Chomsky Hierarchy

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Chomsky Hierarchy

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$$\mathscr{L}(REG) \subset \mathscr{L}(LIN) \subset \mathscr{L}(CF) \subset \mathscr{L}(CS) \subset \mathscr{L}(RE) \subset \mathscr{L}(ALL)$$

$\mathscr{L}(REG)$ family of regular languages (type 3)

$\mathscr{L}(LIN)$ family of linear languages

$\mathscr{L}(CF)$ family of context-free languages (type 2)

$\mathscr{L}(CS)$ family of context-sensitive languages (type 1)

$\mathscr{L}(RE)$ family of recursively enumerable languages (type 0)

$\mathscr{L}(ALL)$ family of all languages

Type 0 Grammars

Type 0 Grammar

$$G = (N, T, P, S)$$

N alphabet of nonterminals

T alphabet of terminals

P finite set of productions (rules) of the form

$$y \rightarrow x$$

with $y, x \in V^*$, alph$(y) \cap N \neq \emptyset$

S the start symbol, $S \in N$

- $V = N \cup T$
- alph(y) is the set of all symbols occurring in $y \in V^*$

Derivation Step

Production Label

Consider a production $p : y \rightarrow x$

$\qquad\quad p$ label

$\qquad y \rightarrow x$ production

Instead of $y \rightarrow x \in P$, we often write $p \in P$

Derivation Step

For every $u, v \in V^*$ and $p : y \rightarrow x$,

$$uyv \Rightarrow uxv \; [p]$$

or simply

$$uyv \Rightarrow uxv$$

Notation

Notation

For every $y_0, y_1, \ldots, y_n$, for some $n \geq 1$, such that

$$y_0 \Rightarrow y_1 \; [p_1] \Rightarrow y_2 \; [p_2] \Rightarrow \; \ldots \; \Rightarrow y_n \; [p_n],$$

where $p_i \in P$, for all $i = 1, \ldots, n$,

- $y_0 \Rightarrow^n y_n \; [p_1 \ldots p_n]$ or $y_0 \Rightarrow^n y_n$
- for every $y \in V^*$,
$$y \Rightarrow^0 y \; [\varepsilon] \text{ or } y \Rightarrow^0 y$$

- if $v \Rightarrow^m w \; [\alpha]$ for some $m \geq 1$, then
$$v \Rightarrow^+ w \; [\alpha] \text{ or } v \Rightarrow^+ w$$

- if $v \Rightarrow^m w \; [\alpha]$ for some $m \geq 0$, then
$$v \Rightarrow^* w \; [\alpha] \text{ or } v \Rightarrow^* w$$

Language of Grammar

Relation of Direct Derivation

$\Rightarrow$ direct derivation relation

$\Rightarrow^*$ transitive and reflexive closure of direct derivation relation

$\Rightarrow^+$ transitive closure of direct derivation relation

Language of Grammar

$$L(G) = \{w \in T^* : S \Rightarrow^* w\}$$

CS, CF, LIN and REG Grammars

CS, CF, LIN and REG Grammars

A type-0 grammar $G = (N, T, P, S)$ is

1. **context-sensitive** if for every $y \rightarrow x \in P$,

$$|y| \leq |x|, \text{ or } y = S, x = \varepsilon$$

2. **context-free** if for every $y \rightarrow x \in P$,

$$y \in N$$

3. **linear** if for every $y \rightarrow x \in P$,

$$y \in N, x \in T^* \cup T^* N T^*$$

4. **regular** if for every $y \rightarrow x \in P$,

$$y \in N, x \in \{\varepsilon\} \cup T \cup TN$$

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