w_{i,j}^{(1)}$ do $\mathbf{g}_{\mathbf{x}/\hat{\mathbf{x}}}\left[\frac{\partial E}{\partial w_{i,j}^{(1)}}\right] = \mathbf{g}_{\mathbf{x}/\hat{\mathbf{x}}}\left[\frac{\partial E}{\partial \vartheta_i^{(1)}}\right]\cdot x_j$
          let $\mathbf{g} = \mathbf{g} + \mathbf{g}_{\mathbf{x}/\hat{\mathbf{x}}}$
       end
       // alter values of thresholds and weights according to the gradient $\mathbf{g}$
       for each threshold $\vartheta_i^{(l)}$ in $\mathbf{w}_+$ do
          let $\mathbf{w}_{next}\left[\vartheta_i^{(l)}\right] = \mathbf{w}\left[\vartheta_i^{(l)}\right] - \lambda\cdot\mathbf{g}\left[\frac{\partial E}{\partial \vartheta_i^{(l)}}\right] + \mu\cdot\left(\mathbf{w}\left[\vartheta_i^{(l)}\right] - \mathbf{w}_{prev}\left[\vartheta_i^{(l)}\right]\right)$
       for each weight $w_{i,j}^{(l)}$ in $\mathbf{w}_+$ do
          let $\mathbf{w}_{next}\left[w_{i,j}^{(l)}\right] = \mathbf{w}\left[w_{i,j}^{(l)}\right] - \lambda\cdot\mathbf{g}\left[\frac{\partial E}{\partial w_{i,j}^{(l)}}\right] + \mu\cdot\left(\mathbf{w}\left[w_{i,j}^{(l)}\right] - \mathbf{w}_{prev}\left[w_{i,j}^{(l)}\right]\right)$
       let $\mathbf{w}_{prev} = \mathbf{w}$ , $\mathbf{w} = \mathbf{w}_{next}$
   end
   let $\mathbf{w}_+ = \mathbf{w}$
end
```

5.5 Heuristic analysis of characters

The segmentation algotithm described in chapter three can sometimes detect redundant elements, which do not correspond to proper characters. The shape of these elements after normalization is often similar to the shape of characters. Because of this, these elements are not reliably separable by traditional OCR methods, although they vary in size as well as in contrast, brightness or hue. Since the feature extraction methods described in chapter four do not consider these properties, there is a need to use additional heuristic analyses to filter non-character elements. The analysis expects all elements to have similar properties. Elements with considerably different properties are treated as invalid and excluded from the recognition process.

The analysis consists of two phases. The first phase deals with statistics of brighness and contrast of segmented characters. Characters are then normalized and processed by the piece extraction algorithm.

Since the piece extraction and normalization of brightness disturbs statistical properties of segmented characters, it is necessary to proceed the first phase of analysis before the application of the piece extraction algorithm.

In addition, the heights of detected segments are same for all characters. Because of this, there is a need to proceed the analysis of dimensions after application of the piece extraction algorithm. The piece extraction algorithm strips off white padding, which surrounds the character.

Respecting the constraints above, the sequence of steps can be assembled as follows:

1. Segment the plate (result is in figure 5.8.a).
2. Analyse the brightness and contrast of segments and exclude faulty ones.
3. Apply the piece extraction algorithm on segments (result is in figure 5.8.b).
4. Analyse the dimensions of segments and exclude faulty ones.

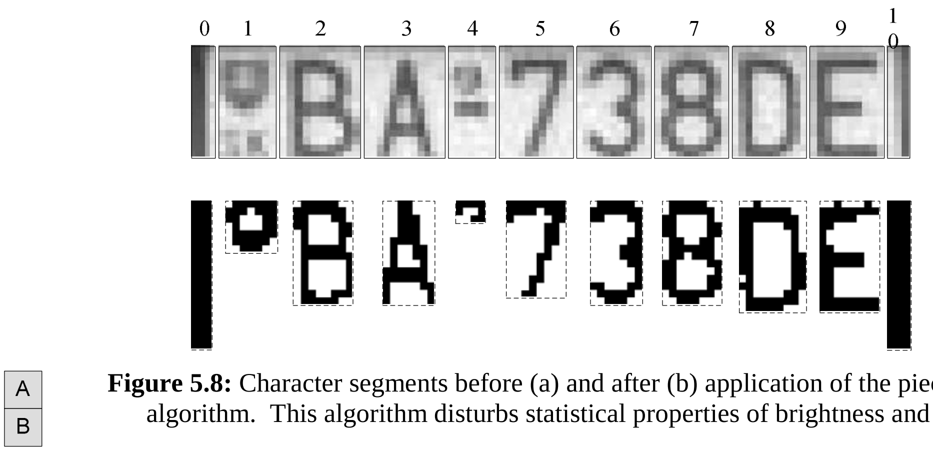


Figure 5.8: Character segments before (a) and after (b) application of the piece extraction algorithm. This algorithm disturbs statistical properties of brightness and contrast.

If we assume that there are not big differences in brightness and contrast of segments, we can exclude the segments, which considerably differs from the mean. Let ith segment of plate be defined by a discrete function $f_i(x,y)$, where w_i and h_i are dimensions of the element. We define the following statistical properties of an element:

The global brightness of such segment is defined as a mean of brightnesses of individual pixels:

$$p_b^{(i)} = \sum_{x=0}^{w_i} \sum_{y=0}^{h_i} f(x,y)$$

The global contrast of the ith segment is defined as a standard deviation of brightnesses of individual pixels:

$$p_c^{(i)} = \sqrt{\frac{\sum_{x=0}^{w_i}\sum_{y=0}^{h_i}\left(p_b^{(i)} - f(x,y)\right)^2}{w_i \cdot h_i}}$$

The function $f(x,y)$ represents only an intensity of grayscale images, but the additional heuristic analysis of colors can be involved to improve the recognition process. This analysis separates character and non-character elements on color basis. If the captured snapshot is represented by a HSV color model, we can directly compute the global hue and saturation of the segments as a mean of hue and saturation of individual pixels:

$$p_h^{(i)} = \sum_{x=0}^{w_i}\sum_{y=0}^{h_i} h(x,y) \; ; \; p_s^{(i)} = \sum_{x=0}^{w_i}\sum_{y=0}^{h_i} s(x,y)$$

where $h(x,y)$ and $s(x,y)$ is a hue and saturation of the certain pixel in the HSV color model. If the captured snapshot is represented by a RGB color model, there is need to transform it to the HSV model first.

To determine the validity of the element, we compute an average value of a chosen property over all elements. For example, the average brightness is computed as $\overline{p}_b = \sum_{i=0}^{n-1} p_b^{(i)}$, where n is a number of elements. The element i is considered as valid, if its global brightness $p_b^{(i)}$ does not differ more than 16 % from the average brightness $\overline{p}_b$. The threshold values of individual properties have been calibrated as follows:

brightness (BRI)	$\frac{p_b^{(i)} - \overline{p}_b}{\overline{p}_b} < 0.16$	Contrast (CON)	$\frac{p_c^{(i)} - \overline{p}_c}{\overline{p}_c} < 0.1$
hue (HUE)	$\frac{p_h^{(i)} - \overline{p}_h}{\overline{p}_h} < 0.145$	Saturation (SAT)	$\frac{p_s^{(i)} - \overline{p}_s}{\overline{p}_s} < 0.24$
Height (HEI)	$\left\lvert\frac{h_i - \overline{h}}{\overline{h}}\right\rvert < 0.2$	width/height ratio (WHR)	$0.1 < \frac{w_i}{h_i} < 0.92$

If the segment violates at least one of the constraints above, it is considered as invalid and excluded from the recognition process. The table 5.1 contains properties of elements from figure 5.8. According to this table, elements 0 and 10 have been refused due to an uncommon width/height ratio, and elements 1 and 4 due to a small height.

