

Cooperating Distributed Grammar Systems

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CD Grammar Systems

CD Grammar System

A cooperating distributed (CD) grammar system of degree n , $n \geq 1$, is a construct

$$\Gamma = (N, T, S, P_1, \dots, P_n)$$

where

N, T, S are defined as usual

P_i is a finite set of context-free productions, called component of Γ ,
for each $i \in \{1, \dots, n\}$

Notation

- the i th grammar $G_i = (N, T, P_i, S)$
- $y \not\Rightarrow z$ – y does not directly derive z

CD Grammar Systems – Derivation Modes I

Terminating Mode

For each $i = 1, \dots, n$, terminating derivation by the i th component

$$x \overset{t}{\Rightarrow}_i y$$

iff

- 1** $x \Rightarrow^* y$ in $G_i = (N, T, P_i, S)$ and
- 2** $y \not\Rightarrow z$ for all $z \in (N \cup T)^*$

Note

$y \in T^*$ is not required

CD Grammar Systems – Derivation Modes II

k-Step Derivation in G_i

$$x \overset{i}{\Rightarrow} =^k y$$

iff $x \Rightarrow^k y$ in G_i

At Most *k*-Step Derivation in G_i

$$x \overset{i}{\Rightarrow} \leq^k y$$

iff $x \Rightarrow^j y$ in G_i for some $j \leq k$

At Least *k*-Step Derivation in G_i

$$x \overset{i}{\Rightarrow} \geq^k y$$

iff $x \Rightarrow^j y$ in G_i for some $j \geq k$

CD Grammar Systems – Generated Language

Set of Derivation Modes

$$D = \{*, t\} \cup \{\leq k, = k, \geq k : k = 1, 2, \dots\}$$

Set of Possible Derivations

$$F(G_j, u, f) = \{v : u \xrightarrow{f} v\}, \text{ where } j \in \{1, \dots, n\}, f \in D, u \in V^*$$

Generated Language

$$L_f(\Gamma) = \{ w \in T^* : \text{there are } v_0, v_1, \dots, v_m \text{ such that} \\ v_i \in F(G_{j_i}, v_{i-1}, f), i = 1, \dots, m, j_i \in \{1, \dots, n\}, \\ v_0 = S, v_m = w, \text{ for some } m \geq 1 \}$$

CD Grammar Systems – Example I

Example

$$\Gamma = (\{S, A, A', B, B'\}, \{a, b, c\}, S, P_1, P_2)$$

where

$$P_1 = \{S \rightarrow S, S \rightarrow AB, A' \rightarrow A, B' \rightarrow B\}$$

$$P_2 = \{A \rightarrow aA'b, B \rightarrow cB', A \rightarrow ab, B \rightarrow c\}$$

$$L_f(\Gamma)_{f \in \{=1, \geq 1, *, t\} \cup \{\leq k : k \geq 1\}} =$$

$$L_{=2}(\Gamma) = L_{\geq 2}(\Gamma) =$$

$$L_{=k}(\Gamma)_{k \geq 3} = L_{\geq 3}(\Gamma) =$$

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$$L_f(\Gamma)_{f \in \{=1, \geq 1, *, t\} \cup \{\leq k : k \geq 1\}} = \{a^n b^n c^m : m, n \geq 1\}$$

$$L_{=2}(\Gamma) = L_{\geq 2}(\Gamma) =$$

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$$L_{=2}(\Gamma) = L_{\geq 2}(\Gamma) = \{a^n b^n c^n : n \geq 1\}$$

