

Random Context Grammars

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Random Context Grammar

Random Context Grammar

A (permitting) random context grammar is a pair

$$H = (G, R)$$

where

- $G = (N, T, P, S)$ is a context-free grammar
- R is a finite relation from P to N

Notation

If $p : A \rightarrow x \in P$, $R(p) = Q$, we write

$$(p : A \rightarrow x, Q)$$

where $Q \subseteq N$ is called a permitting context

Derivation Step of Random Context Grammar

Derivation Step

For $x, y \in V^*$, $p \in P$,

$$x \Rightarrow y [p] \text{ in } H$$

if

- 1** $x \Rightarrow y [p] \text{ in } G$ and
- 2** $R(p) \subseteq \text{alph}(x)$

Random Context Grammar with Appearance Checking

Random Context Grammar with Appearance Checking

A random context grammar with appearance checking is a triple

$$H = (G, R, F)$$

where

- $G = (N, T, P, S)$ is a context-free grammar
- R, F are two finite relations from P to N

Notation

If $p : A \rightarrow x \in P$, $R(p) = Q$, and $F(p) = K$, we write

$$(p : A \rightarrow x, Q, K)$$

where Q and K are permitting and forbidding contexts, respectively

Derivation Step of Random Context Grammar with Appearance Checking

Forbidding Grammar

If every $(p : A \rightarrow x, Q, K)$ satisfies $Q = \emptyset$, then H is called a forbidding grammar.

Derivation Step

For $x, y \in V^*$, $p \in P$,

$$x \Rightarrow y [p] \text{ in } H$$

if

- 1** $x \Rightarrow y [p] \text{ in } G$
- 2** $R(p) \subseteq \text{alph}(x)$
- 3** $F(p) \cap \text{alph}(x) = \emptyset$

Example – Permitting Grammar I

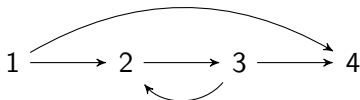
Example

1 ($S \rightarrow ABC, \emptyset$)

2 ($A \rightarrow aA', \{B\}$)
($B \rightarrow bB', \{C\}$)
($C \rightarrow cC', \{A'\}$)

3 ($A' \rightarrow A, \{B'\}$)
($B' \rightarrow B, \{C'\}$)
($C' \rightarrow C, \{A\}$)

4 ($A \rightarrow a, \{B\}$)
($B \rightarrow b, \{C\}$)
($C \rightarrow c, \emptyset$)



Example – Permitting Grammar II

Example

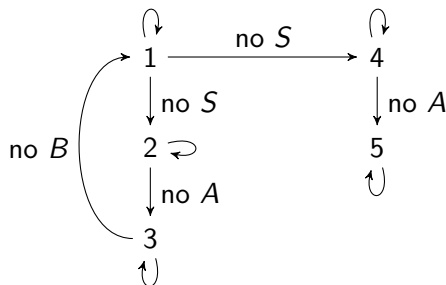
$$\begin{array}{ll} S \Rightarrow ABC & 1 \\ \Rightarrow^3 aA'bB'cC' & 2 \\ \Rightarrow^3 aAbBcC & 3 \\ \Rightarrow^3 aaA'bbB'ccC' & 2 \\ \Rightarrow^3 aaAbbBccC & 3 \\ & \vdots \\ \Rightarrow^3 a^n Ab^n Bc^n C & 3 \\ \Rightarrow^3 a^{n+1} b^{n+1} c^{n+1} & 4 \end{array}$$

$$L(H) = \{a^k b^k c^k : k \geq 1\}$$

Example – Forbidding Grammar I

Example

- (1 : $S \rightarrow AA, \emptyset, \{B, D\}$)
- (2 : $A \rightarrow B, \emptyset, \{S, D\}$)
- (3 : $B \rightarrow S, \emptyset, \{A, D\}$)
- (4 : $A \rightarrow D, \emptyset, \{S, B\}$)
- (5 : $D \rightarrow a, \emptyset, \{S, A, B\}$)



Example – Forbidding Grammar II

Example

		rules
$S \Rightarrow$	AA	1
\Rightarrow^2	BB	2
\Rightarrow^2	SS	3
\Rightarrow^2	$AAAA$	1
\Rightarrow^4	$BBBB$	2
\Rightarrow^4	$SSSS$	3
	\vdots	
\Rightarrow^{2^i}	$A^{2^{i+1}}$	1
$\Rightarrow^{2^{i+1}}$	$D^{2^{i+1}}$	4
$\Rightarrow^{2^{i+1}}$	$a^{2^{i+1}}$	5

$$L(H) = \{a^{2^n} : n \geq 1\}$$

Bibliography



J. Dassow and Gh. Păun.

Regulated Rewriting in Formal Language Theory.

Akademie-Verlag, Berlin, 1989.



A. P. J. van der Walt.

Random context grammars.

In *Proceedings of Symposium on Formal Languages*, 1970.