	i	BRI	CON	HUE	SAT	HEI	WHR	Violated constraints
	0	0.247	0.038	0.152	0.236	0.189	0.093	BRI,HUE,WHR
	1	0.034	0.096	0.181	0.134	-0.554	0.833	HUE,HEI
	2	0.002	0.018	0.030	0.038	0.040	0.642	
	3	0.084	0.012	0.003	0.061	0.189	0.625	
	4	0.001	0.003	0.021	0.059	-0.777	1.666	HEI,WHR
	5	0.117	0.016	0.002	0.063	0.189	0.625	
	6	0.063	0.016	0.007	0.056	0.189	0.562	
	7	0.025	0.011	0.025	0.028	0.114	0.533	
	8	0.019	0.025	0.012	0.034	0.114	0.600	
	9	0.019	0.048	0.009	0.045	0.114	0.533	
	10	0.062	0.009	0.041	0.018	0.189	0.095	WHR

Table 5.1: Properties of segments in figure 5.8. The meaning of abbreviations is as follows: BRI=brightness, CON=contrast, HUE=hue, SAT=saturation, HEI=height, WHR=width/height ratio.

Chapter 6

Syntactical analysis of recognized plate

6.1 Principle and algorithms

In some situations when the recognition mechanism fails, there is a possibility to detect a failure by a syntactical analysis of the recognized plate. If we have country-specific rules for the plate, we can evaluate the validity of that plate towards these rules. Automatic syntax-based correction of plate numbers can increase recognition abilities of the whole ANPR system.

For example, if the recognition software is confused between characters „8“ and „B“, the final decision can be made according to the syntactical pattern. If the pattern allows only digits for that position, the character „8“ will be used rather than the character „B“.

Another good example is a decision between the digit „0“ and the character „O“. The very small difference between these characters makes their recognition extremely difficult, in many cases impossible.

6.1.1 Recognized character and its cost

In most cases, characters are recognized by neural networks. Each neuron in an output layer of a neural network typically represents one character. Let $\mathbf{y} = \left(y_0, \ldots, y_9, y_A, \ldots, y_Z \right)$ be a vector of output activities. If there are 36 characters in the alphabet, the vector $\mathbf{y}$ will be also 36-dimensional.

Let y_i be an i^th component of the vector $\mathbf{y}$. Then, y_i means how much does the input character corresponds to the i^th character in the alphabet, which is represented by this component. The recognized character χ is represented by the greatest component of the vector $\mathbf{y}$:

$$\chi = chr\left(\max_{0 \le i \le z} \left\{ y_i \right\} \right)$$

where $chr\left(y_i \right)$ is the character, which is represented by the i^th component of vector $\mathbf{y}$.

Let $\mathbf{y}^{(S)}$ be a vector $\mathbf{y}$ descendingly sorted according to the values of components. Then, the recognized character is represented by the first component of so sorted vector:

$$\chi = chr\left(y_0^{(S)} \right)$$

When the recognition process fails, the first component of $\mathbf{y}^{(S)}$ can contain invalid character, which does not match the syntax pattern. Then, it is necessary to use the next valid character with a worse cost.

6.1.2 Syntactical patterns

In praxis, ANPR systems must deal with many different types of plate numbers. Number plates are not unified, so each country has own type. Because of this, number plate recognition system should be able to recognize a type of the number plate, and automatically assign the correct syntactical pattern to it. The assignation of the right syntactical pattern is a fundamental problem in syntactical analysis.

Syntactical pattern is a set of rules defining characters, which can be used on a certain position in a plate number. If the plate number $\mathbf{P}$ is a sequence of n alphanumerical characters $\mathbf{P} = \left(p^{(0)} \ldots p^{(n-1)} \right)$, then the syntactical pattern $`\mathbf{P}$ is a $n$-tuple of sets $`\mathbf{P} = \left(`p^{(0)} \ldots `p^{(n-1)} \right)$, and $`p^{(i)}$ is a set of all allowed characters for the ith position in a plate.

For example, czech number plates can contain digit on a first position followed by a character denoting the region, where the plate has been registered and five other digits for a registration number of a car. Formally, the syntactical pattern $`\mathbf{P}$ for czech number plates can looks like this:

$$`\mathbf{P} = \begin{pmatrix} \{0,1,2,3,4,5,6,7,8,9\}, \{\mathrm{C,B,K,H,L,T,N,E,P,A,S,U,J,Z}\}, \\ \{0,1,2,3,4,5,6,7,8,9\}, \{0,1,2,3,4,5,6,7,8,9\}, \{0,1,2,3,4,5,6,7,8,9\}, \\ \{0,1,2,3,4,5,6,7,8,9\}, \{0,1,2,3,4,5,6,7,8,9\}, \{0,1,2,3,4,5,6,7,8,9\} \end{pmatrix}$$

6.1.3 Choosing the right pattern

If there are n syntactical patterns $`\mathbf{P}^{(0)} \ldots `\mathbf{P}^{(n-1)}$, we have to choose the most suitable one for the evaluated plate number $\mathbf{P}$. For this purpose, we define a metrics (or a cost) δ for a computation of a similarity between the evaluated plate number and the corresponding syntactical pattern:

$$\delta(`\mathbf{P}) = \left| \left\{ p^{(i)} \middle| p^{(i)} \notin `p^{(i)} \right\} \right| + \sum_{i=0}^{n-1} \left(\frac{1}{\max\limits_{0 \le j \le z} \left\{ y_j^{(i)} \right\}} \times 10^{-2} \right),$$

where $\left| \left\{ p^{(i)} \middle| p^{(i)} \notin `p^{(i)} \right\} \right|$ is a number of characters, which do not match to corresponding positions in the syntactical pattern $`\mathbf{P}$. Let $\mathbf{y}^{(i)}$ be an output vector for the ith-recognized character in a plate. The greatest component of that vector $\max\limits_{0 \le j \le z} \left\{ y_j^{(i)} \right\}$ then indicates how successfully the plate has been recognized. Then, the reciprocal value of $\max\limits_{0 \le j \le z} \left\{ y_j^{(i)} \right\}$ is a cost of the character. Another way of the cost evaluation is a usage of the Smith-Waterman algorithm to compute the difference between the recognized plate number and the syntactical pattern.

For example, assume that plate number '0B01234' has been recognized as '0801234', and the recognition pattern does not allow digit at the second position of a plate. If the character "8" has been recognized with similarity ratio of 0.90, and other characters with the ratio of 0.95, the metrics for this pattern is determined as follows.

$$\delta(`\mathbf{P}) = (1) + \left(\frac{10^{-2}}{0.95} + \frac{10^{-2}}{0.90} + \frac{10^{-2}}{0.95} + \frac{10^{-2}}{0.95} + \frac{10^{-2}}{0.95} + \frac{10^{-2}}{0.95} + \frac{10^{-2}}{0.95} \right) = 1{,}07426$$

If there is a pattern that exactly matches to the evaluated plate number, we can say that number has been correctly recognized, and no further corrections are needed. In addition, it is not possible to detect a faulty number plate, if it does not break rules of a syntactical pattern. Otherwise, it is necessary to correct detected plate using the pattern with lowest cost δ:

$$`\mathbf{P}^{(sel)} = \arg\min_{0\le i<n} \left\{ \delta\left( `\mathbf{P}^{(i)} \right) \right\}$$

The correction of a plate means the replacement of each invalid character by another one. If the character $p^{(i)}$ at the ith position of the plate $\mathbf{P}$ does not match the selected pattern $`\mathbf{P}^{(sel)}$, it will be replaced by the first valid one from $\mathbf{y}^{(s)}$. $\mathbf{y}^{(s)}$ is a sorted vector of output activities denoting how much the recognized character is similar to an individual character from the alphabet.

Heuristic analysis of a segmented plate can sometimes incorrectly evaluate non-character elements as characters. Acceptance of the non-character elements causes that the recognized plate will contain redundant characters. Redundant characters occur usually on sides of the plate, but rarely in the middle.

If the recognized plate number is longer than the longest syntax pattern, we can select the nearest pattern, and drop the redundant characters according to it.

