

Cooperating Distributed Grammar Systems

Jiří Techet Tomáš Masopust Alexander Meduna

Department of Information Systems
Faculty of Information Technology
Brno University of Technology
Božetěchova 2, Brno 61266, Czech Republic

Modern Formal Language Theory, 2007

CD Grammar Systems

CD Grammar System

A cooperating distributed (CD) grammar system of degree n , $n \geq 1$, is a construct

$$\Gamma = (N, T, S, P_1, \dots, P_n)$$

where

N, T, S are defined as usual

P_i is a finite set of context-free productions, called component of Γ ,
for each $i \in \{1, \dots, n\}$

Notation

- the i th grammar $G_i = (N, T, P_i, S)$
- $y \not\Rightarrow z$ – y does not directly derive z

CD Grammar Systems – Derivation Modes I

Terminating Mode

For each $i = 1, \dots, n$, terminating derivation by the i th component

$$x \Rightarrow_i^t y$$

iff

- 1** $x \Rightarrow^* y$ in $G_i = (N, T, P_i, S)$ and
- 2** $y \not\Rightarrow z$ for all $z \in (N \cup T)^*$

Note

$y \in T^*$ is not required

CD Grammar Systems – Derivation Modes II

k -Step Derivation in G_i

$$x \Rightarrow_i^k y$$

iff $x \Rightarrow^k y$ in G_i

At Most k -Step Derivation in G_i

$$x \Rightarrow_i^{\leq k} y$$

iff $x \Rightarrow^j y$ in G_i for some $j \leq k$

At Least k -Step Derivation in G_i

$$x \Rightarrow_i^{\geq k} y$$

iff $x \Rightarrow^j y$ in G_i for some $j \geq k$

CD Grammar Systems – Generated Language

Set of Derivation Modes

$$D = \{*, t\} \cup \{\leq k, = k, \geq k : k = 1, 2, \dots\}$$

Set of Possible Derivations

$$F(G_j, u, f) = \{v : u \xRightarrow{f} v\}, \text{ where } j \in \{1, \dots, n\}, f \in D, u \in V^*$$

Generated Language

$$L_f(\Gamma) = \{ w \in T^* : \text{there are } v_0, v_1, \dots, v_m \text{ such that} \\ v_i \in F(G_{j_i}, v_{i-1}, f), i = 1, \dots, m, j_i \in \{1, \dots, n\}, \\ v_0 = S, v_m = w, \text{ for some } m \geq 1 \}$$

CD Grammar Systems – Example I

Example

$$\Gamma = (\{S, A, A', B, B'\}, \{a, b, c\}, S, P_1, P_2)$$

where

$$P_1 = \{S \rightarrow S, S \rightarrow AB, A' \rightarrow A, B' \rightarrow B\}$$

$$P_2 = \{A \rightarrow aA'b, B \rightarrow cB', A \rightarrow ab, B \rightarrow c\}$$

$$L_f(\Gamma)_{f \in \{=1, \geq 1, *, t\} \cup \{\leq k : k \geq 1\}} =$$

$$L_{=2}(\Gamma) = L_{\geq 2}(\Gamma) \quad =$$

$$L_{=k}(\Gamma)_{k \geq 3} = L_{\geq 3}(\Gamma) \quad =$$

CD Grammar Systems – Example I

Example

$$\Gamma = (\{S, A, A', B, B'\}, \{a, b, c\}, S, P_1, P_2)$$

where

$$P_1 = \{S \rightarrow S, S \rightarrow AB, A' \rightarrow A, B' \rightarrow B\}$$

$$P_2 = \{A \rightarrow aA'b, B \rightarrow cB', A \rightarrow ab, B \rightarrow c\}$$

$$L_f(\Gamma)_{f \in \{=1, \geq 1, *, t\} \cup \{\leq k : k \geq 1\}} = \{a^n b^n c^m : m, n \geq 1\}$$

$$L_{=2}(\Gamma) = L_{\geq 2}(\Gamma) =$$

$$L_{=k}(\Gamma)_{k \geq 3} = L_{\geq 3}(\Gamma) =$$

CD Grammar Systems – Example I

Example

$$\Gamma = (\{S, A, A', B, B'\}, \{a, b, c\}, S, P_1, P_2)$$

where

$$P_1 = \{S \rightarrow S, S \rightarrow AB, A' \rightarrow A, B' \rightarrow B\}$$

$$P_2 = \{A \rightarrow aA'b, B \rightarrow cB', A \rightarrow ab, B \rightarrow c\}$$

$$L_f(\Gamma)_{f \in \{=1, \geq 1, *, t\} \cup \{\leq k : k \geq 1\}} = \{a^n b^n c^m : m, n \geq 1\}$$

$$L_{=2}(\Gamma) = L_{\geq 2}(\Gamma) = \{a^n b^n c^n : n \geq 1\}$$

$$L_{=k}(\Gamma)_{k \geq 3} = L_{\geq 3}(\Gamma) =$$

CD Grammar Systems – Example I

Example

$$\Gamma = (\{S, A, A', B, B'\}, \{a, b, c\}, S, P_1, P_2)$$

where

$$P_1 = \{S \rightarrow S, S \rightarrow AB, A' \rightarrow A, B' \rightarrow B\}$$

$$P_2 = \{A \rightarrow aA'b, B \rightarrow cB', A \rightarrow ab, B \rightarrow c\}$$

$$L_f(\Gamma)_{f \in \{=1, \geq 1, *, t\} \cup \{\leq k : k \geq 1\}} = \{a^n b^n c^m : m, n \geq 1\}$$

