Left Forbidding Grammar Systems

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Contents of presentation

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Introduction

Our (AM, TM, FG) research:

- Itroduce new formalism.
  - Left forbidding grammars
  - Left forbidding grammar systems
- Determine generative power of new formalism.
  - Simulation of some formal system by LFG and LFGS
  - Generative power of simulated formal system is known

State grammars

State grammar is

$$G = (T, N, Q, (S, q_0), P)$$

where

- T is finite set of terminal symbols
- N is finite set of nonterminal symbols
- Q is finite set of states
- (S, q_0) is starting nonterminal and state $(S \in N, q_0 \in Q)$
- P finite set of productions

Configuration:

- Formally configuration is $(N \cup T)^* \times Q$.
- E. g. $(aaAbbBc, q_B) \Rightarrow (aaAbbBcc, q_A)$ using rule $(B, q_B) \rightarrow (Bc, q_A)$.

State grammars II.

Finite set of production is defined

$$P \subseteq N \times Q \times V^* \times Q$$

where $V = N \cup T$.

Form of productions

- $(X, q) \rightarrow (v, r)$, where $X \rightarrow v$ is CF rule and $r, q \in Q$.

Relation $\Rightarrow$ is defined

- $(uXw, q) \Rightarrow (uvw, r)$ if $(X, q) \rightarrow (v, r) \in P$

Language $L(G) = \{w \in T^* | (S, q_0) \Rightarrow^* (w, q)\}$.

State grammars - Example

$$G = (\{S, A, B\}, \{q_A, q_B, q_0, q_f\}, \{a, b, c\}, (S, q_0), P)$$

$$\begin{aligned} P = \{(S, q_0) &\rightarrow (AB, q_A), \\ (A, q_A) &\rightarrow (aAb, q_B), \\ (B, q_B) &\rightarrow (cB, q_A), \\ (A, q_A) &\rightarrow (ab, q_f), \\ (B, q_f) &\rightarrow (c, q_f)\}. \end{aligned}$$

$L(G) = \{a^n b^n c^n | n \geq 1\}$

Generative power of SG: $L(SG) = RE$.

Left-forbidding grammars

Left-forbidding grammar is

$$G = (N, T, S, P)$$

where

- T is finite set of terminal symbols
- N is finite set of nonterminal symbols
- S is starting nonterminal
- P is finite set of productions of the form
  - $P \subseteq N \times (N \cup T)^* \times \mathcal{P}(N)$,
  - More commonly written $p : (X \rightarrow w, M)$ where $X \rightarrow w$ is CF production and $M \subseteq N$ is set of forbidding symbols.

Left-forbidding grammars II.

Operation $\Rightarrow$:

- For production $p : (X \rightarrow v, M)$

$$uXw \Rightarrow uvw[p] \Leftrightarrow alph(u) \cap M = \emptyset$$

Example:

- Configuration $aABA \Rightarrow aaBA[p]$, $p : (A \rightarrow a, \{B\})$.

Language of G:

- $L(G) = \{w \in T^* | S \Rightarrow^* w\}$

Special case:

- If forbidding set M in all rules is N then in every derivation step we rewrite leftmost nonterminal.

Left-forbidding grammar - Example

In production with AM, TM:

- ??Can we generate a^{2^n} or $a^n b^n c^n$ with left-forbidding grammars??
- ??Generative power of Left-Forbidding grammar if $M \subseteq (N \cup N^2)$ (forbidding set)??
- We assume that $L(LFG) = CF$.

CD grammar systems

CD grammar system of degree $n \geq 1$ is $(n+3)$-touple

$$\Gamma = (N, T, S, P_1, \ldots, P_n)$$

where each $P_1, \ldots, P_n$ is finite set of productions.
Notation:

- i-th CF grammar $G_i = (N, T, S, P_i)$

Modes of derivation ($\Rightarrow$):

- $\Rightarrow^t$ – terminating derivation
- $\Rightarrow^{=k}$ – k-step derivation
- $\Rightarrow^{\leq k}$ – at most k-step derivation
- $\Rightarrow^{\geq k}$ – at least k-step derivation

Left Forbidding CD Grammar Systems

Left Forbidding CD grammar system of degree $n \geq 1$ is $(n+3)$-touple

$$\Gamma = (N, T, S, P_1, \ldots, P_n)$$

where each $P_1, \ldots, P_n$ is finite set of left forbidding productions.

Notation:

- i-th Left Forbidding Grammar $G_i = (N, T, S, P_i)$

LFCD Grammar system has same modes of derivation as CD grammar systems.

LFCD Grammar System - Example

$G = (\{S, A, B, \langle 1\rangle, \langle 2\rangle, \langle T\rangle, \langle\times\rangle\}, a, S, P_1, P_2)$, grammar works in t-mode.

$$\begin{array}{rcccl}
P_1 = \{(& S & \rightarrow & \langle 2\rangle A, & \emptyset), \\
(& \langle 1\rangle & \rightarrow & \langle 2\rangle, & \emptyset), \\
(& B & \rightarrow & A, & \{\langle T\rangle, \langle 1\rangle\}), \\
(& B & \rightarrow & a, & \{\langle 1\rangle, \langle 2\rangle\}), \\
(& \langle T\rangle & \rightarrow & \varepsilon, & \emptyset), \\
(& B & \rightarrow & \langle\times\rangle, & \{\langle T\rangle, \langle 1\rangle, \langle 2\rangle\})\} \\
P_2 = \{(& \langle 2\rangle & \rightarrow & \langle 1\rangle, & \emptyset), \\
(& \langle 2\rangle & \rightarrow & \langle T\rangle, & \emptyset), \\
(& A & \rightarrow & BB, & \langle 2\rangle)\}
\end{array}$$

$$L(G) = \{a^n | n \geq 1\}$$

Generative Power of LFCD GS

Let Γ is LFCD Grammar System, we have proven that

$$L(\Gamma) = RE$$

Idea:

- $L(SG) = RE$.
- Simulation of a State Grammar by LFCD Grammar System.
- Components of Grammar System works in t-mode.

Corollary:

- Grammar System has only 2 components.
- Components of Grammar System can work in ≤ 4-mode.

Sketch of Proof

Summary:

- record configuration of State Grammar in each derivation sentence of LFCDGS,
- simulate leftmost derivation of State Grammar.

Forms of nonterminals:

- $[X, p, q, i]$ – First nonterminal of derivation sentence. $(i \in \{1, 2, 3\})$
  - p, q states of State Grammar.
- $\overline{X}$ – Nonterminal representating limit in sentence.
- $\#$ – Nonterminal representating limit in sentence if state grammar contains a production $X \rightarrow \varepsilon$.

Note $X \in (T \cup N)$.

Sketch of Proof II.

Forms of derivation sentence:

$$[X, p, q, i]x_1 \dots x_n$$

$$[X, p, q, i]x_1 \dots \overline{x_j} \dots x_n$$

$$[X, p, q, i]x_1 \dots \# \dots x_n$$

Conclusion

Our research results:

- Generative power of one left-forbidding grammar equivalent to CF grammar.
- Generative power of left-forbidding CD grammar system is equivalent to any grammar – power of turing machine.

References

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