Chapter 7

Tests and final considerations

7.1 Choosing the representative set of snapshots

I have captured many of static snapshots of vehicles for the test purposes. Random moving and standing vehicles with Slovak and Czech number plates have been included. At first, my objective was to find a representative set of number plates, which are recognizable by humans. Of course, the set like this contains extremely wide spectrum of plates, such as clear and easy recognizable as well as plates degraded by the significant motion blur or skew.

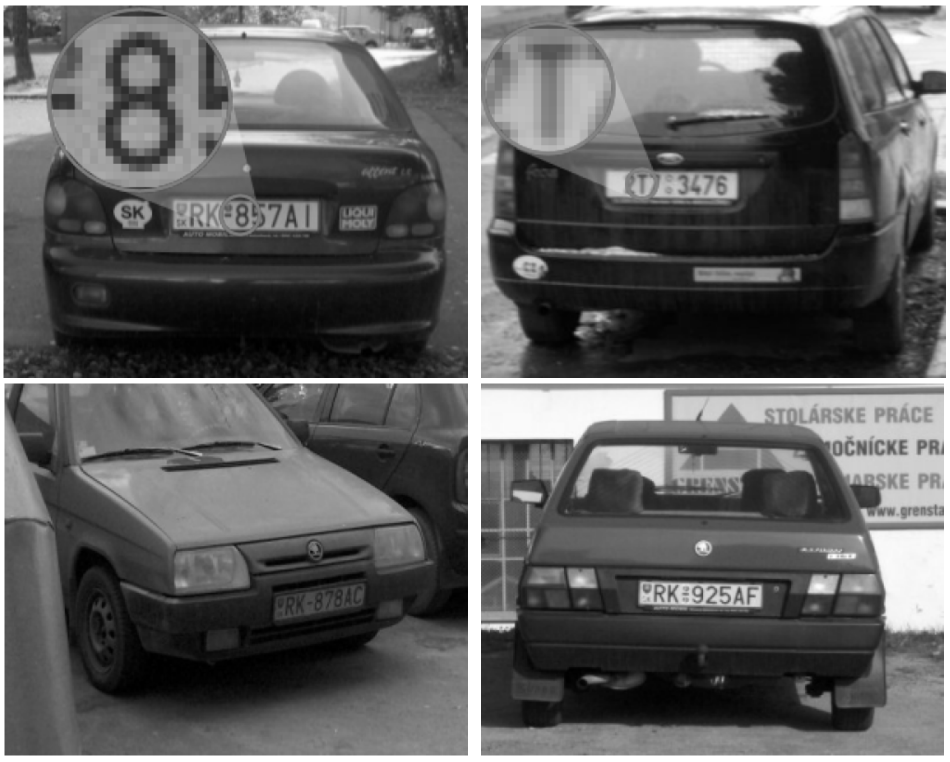
Then, a recognition ability of a machine is represented by a ratio between the number of plates, which have been recognized by the machine, and the number of plates recognized by a human. Practically, it is impossible to build a machine with the same recognition abilities as a human has. Because of this, the test like this is extremely difficult and useless.

In praxis, it is more useful to find a representative set of number plates, which can be captured by an ANPR camera. The position of the camera has a significantly affects the quality of captured images, and a successfulness of the whole recognition process. The suitable position of the camera towards the lane can lead to a better set of all possible snapshots. In some situations, we can avoid of getting skewed snapshots by a suitable positioning of the camera. Sometimes, this is cleverer than a development of the robust de-skewing mechanisms.

Let S be a representative set of all snapshots, which can be captured by a concrete instance of the ANPR camera. Some of the snapshots in this set can be blurred, some of them can be too small, too big, too skewed or too deformed. Because of this, I have divided the whole set into a following subsets:

$$S = S_c \cup S_b \cup S_s \cup S_e \cup S_l$$

where S_c is a subset of “clear” plates, S_b is a subset of blurred plates, S_s is a subset of skewed plates, S_e is a subset of plates, which has a difficult surrounding environment, and S_l is a subset of plates with little characters.



A	B
C	D

Figure 7.1: Typical snapshot from the set of (a) clear plates (b) plates with little, or blurred characters (c) skewed plates (d) plates with difficult surrounding environment

7.2 Evaluation of a plate number correctness

Plate numbers recognized by a machine can sometimes differ from the correct ones. Because of this, there is a need to define formulas and rules, which will be used to evaluate a degree of plate correctness.

Let $\mathbf{P}$ be a plate number, and $S = \left\{ \mathbf{P}^{(0)}, \ldots, \mathbf{P}^{(n\text{-}1)} \right\}$ be a set of all tested plate numbers. Then, recognition rate $R\left(S\right)$ of the ANPR system tested on set S is calculated as:

$$R\left(S\right) = \frac{1}{n} \sum_{i=0}^{n-1} s\left(\mathbf{P}^{(i)}\right),$$

where n is a cardinality of the set S, and $s(\mathbf{P})$ is a correctness score of the plate $\mathbf{P}$. The correctness score is a value, which express how successfully the plate has been recognized.

Now the question is how to define the correctness score of individual plates. There are two different approaches, how to evaluate it. The first is a binary score, and the second is a weighted score.

7.2.1 Binary score

Let us say, that plate number $\mathbf{P}$ is a sequence of n alphanumerical characters $\mathbf{P} = \left(p^{(0)}, \ldots, p^{(n-1)} \right)$. If $\mathbf{P}^{(r)}$ is the plate number recognized by a machine, and $\mathbf{P}^{(c)}$ is the correct one, then binary score s_b of plate $\mathbf{P}^{(r)}$ is evaluated as follows:

$$s_b\left(\mathbf{P}^{(r)}\right) = \begin{cases} 0 & if \quad \mathbf{P}^{(r)} \neq \mathbf{P}^{(c)} \\ 1 & if \quad \mathbf{P}^{(r)} = \mathbf{P}^{(c)} \end{cases}$$

Two plate numbers are equal, if all characters on corresponding positions are equal:

$$\left(\mathbf{P}^{(r)} = \mathbf{P}^{(c)}\right) \Leftrightarrow \left(\forall i \forall j : p_i^{(r)} \neq p_j^{(r)} \Rightarrow i \neq j\right),$$

where $p_i^{(r)}$ is the ith character of plate number $\mathbf{P}^{(r)}$

7.2.2 Weighted score

If $\mathbf{P}^{(r)}$ is a plate number recognized by a machine, and $\mathbf{P}^{(c)}$ is the correct one, then weighted score s_w of plate $\mathbf{P}^{(r)}$ is given as:

$$s_w\left(\mathbf{P}^{(r)}\right) = \frac{\left|\left\{p_i^{(r)} \middle| p_i^{(r)} = p_i^{(c)}\right\}\right|}{\left|\left\{p_i^{(r)}\right\}\right|} = \frac{m}{n}$$

where m is the number of correctly recognized characters, and n is the number of all characters in plate.

For example if the plate “`KE123AB`” has been recognized as “`KE128AB`”, the weighted correctness score for this plate is 0.85, but the binary score is 0.

7.3 Results

The table 7.1 shows recognition rates, which has been achieved while testing on various set of number plates. According to the results, this system gives good responses only to clear plates, because skewed plates and plates with difficult surrounding environment causes significant degradation of recognition abilities.

	Total number of plates	Total number of characters	Weighted score
Clear plates	68	470	87.2
Blurred plates	52	352	46.87
Skewed plates	40	279	51.64
Average plates	177	1254	73.02

Table 7.1: Recognition rates of the ANPR system.

