LR Parsing

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Outline

- **LR Parsing Algorithm**

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- **Construction of LR Table**

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- **Handling Errors in LR Parsing**

LR Parsers

- ***L****eft-to-right* scan of tokens
- ***R****ightmost* derivation
- Uses right parse – reverse sequence of rules
- Bottom-up parsing
- Based on *LR tables* constructed from *LR grammars*
    - LR grammar – context-free grammar for which LR table can be built

Advantages

- LR parsers are fast
- Easy way of handling syntax errors
- Ultimately powerful
    - The family of LR languages equals the family of languages accepted by deterministic pushdown automata (DPDA)

Topic

- **LR Parsing Algorithm**
- **Construction of LR Table**
- **Handling Errors in LR Parsing**

LR table

Consider LR grammar $G = (N, T, P, S)$. Then G-based **LR table** consists of:

- G-based action part *action*
- G-based goto part *goto*

- Rows are denoted by the symbols of ${}_G\Theta = \{\theta_1, \dots, \theta_m\}$
  - States of extended pushdown automata (LR parser is EPDA)
- Columns of *action* are denoted by the symbols of T
  - Terminal symbols
- Columns of *goto* are denoted by the symbols of N
  - Nonterminal symbols

Configuration of the parser

$$\triangleright q_0 Y_1 q_1 \dots Y_{m-1} q_{m-1} Y_m q_m \diamond v \triangleleft$$

where $q_i \in {}_G\Theta$, $Y_i \in N \cup T$, $v \in suffixes(w)$, $w \in L(G)$

goto and *action* Part of LR Table

Table: *action* part[1]

	t_1	$\dots$	t_i	$\dots$	t_n
θ_1					
$\vdots$					
θ_j			$action[\theta_j, t_i] \in {}_G\Theta \cup P \cup \{☺, \square\}$		
$\vdots$					
θ_m					

Table: *goto* part[1]

	A_1	$\dots$	A_i	$\dots$	A_k
θ_1					
$\vdots$					
θ_j			$goto[\theta_j, A_i] \in N \cup \{\square\}$		
$\vdots$					
θ_m					

[1] Empty cell represents blank symbol written as $\square$.

LR-REDUCE

If

- $p\colon A \to X_1 X_2 \ldots X_n \in P$
  - for some $n \geq 0$, $X_j \in N \cup T$, $1 \leq j \leq n$
- $o_0 X_1 o_1 X_2 o_2 \ldots o_{n-1} X_n o_n$ is the pushdown top
  - o_n is topmost symbol, $o_k \in {}_G\Theta, 0 \leq k \leq n$

then **LR-REDUCE**(p) replaces $o_0 X_1 o_1 X_2 o_2 \ldots o_{n-1} X_n o_n$ with Ah on the pushdown top

- $h \in {}_G\Theta$ is defined as $h = goto[o_0, A]$, otherwise **REJECT**

LR-SHIFT

- Let $ins_1 = t, t \in N \cup T$ be the head of input tape, pd_1 be the topmost pushdown symbol, and $action[pd_1, t] = o, o \in {}_G\Theta$
- **LR-SHIFT** extends pushdown *pd* by *to* and advances to the next input
  - *to* now occurs at the top of the pushdown (*o* is the topmost) and ins_1 refers to the input symbol occurring right behind t in the input string

Algorithm 1.1: LR Parser

- **Input**: An LR grammar, $G = (N, T, P, S)$,
  an input string w, $w \in T^*$ and
  G-based LR table.
- **Output: ACCEPT** if $w \in L(G)$, or
  REJECT if $w \notin L(G)$.

