

Chomsky Hierarchy

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Chomsky Hierarchy

$$\mathcal{L}(\text{REG}) \subset \mathcal{L}(\text{LIN}) \subset \mathcal{L}(\text{CF}) \subset \mathcal{L}(\text{CS}) \subset \mathcal{L}(\text{RE}) \subset \mathcal{L}(\text{ALL})$$

$\mathcal{L}(\text{REG})$ family of regular languages (type 3)

$\mathcal{L}(\text{LIN})$ family of linear languages

$\mathcal{L}(\text{CF})$ family of context-free languages (type 2)

$\mathcal{L}(\text{CS})$ family of context-sensitive languages (type 1)

$\mathcal{L}(\text{RE})$ family of recursively enumerable languages (type 0)

$\mathcal{L}(\text{ALL})$ family of all languages

Type 0 Grammars

Type 0 Grammar

$$G = (N, T, P, S)$$

N alphabet of **nonterminals**

T alphabet of **terminals**

P finite set of **productions** (**rules**) of the form

$$y \rightarrow x$$

with $y, x \in V^*$, $\text{alph}(y) \cap N \neq \emptyset$

S the **start symbol**, $S \in N$

- $V = N \cup T$

- **alph**(y) is the set of all symbols occurring in $y \in V^*$

Derivation Step

Production Label

Consider a production $p : y \rightarrow x$

p label

$y \rightarrow x$ production

Instead of $y \rightarrow x \in P$, we often write $p \in P$

Derivation Step

For every $u, v \in V^*$ and $p : y \rightarrow x$,

$$uyv \Rightarrow uxv [p]$$

or simply

$$uyv \Rightarrow uxv$$

Notation

For every y_0, y_1, \dots, y_n , for some $n \geq 1$, such that

$$y_0 \Rightarrow y_1 [p_1] \Rightarrow y_2 [p_2] \Rightarrow \dots \Rightarrow y_n [p_n],$$

where $p_i \in P$, for all $i = 1, \dots, n$,

- $y_0 \Rightarrow^n y_n [p_1 \dots p_n]$ or $y_0 \Rightarrow^n y_n$

- for every $y \in V^*$,

$$y \Rightarrow^0 y [\varepsilon] \text{ or } y \Rightarrow^0 y$$

- if $v \Rightarrow^m w [\alpha]$ for some $m \geq 1$, then

$$v \Rightarrow^+ w [\alpha] \text{ or } v \Rightarrow^+ w$$

- if $v \Rightarrow^m w [\alpha]$ for some $m \geq 0$, then

$$v \Rightarrow^* w [\alpha] \text{ or } v \Rightarrow^* w$$

Relation of Direct Derivation

- \Rightarrow direct derivation relation
- \Rightarrow^* **transitive** and **reflexive** closure of direct derivation relation
- \Rightarrow^+ **transitive** closure of direct derivation relation

Language of Grammar

$$L(G) = \{w \in T^* : S \Rightarrow^* w\}$$

CS, CF, LIN and REG Grammars

CS, CF, LIN and REG Grammars

A type-0 grammar $G = (N, T, P, S)$ is

- 1 **context-sensitive** if for every $y \rightarrow x \in P$,

$$|y| \leq |x|, \text{ or } y = S, x = \varepsilon$$

- 2 **context-free** if for every $y \rightarrow x \in P$,

$$y \in N$$

- 3 **linear** if for every $y \rightarrow x \in P$,

$$y \in N, x \in T^* \cup T^*NT^*$$

- 4 **regular** if for every $y \rightarrow x \in P$,

$$y \in N, x \in \{\varepsilon\} \cup T \cup TN$$

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