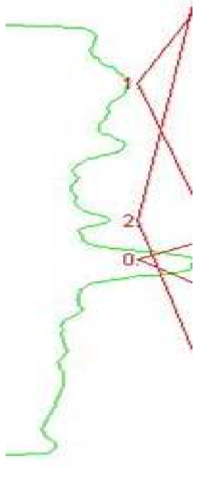
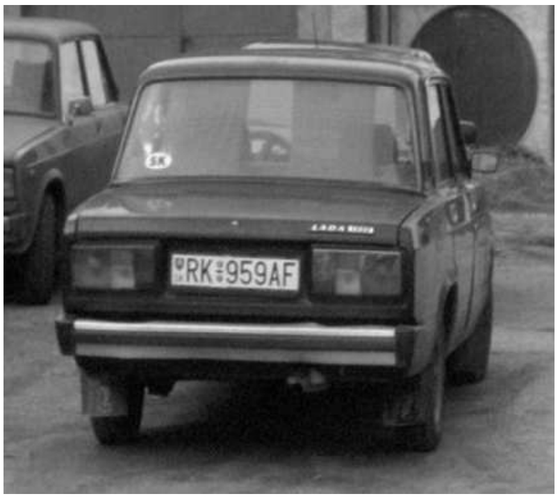
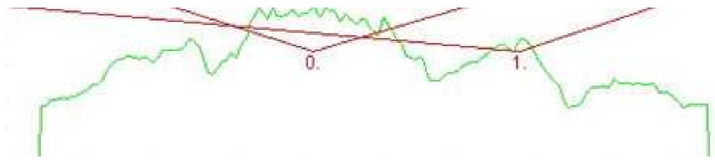
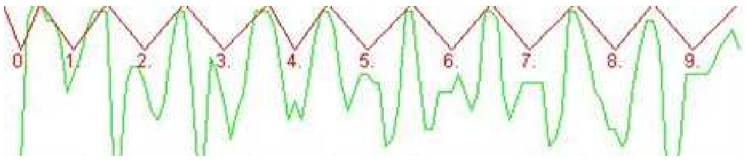
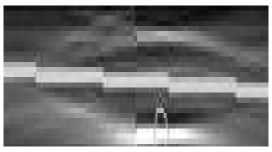
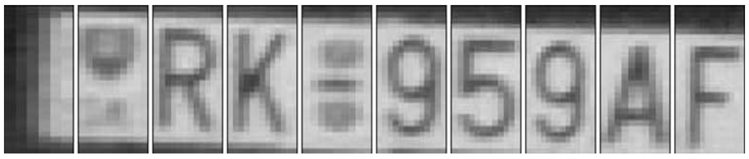
Summary

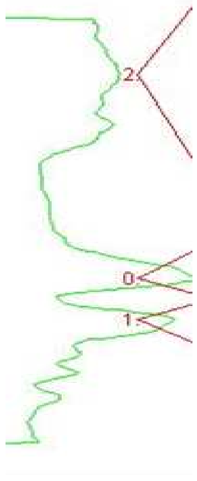
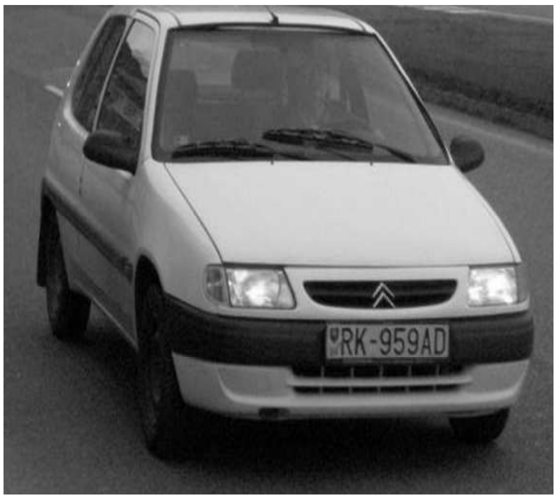
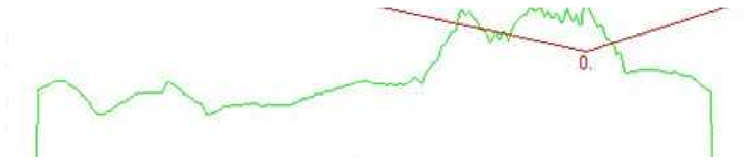
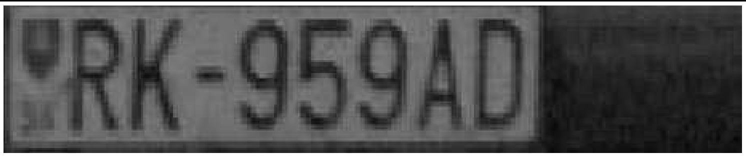
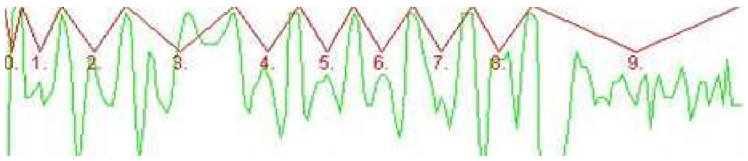
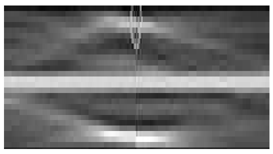
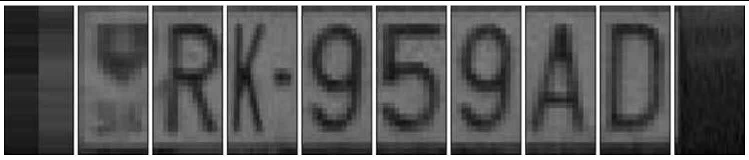
The objective of this thesis was to study and resolve algorithmic and mathematical aspects of the automatic number plate recognition systems, such as problematic of machine vision, pattern recognition, OCR and neural networks. The problematic has been divided into several chapters, according to a logical sequence of the individual recognition steps. Even though there is a strong succession of algorithms applied during the recognition process, chapters can be studied independently.

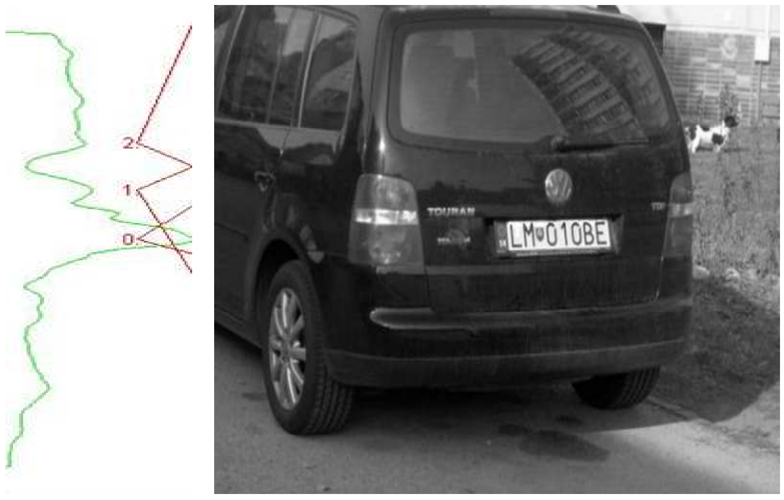
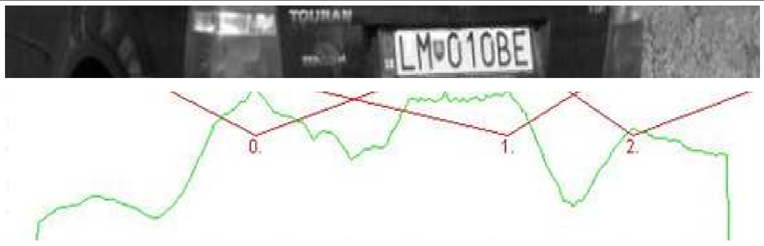
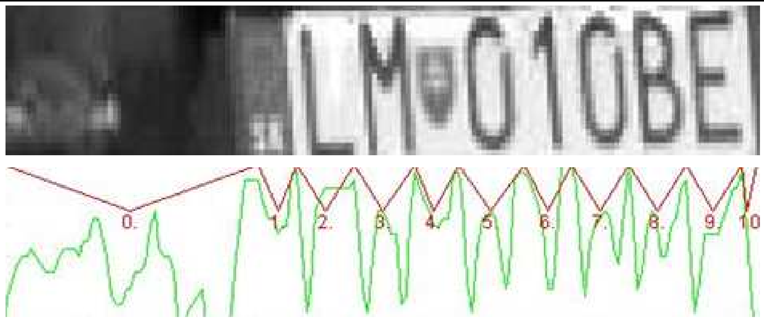
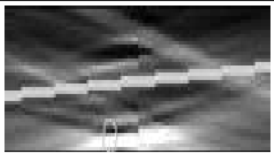
This work also contains demonstration ANPR software, which comparatively demonstrates all described algorithms. I had more choices of programming environment to choose from. Mathematical principles and algorithms should not be studied and developed in a compiled programming language. I have considered usage of the Matlab™ and the Java™. Finally, I implemented ANPR in Java rather than in Matlab, because Java™ is a compromise programming environment between the Matlab™ and compiled programming language, such as C++. Otherwise, I would have to develop algorithms in Matlab, and then rewrite them into a compiled language as a final platform for their usage in the real environment.

