

# Programmed Grammars

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# Programmed Grammar

## Programmed Grammar

A programmed grammar is a pair

$$H = (G, R)$$

where

- $G = (N, T, P, S)$  is a context-free grammar
- $R$  is a finite relation on  $P$

## Notation

If  $p : A \rightarrow x \in P$ ,  $R(p) = Q$ , we write

$$(p : A \rightarrow x, Q)$$

# Generated Language

## Derivation Step

For  $(x, p), (y, q) \in V^* \times P$ ,

$$(x, p) \Rightarrow (y, q) \text{ in } H$$

if

**1**  $x \Rightarrow y [p]$  in  $G$

**2**  $q \in R(p)$

## Generated Language

$$L(H) = \{x \in T^* : (S, p) \Rightarrow^* (x, p') \text{ for some } p, p' \in P\}$$

# Programmed Grammar – Example

## Example

$$(1 : S \rightarrow ABC, \{2, 5\})$$

$$(2 : A \rightarrow aA, \{3\})$$

$$(3 : B \rightarrow bB, \{4\})$$

$$(4 : C \rightarrow cC, \{2, 5\})$$

$$(5 : A \rightarrow a, \{6\})$$

$$(6 : B \rightarrow b, \{7\})$$

$$(7 : C \rightarrow c, \{7\})$$

$$(S, 1) \Rightarrow (ABC, 2)$$

$$\Rightarrow (aABC, 3)$$

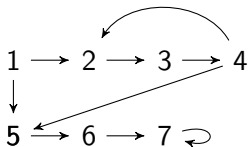
$$\Rightarrow (aAbBC, 4)$$

$$\Rightarrow (aAbBcC, 5)$$

$$\Rightarrow (aabBcC, 6)$$

$$\Rightarrow (aabbcc, 7)$$

$$\Rightarrow (aabbcc, 7)$$



$$L(H) = \{a^n b^n c^n : n \geq 1\}$$

# Programmed Grammar with Appearance Checking

## Programmed Grammar with Appearance Checking

A programmed grammar with appearance checking is a triple

$$H = (G, R, F)$$

where

- $G = (N, T, P, S)$  is a context-free grammar
- $R, F$  are finite relations on  $P$

### Notation

If  $p : A \rightarrow x \in P$ ,  $R(p) = U$ , and  $F(p) = V$ , we write

$$(p : A \rightarrow x, U, V)$$

where  $U$  and  $V$  are success and failure fields, respectively

# Derivation Step

## Derivation Step

For  $(x, p), (y, q) \in V^* \times P$ ,

$$(x, p) \Rightarrow (y, q) \text{ in } H$$

if

- either  $x \Rightarrow y [p]$  in  $G$  and  $q \in R(p)$
- or  $x = y$ ,  $q \in F(p)$ ,  $p$  is not applicable to  $x$

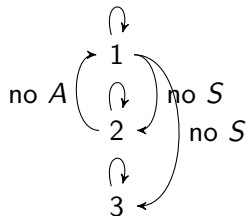
# Example I

## Example

(1 :  $S \rightarrow AA, \{1\}, \{2, 3\}$ )

(2 :  $A \rightarrow S, \{2\}, \{1\}$ )

(3 :  $A \rightarrow a, \{3\}, \emptyset$ )



$L(H) = \{a^{2^n} : n \geq 1\}$

$(S, 1) \Rightarrow (AA, 1) \Rightarrow (AA, 3)$   
 $\Rightarrow (Aa, 3)$   
 $\Rightarrow (aa, 3)$

$\Rightarrow (AA, 2)$   
 $\Rightarrow (AS, 2)$   
 $\Rightarrow (SS, 2)$   
 $\Rightarrow (SS, 1)$   
 $\Rightarrow (SAA, 1)$   
 $\Rightarrow (AAAA, 1) \Rightarrow (AAAA, 3)$   
 $\Rightarrow (AaAA, 3)$   
 $\Rightarrow (AaaA, 3)$   
 $\Rightarrow (aaaA, 3)$   
 $\Rightarrow (aaaa, 3)$

$\Rightarrow (AAAA, 2)$   
 $\Rightarrow (ASAA, 2) \dots$

# Example II

## Example

(1 :  $S \rightarrow SC, \{1, 2\}, \emptyset$ )

(2 :  $S \rightarrow AA, \{3\}, \emptyset$ )

(3 :  $A \rightarrow B, \{4\}, \{5\}$ )

(4 :  $C \rightarrow D, \{3\}, \{7\}$ )

(5 :  $C \rightarrow C, \{6\}, \emptyset$ )

(6 :  $B \rightarrow A, \{6\}, \{3\}$ )

(7 :  $B \rightarrow A, \{7\}, \{8\}$ )

(8 :  $D \rightarrow A, \{9\}, \{10\}$ )

(9 :  $D \rightarrow C, \{9\}, \{3\}$ )

(10 :  $A \rightarrow a, \{10\}, \emptyset$ )

Which language generates this grammar?



# Bibliography



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