$$L_{=k}(\Gamma)_{k \geq 3} = L_{\geq 3}(\Gamma) = \emptyset$$

CD Grammar Systems – Example II

Example

$$\Gamma = (\{S, A\}, \{a\}, S, P_1, P_2, P_3)$$

where

$$P_1 = \{S \rightarrow AA\}$$

$$P_2 = \{A \rightarrow S\}$$

$$P_3 = \{A \rightarrow a\}$$

$$L_t(\Gamma) =$$

CD Grammar Systems – Example II

Example

$$\Gamma = (\{S, A\}, \{a\}, S, P_1, P_2, P_3)$$

where

$$P_1 = \{S \rightarrow AA\}$$

$$P_2 = \{A \rightarrow S\}$$

$$P_3 = \{A \rightarrow a\}$$

$$L_t(\Gamma) = \{a^{2^n} : n \geq 1\}$$

Denotation of CD Language Families

Denotation of CD Language Families

$$CD_x^y(f)$$

where

f derivation mode, $f \in D$

y ■ nothing – no ε -productions

■ ε – ε -productions allowed

x ■ n – degree at most n , $n \geq 1$

■ ∞ – the number of components is not limited

Note

$CD_\infty(=)$ the union of all families $CD_\infty(= k)$ for $k = 1, 2, \dots$

$CD_\infty(\geq)$ the union of all families $CD_\infty(\geq k)$ for $k = 1, 2, \dots$

CD Grammar Systems – Generative Power

Theorem

- $CD_{\infty}^y(f) = \mathcal{L}(CF)$, for all $f \in \{= 1, \geq 1, *\} \cup \{\leq k : k \geq 1\}$
- $\mathcal{L}(CF) = CD_1^y(f) \subset CD_2^y(f) \subseteq CD_r^y(f) \subseteq CD_{\infty}^y(f) \subseteq \mathcal{L}(M)$, for all $f \in \{= k, \geq k : k \geq 2\}, r \geq 3$
- $CD_r^y(\geq k) \subseteq CD_r^y(\geq k + 1)$
- $CD_{\infty}^y(\geq) \subseteq CD_{\infty}^y(=)$
- $\mathcal{L}(CF) = CD_1^y(t) = CD_2^y(t) \subset CD_3^y(t) = CD_{\infty}^y(t) = \mathcal{L}(ETOL)$

Hybrid CD Grammar Systems

- combination of different modes is possible

Hybrid CD Grammar System

$$\Gamma = (N, T, S, (P_1, f_1), \dots, (P_n, f_n))$$

where

N, T, S, P_i are defined as in the case of CD grammar systems

f_i is the mode of the i th component, $f_i \in D$ for all $i \in \{1, \dots, n\}$

Generated Language

$$L(\Gamma) = \{w \in T^* : \text{there are } v_0, v_1, \dots, v_m \text{ such that} \\ v_i \in F(G_{j_i}, v_{i-1}, f_{j_i}), i = 1, \dots, m, j_i \in \{1, \dots, n\}, \\ v_0 = S, v_m = w, \text{ for some } m \geq 1\}$$

Denotation of Hybrid CD Language Families

Denotation of Hybrid CD Language Families

$$XCD_{x,v}^y(f)$$

where

x, y, f are defined as in the case of CD grammar systems

- v
 - m – each P_i contains at most m productions, $m \geq 1$
 - ∞ , nothing – any number of productions
- X
 - nothing – nondeterministic
 - D – deterministic (for each P_i , $A \rightarrow u, A \rightarrow w \in P_i$ satisfy $u = w$)
 - H – hybrid (then, (f) is not written)

Example

HCD_n^ϵ is a hybrid CD grammar system with at most n components

Hybrid CD Grammar Systems – Generative Power


Theorem


- $CD_{\infty,\infty}(f) = CD_{\infty,1}(f) = \mathcal{L}(CF)$, for all $f \in \{=, \geq, *\} \cup \{\leq k : k \geq 1\}$
- $\mathcal{L}(CF) \subset CD_{\infty,1}^{\varepsilon}(t) \subset CD_{\infty,2}^{\varepsilon}(t) \subseteq CD_{\infty,3}^{\varepsilon}(t) \subseteq CD_{\infty,4}^{\varepsilon}(t) \subseteq CD_{\infty,5}^{\varepsilon}(t) = CD_{\infty,\infty}^{\varepsilon}(t) = \mathcal{L}(ETOL)$
- $CD_{n,m}(f) \subset CD_{n+1,m}(f)$, $f \in \{*, t\}$
- $CD_{n,m}(f) \subset CD_{n,m+1}(f)$, $f \in \{*, t\}$


Theorem

- $\mathcal{L}(CF) = HCD_1 \subset HCD_2 \subseteq HCD_3 \subseteq HCD_4 = HCD_{\infty} = \mathcal{L}(M, ac)$
- $\mathcal{L}(ETOL) \subset HCD_4$
- $CD_{\infty}(=) \subset HCD_3$

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