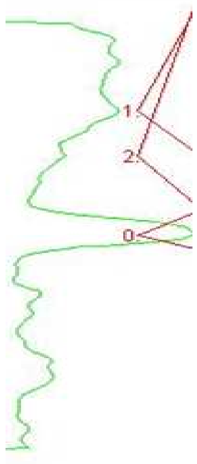
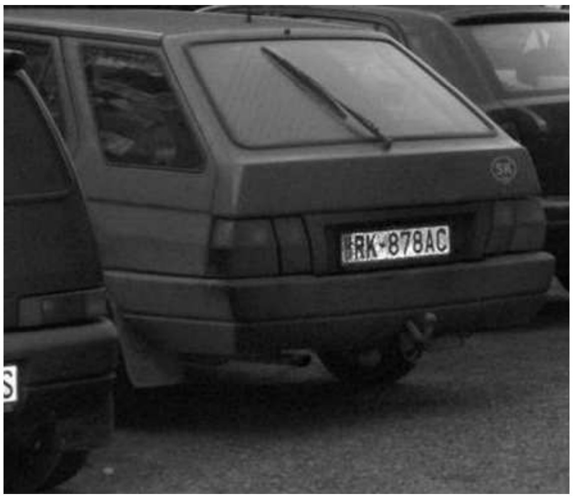
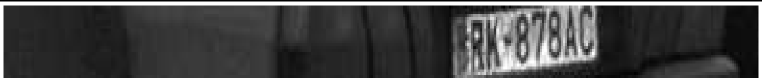
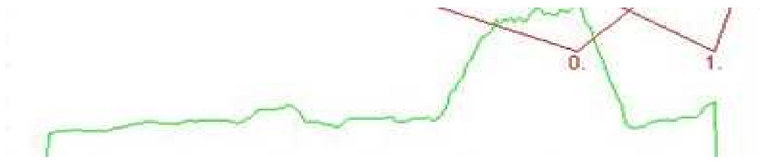
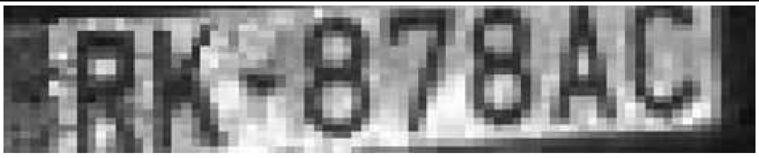
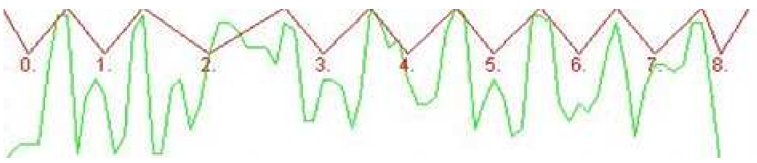
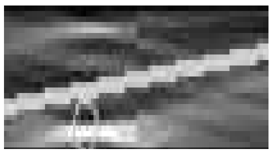
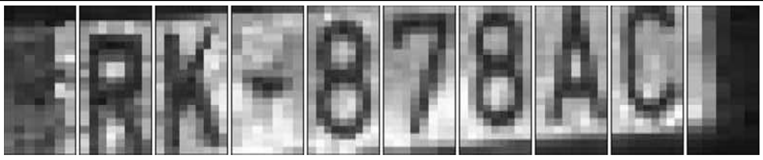
ANPR solution has been tested on static snapshots of vehicles, which has been divided into several sets according to difficultness. Sets of blurry and skewed snapshots give worse recognition rates than a set of snapshots, which has been captured clearly. The objective of the tests was not to find a one hundred percent recognizable set of snapshots, but to test the invariance of the algorithms on random snapshots systematically classified to the sets according to their properties.

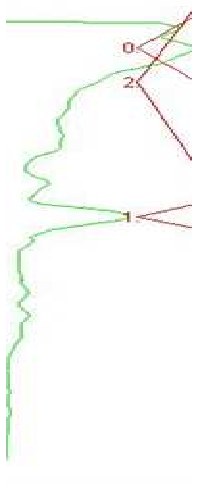
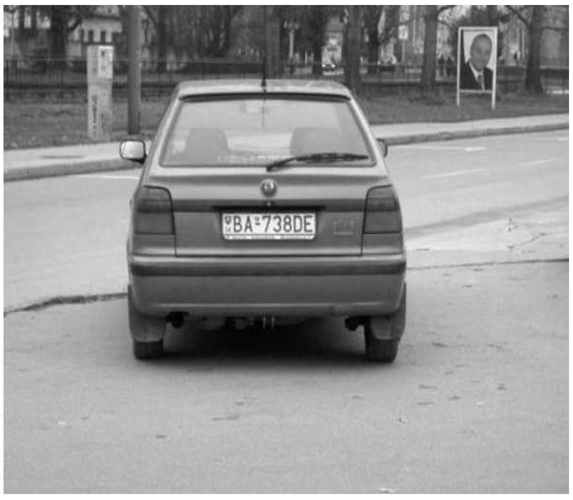
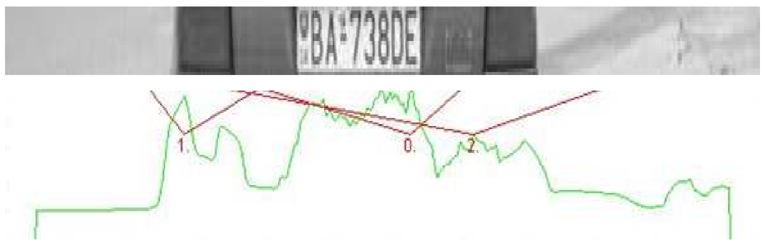
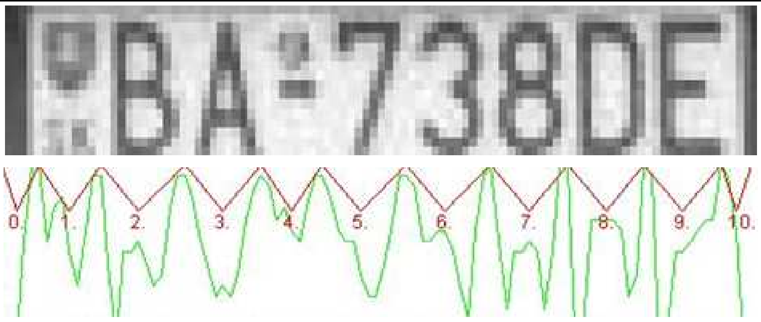
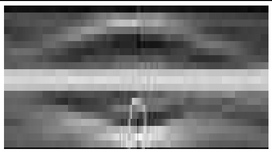
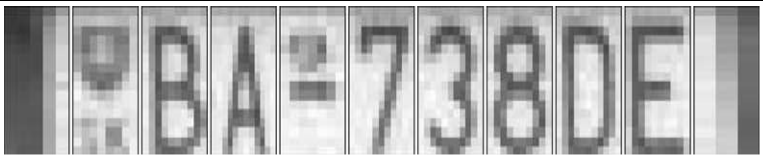
Appendix A: Case study

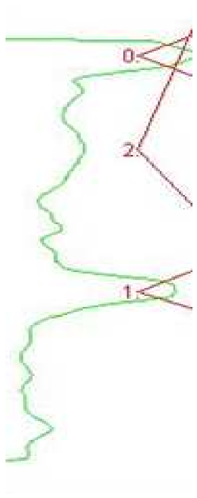
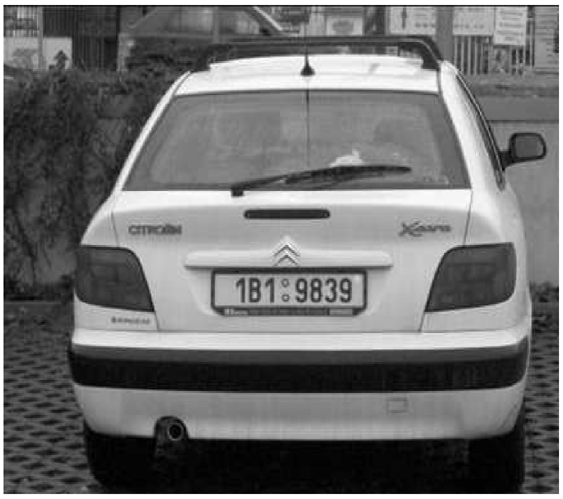
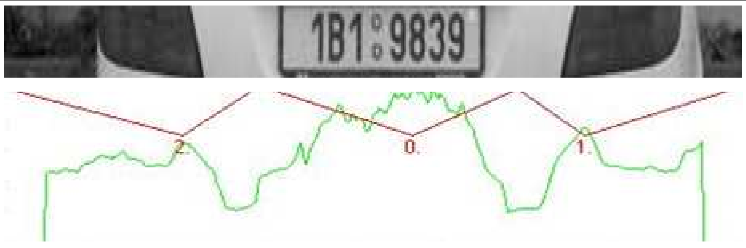
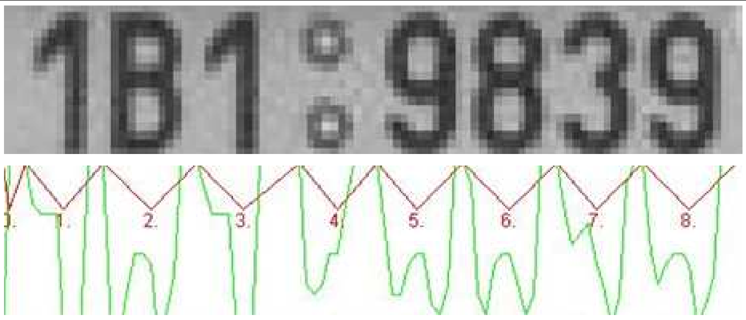