$$L_{=2}(\Gamma) = L_{\geq 2}(\Gamma) = \{a^n b^n c^n : n \geq 1\}$$

$$L_{=k}(\Gamma)_{k \geq 3} = L_{\geq 3}(\Gamma) = \emptyset$$

CD Grammar Systems – Example II

Example

$$\Gamma = (\{S, A\}, \{a\}, S, P_1, P_2, P_3)$$

where

$$P_1 = \{S \rightarrow AA\}$$

$$P_2 = \{A \rightarrow S\}$$

$$P_3 = \{A \rightarrow a\}$$

$$L_t(\Gamma) =$$

CD Grammar Systems – Example II

Example

$$\Gamma = (\{S, A\}, \{a\}, S, P_1, P_2, P_3)$$

where

$$P_1 = \{S \rightarrow AA\}$$

$$P_2 = \{A \rightarrow S\}$$

$$P_3 = \{A \rightarrow a\}$$

$$L_t(\Gamma) = \{a^{2^n} : n \geq 1\}$$

Denotation of CD Language Families

Denotation of CD Language Families

$$CD_x^y(f)$$

where

f derivation mode, $f \in D$

y ■ nothing – no ε -productions

 ■ ε – ε -productions allowed

x ■ n – degree at most n , $n \geq 1$

 ■ ∞ – the number of components is not limited

Note

$CD_\infty(=)$ the union of all families $CD_\infty(=k)$ for $k = 1, 2, \dots$

$CD_\infty(\geq)$ the union of all families $CD_\infty(\geq k)$ for $k = 1, 2, \dots$

CD Grammar Systems – Generative Power

Theorem

- $CD_{\infty}^y(f) = \mathcal{L}(CF)$, for all $f \in \{= 1, \geq 1, *\} \cup \{\leq k : k \geq 1\}$
- $\mathcal{L}(CF) = CD_1^y(f) \subset CD_2^y(f) \subseteq CD_r^y(f) \subseteq CD_{\infty}^y(f) \subseteq \mathcal{L}(M)$, for all $f \in \{= k, \geq k : k \geq 2\}$, $r \geq 3$
- $CD_r^y(\geq k) \subseteq CD_r^y(\geq k + 1)$
- $CD_{\infty}^y(\geq) \subseteq CD_{\infty}^y(=)$
- $\mathcal{L}(CF) = CD_1^y(t) = CD_2^y(t) \subset CD_3^y(t) = CD_{\infty}^y(t) = \mathcal{L}(ET0L)$

Hybrid CD Grammar Systems

- combination of different modes is possible

Hybrid CD Grammar System

$$\Gamma = (N, T, S, (P_1, f_1), \dots, (P_n, f_n))$$

where

N, T, S, P_i are defined as in the case of CD grammar systems

f_i is the mode of the i th component, $f_i \in D$ for all $i \in \{1, \dots, n\}$

Generated Language

$$L(\Gamma) = \{w \in T^* : \text{there are } v_0, v_1, \dots, v_m \text{ such that} \\ v_i \in F(G_{j_i}, v_{i-1}, f_{j_i}), i = 1, \dots, m, j_i \in \{1, \dots, n\}, \\ v_0 = S, v_m = w, \text{ for some } m \geq 1\}$$

Denotation of Hybrid CD Language Families

Denotation of Hybrid CD Language Families

$$XCD_{x,v}^y(f)$$

where

x, y, f are defined as in the case of CD grammar systems

- v
 - m – each P_i contains at most m productions, $m \geq 1$
 - ∞ , nothing – any number of productions
- X
 - nothing – nondeterministic
 - D – deterministic (for each P_i , $A \rightarrow u, A \rightarrow w \in P_i$ satisfy $u = w$)
 - H – hybrid (then, (f) is not written)

Example

HCD_n^ε is a hybrid CD grammar system with at most n components

Hybrid CD Grammar Systems – Generative Power


Theorem


- $CD_{\infty,\infty}(f) = CD_{\infty,1}(f) = \mathcal{L}(CF)$, for all $f \in \{=, 1, \geq 1, *\} \cup \{\leq k : k \geq 1\}$
- $\mathcal{L}(CF) \subset CD_{\infty,1}^{\varepsilon}(t) \subset CD_{\infty,2}^{\varepsilon}(t) \subseteq CD_{\infty,3}^{\varepsilon}(t) \subseteq CD_{\infty,4}^{\varepsilon}(t) \subseteq CD_{\infty,5}^{\varepsilon}(t) = CD_{\infty,\infty}^{\varepsilon}(t) = \mathcal{L}(ET0L)$
- $CD_{n,m}(f) \subset CD_{n+1,m}(f)$, $f \in \{*, t\}$
- $CD_{n,m}(f) \subset CD_{n,m+1}(f)$, $f \in \{*, t\}$


Theorem

- $\mathcal{L}(CF) = HCD_1 \subset HCD_2 \subseteq HCD_3 \subseteq HCD_4 = HCD_{\infty} = \mathcal{L}(M, ac)$
- $\mathcal{L}(ET0L) \subset HCD_4$
- $CD_{\infty}(=) \subset HCD_3$

Bibliography

 Grammar systems.
<http://www.sztaki.hu/mms/bib.html>.

 E. Csuhaj-Varjú and J. Dassow.
On cooperating distributed grammar systems.
Elektronische Informationsverarbeitung und Kybernetik, 26:49–63,
1990.

 R. Meersman and G. Rozenberg.
Cooperating grammar systems.
In *MFCS 78*, pages 364–373, 1978.