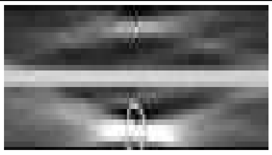
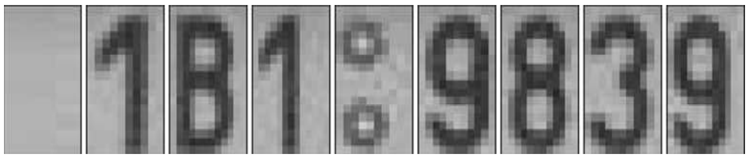
134.jpg	width:488 px	height:366 px	
			The plate has been successfully recognized - no further comment needed.
Detected band	width: 488 px	height: 30 px	
			
Detected plate			Skew detection
			 **1.44°**
Segmentation	Number of detected characters: 10		
			Recognized plate **RK959AF**

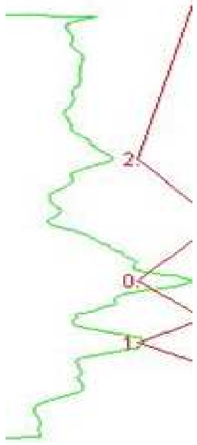
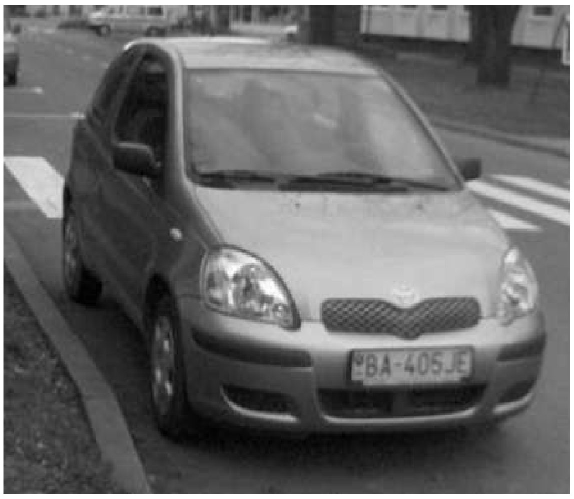
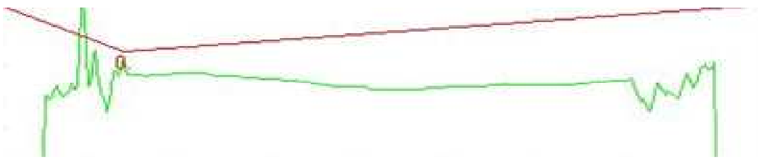
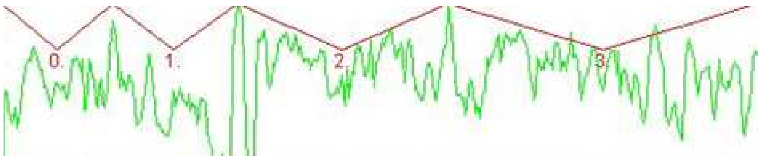
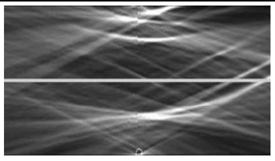
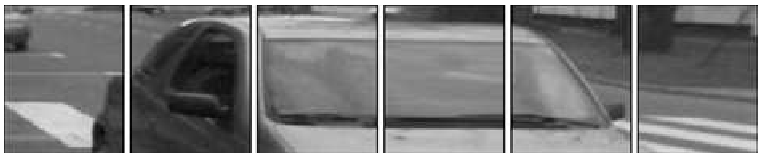
149.jpg	width:526 px	height:350 px	
			The plate has been successfully recognized - no further comment needed.
Detected band	width: 526 px	height: 28 px	
			
Detected plate			Skew detection
 			 **0.0°**
Segmentation	Number of detected characters: 10		
			Recognized plate **RK959AD**

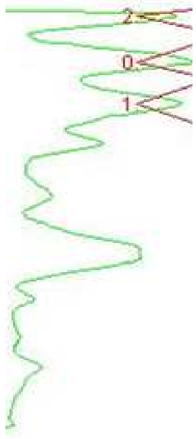
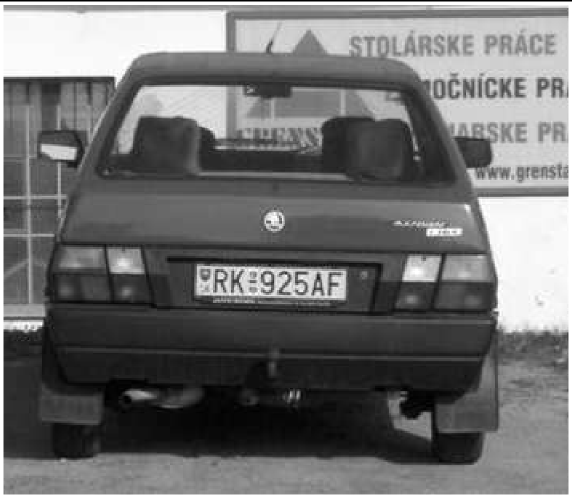
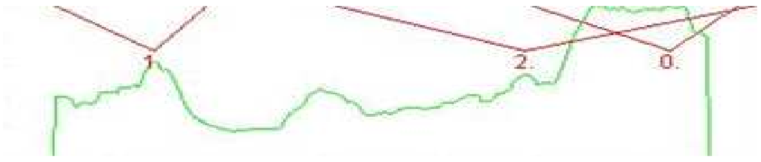
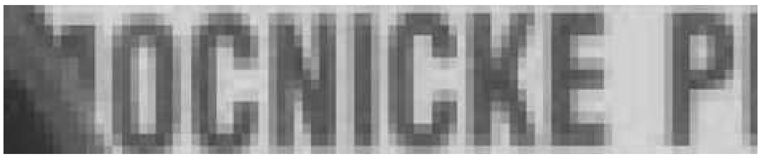
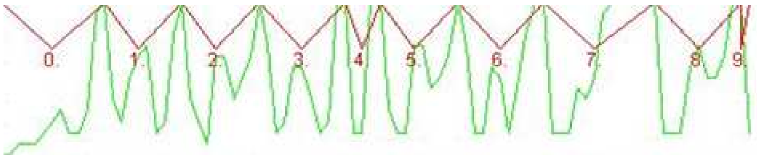
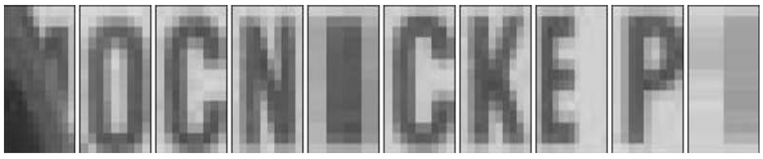
034.jpg	width:547 px	height:410 px	
			The plate has been successfully recognized - no further comment needed.
Detected band	width: 547 px	height: 42 px	
			
Detected plate			Skew detection
			 **-2.656°**
Segmentation	Number of detected characters: 11		
			Recognized plate **LM010BE**

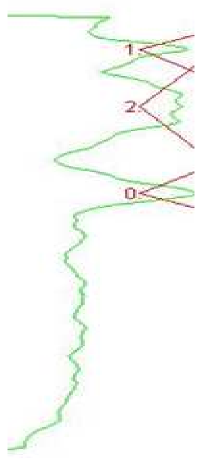
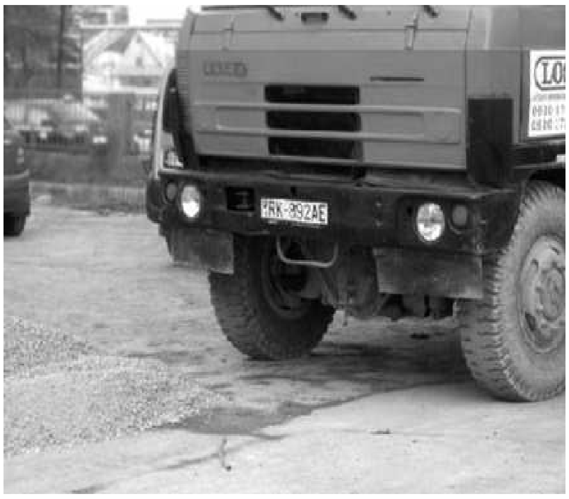
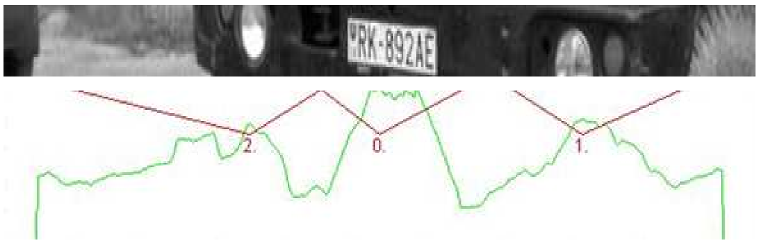
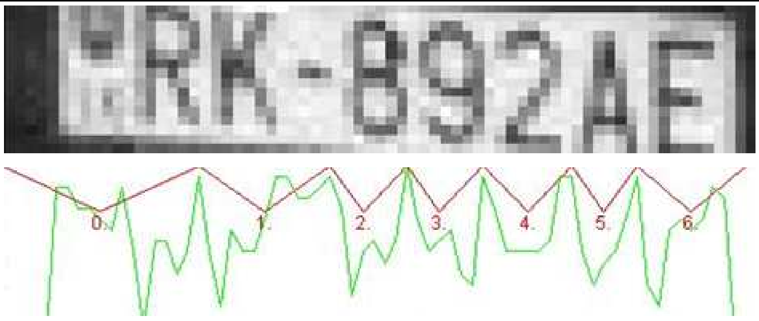
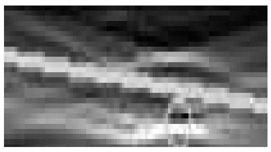
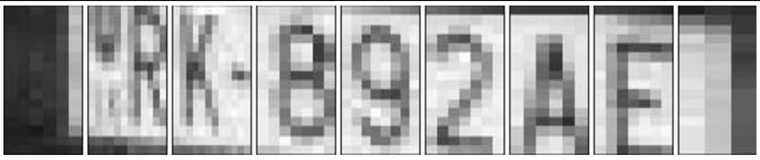
049.jpg	width:410 px	height:360 px	
			The plate has been successfully recognized - no further comment needed.
Detected band	width: 410 px	height: 27 px	
 			
Detected plate			Skew detection
 			 **-5.762°**
Segmentation	Number of detected characters: 10		
			Recognized plate **RK878AC**

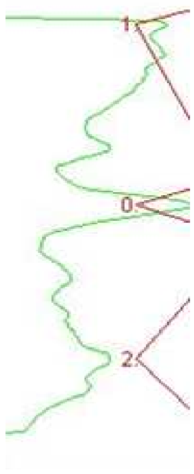
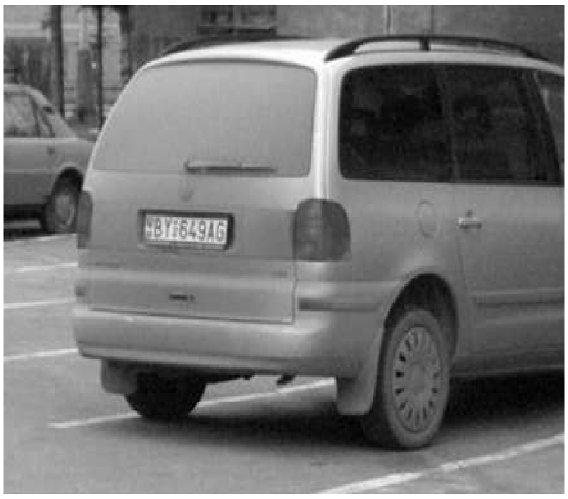
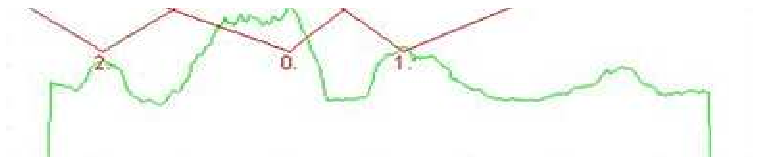
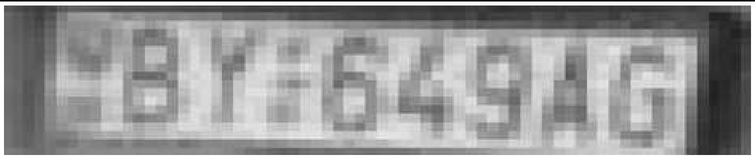
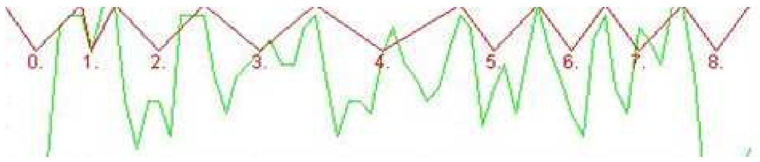
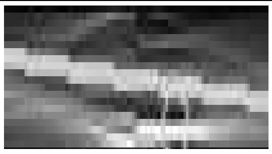
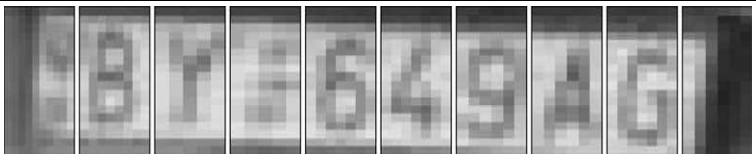
040.jpg	width:576 px	height:432 px	
 			The plate has been successfully recognized - no further comment needed.
Detected band	width: 576 px	height: 21 px	
			
Detected plate			Skew detection
			 **0.0°**
Segmentation	Number of detected characters: 11		
			Recognized plate **BA738DE**

098.jpg	width: 425 px	height: 330 px	
 			The plate has been successfully recognized - no further comment needed.
Detected band	width: 425 px	height: 28 px	
			
Detected plate			Skew detection
			 **0.0°**
Segmentation	Number of detected characters: 9		
			Recognized plate **1B19839**

60.jpg	width:424 px	height:336 px	Class: blurred characters
			A significant blur caused improper detection of the band in a graph of vertical projection. In addition, further heuristic analyses did not detect this fault. Because of this, the incorrect candidate to number plate has been deskewed and then segmented even though it does not have any semantics. Point of failure: vertical projection and heuristic analysis of band.
Detected band	width: 424 px	height: 141 px	
			
Detected plate			Skew detection
			 **0.136°**
Segmentation	Number of detected characters: 6		
			Recognized plate **N/A**

023.jpg	width:354 px	height:308 px	Class: difficult environment
	 STOLÁRSKE PRÁCE OČNÍCKE PR ARSKE PR www.grensta RK 925AF		This is a typical snapshot with a difficult surrounding environment. The table in the background contains more characters in one line than a number plate in the foreground. This fact causes a bigger amount of horizontal edges in an area of the table. The three detected peaks in the graph of the horizontal projection correspond to three rows in the table. Although the number plate candidate is wrong, the further analysis did not refuse it, because the number of characters (10) is within the allowed range.
Detected band	width: 354 px	height: 19 px	
			
Detected plate			Skew detection
			**-1.169°**
Segmentation	Number of detected characters: 10		
			Recognized plate **0CNCKEP**

044.jpg	width:530 px	height:397 px	Class: skewed plates
			The part of graph corresponding to the peak “2” (in vertical graph) has a wider distribution due to the improper vertical projection caused by a skew of the plate. Because of this, the bottom of the last character “E” has been cut off improperly. Point of failure : vertical projection – band clipping
Detected band	width: 530 px	height: 30 px	
			
Detected plate			Skew detection
			 **5.599°**
Segmentation	Number of detected characters: 7		
			Recognized plate **RK892AF**

067.jpg	width:402 px	height:298 px	Class: extremely small characters
 			This snapshot contains extremely small characters, which are not distinguishable by a machine. The number plate has been properly detected and characters have been segmented as well, but it is very hard to distinguish between the “B” and “8” on the first position and between the “6” and “8” on the third position of plate. Point of failure : character recognition
Detected band	width: 402 px	height: 21 px	
			
Detected plate			Skew detection
 			 **4.00°**
Segmentation	Number of detected characters: 10		
			Recognized plate **8Y849A4**

Appendix B: Demo recognition software - User’s manual

JavaANPR is an ANPR recognition software that demonstrates principles described in this thesis. It is written in the Java programming language. If you want to run it, you will need the Java 1.5.0 SE runtime environment (or higher).

After downloading the distribution package, please unpack it into a chosen directory. The distribution package contains compiled program classes, jar archive, source codes and additional program resources such as bitmaps, neural networks etc.

`build`	Compiled classes
`dist`	Distribution directory, contains JAR file and additional resources
`lib`	Compile-time libraries
`nbproject`	Project metadata and build configuration
`resources`	Resources, configuration file, bitmaps, neural networks
`src`	Source files

1. Cleaning, compiling and building the project (optional)

Normally, you do not have to compile the project, because distribution package already contains precompiled binaries. If you want to recompile it again, you can do it using the “Apache Ant” utility.

At first, change a working directory to the “javaanpr”, and type the following command to clean the previous build of the JavaANPR. Issuing this command will delete whole content of the `build` and `dist` directories:

```
javaanpr # ant clean
```

Then, issue the “`ant compile`” and “`ant jar`” commands. The “`compile`” target will compile all source files in the `src` directory. The “`jar`” target will create the “`dist`” directory with a jar archive and additional run-time resources.

```
javaanpr # ant compile
javaanpr # ant jar
```

2. Running the viewer

You can run the interactive ANPR viewer using the ant by typing the following command:

```
javaanpr # ant run
```

If you do not have installed the ANT utility, you can run viewer manually by the following commands:

```
javaanpr # cd ./dist
dist # java –jar javaanpr.jar
```

Another way to run the viewer is a double-click to a `javaanpr.jar` archive (in the MS Explorer)

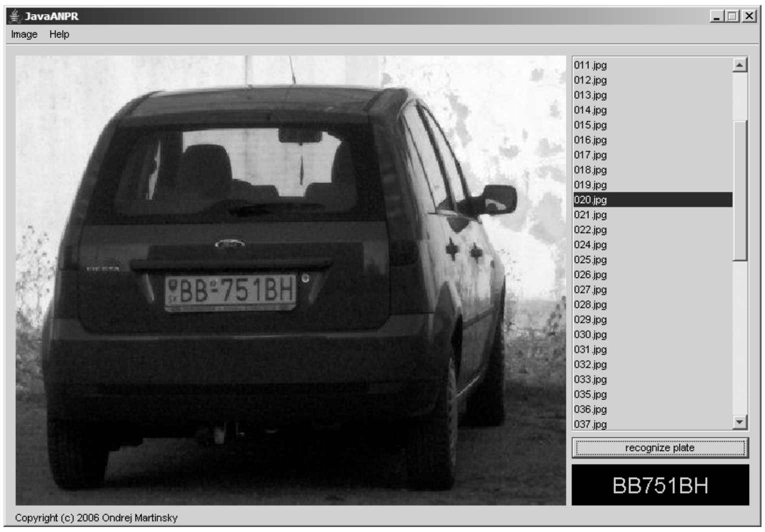


Figure B.1: Graphical user interface of the JavaANPR viewer

Important: By default, the program expects the configuration file “config.xml” and other resources in the working directory. Because of this, please do not run the jar archive from other directories. Otherwise, the program will not be able to start.

3. Using command-line arguments

Besides the graphical user interface, program also contains additional functions, which are accessible using the command-line arguments. For more information about it, please run the jar file with a “-help” command:

```
Automatic number plate recognition system
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Usage : java -jar javaanpr.jar [-options]

Where options include:

    -help          Displays this help
    -gui           Run GUI viewer (default choice)
    -recognize -i <snapshot>
                   Recognize single snapshot
    -recognize -i <snapshot> -o <dstdir>
                   Recognize single snapshot and save report html into
                   specified directory
    -newconfig -o <file>
                   Generate default configuration file
    -newnetwork -o <file>
                   Train neural network according to specified feature
                   extraction method and learning parameters (in config.
                   file) and saves it into output file
    -newalphabet -i <srcdir> -o <dstdir>
                   Normalize all images in <srcdir> and save it to <dstdir>.
```

3.1 Command-line recognition

If you do not want to use the GUI viewer, you can recognize snapshot by issuing the following command. The recognized plate will be written to standard output.

```
dist # java –jar javaanpr.jar –recognize –i <name of image>
```

3.2 Recognition report

Sometimes, it is good to see inside the recognition process of concrete image. Because of this, JavaANPR supports a generation of HTML reports. The recognition report contains images and verbose debugging information about each step of the recognition process. HTML report can be used to determine a point, in which the recognition process failed.

The following command will recognize the image specified by its name, and save the report into a specified destination directory:

```
dist # java –jar javaanpr.jar –recognize –i <name of image>
-o <destination directory>
```

3.3 Creating the default configuration file

Configuration file contains settings and parameters, which are needed during the recognition process. If configuration file does not exist, program will not be able to start. Because of this, JavaANPR is able to generate a default configuration file with recommended configuration settings by the following command:

```
dist # java –jar javaanpr.jar -newconfig -o <file>
```

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