Method

$pd := \triangleright\theta_1$
repeat
  case action[pd_1, ins_1] **of**
    in $_G\Theta$: **LR-SHIFT**
    in P: **LR-REDUCE**(p) with $p = action[pd_1, ins_1]$
    $\Box$: **REJECT** {$\Box$ denotes blank symbol (undefined action)}
    ☺ : **ACCEPT**
  end case
until ACCEPT or REJECT

LR Table Example

- Consider grammar G with the following rules:

$$1: S \rightarrow S \vee A \quad 2: S \rightarrow A \quad 3: A \rightarrow A \wedge B$$
$$4: A \rightarrow B \quad 5: B \rightarrow (S) \quad 6: B \rightarrow i$$

where S is the start symbol, $T = \{\vee, \wedge, (,), i\}$ and $N = \{A, B\}$

Table: *G*-based LR table example

	$\wedge$	$\vee$	i	(	)	$\triangleleft$	S	A	B
θ_1			θ_6	θ_5			θ_2	θ_3	θ_4
θ_2		θ_7				☺			
θ_3	θ_8	2			2	2			
θ_4	4	4			4	4			
θ_5			θ_6	θ_5			θ_9	θ_3	θ_4
θ_6	6	6			6	6			
θ_7			θ_6	θ_5				θ_{10}	θ_4
θ_8			θ_6	θ_5					θ_{11}
θ_9		θ_7			θ_{12}				
θ_{10}	θ_7	1			1	1			
θ_{11}	3	3			3	3			
θ_{12}	5	5			5	5			
	action part						goto part		

- Consider an expression

$$i \wedge i \in L(G)$$

- We make a parse by Algorithm 1.1
- The sequence of configurations is given in following table

Configuration	Table Entry	Parsing Action
$\triangleright\theta_1\diamond i \wedge i\triangleleft$	$action[\theta_1, i] = \theta_6$	**LR-SHIFT**(i)
$\triangleright\theta_1 i\theta_6\diamond \wedge i\triangleleft$	$action[\theta_6, \wedge] = 6, goto[\theta_1, B] = \theta_4$	**LR-REDUCE**(6)
$\triangleright\theta_1 B\theta_4\diamond \wedge i\triangleleft$	$action[\theta_4, \wedge] = 4, goto[\theta_1, A] = \theta_3$	**LR-REDUCE**(4)
$\triangleright\theta_1 A\theta_3\diamond \wedge i\triangleleft$	$action[\theta_3, \wedge] = \theta_8$	**LR-SHIFT**($\wedge$)
$\triangleright\theta_1 A\theta_3 \wedge \theta_8\diamond i\triangleleft$	$action[\theta_8, i] = \theta_8$	**LR-SHIFT**(i)
$\triangleright\theta_1 A\theta_3 \wedge \theta_8 i\theta_6\diamond\triangleleft$	$action[\theta_6, \triangleleft] = 6, goto[\theta_8, B] = \theta_{11}$	**LR-REDUCE**(6)
$\triangleright\theta_1 A\theta_3 \wedge \theta_8 B\theta_{11}\diamond\triangleleft$	$action[\theta_{11}, \triangleleft] = 3, goto[\theta_1, A] = \theta_3$	**LR-REDUCE**(3)
$\triangleright\theta_1 A\theta_3\diamond\triangleleft$	$action[\theta_3, \triangleleft] = 2, goto[\theta_1, S] = \theta_2$	**LR-REDUCE**(2)
$\triangleright\theta_1 S\theta_2\diamond\triangleleft$	$action[\theta_2, \triangleleft] = ☺$	**ACCEPT**

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Item

$$A \rightarrow x \diamond y$$

for each rule $A \rightarrow z$ and any two strings x and y such that $z = xy$

- x – handle prefix on the *pd* top
- Start item: $A \rightarrow \diamond z$
- End item: $A \rightarrow z \diamond$

Example

- Rule: $S \rightarrow S \vee A$
- Items: $S \rightarrow \diamond S \vee A$, $S \rightarrow S \diamond \vee A$, $S \rightarrow S \vee \diamond A$, $S \rightarrow S \vee A \diamond$

Convention

- ${}_G I$ – set of all items for LR grammar G
- ${}_G I_{start}$ – set of start items, ${}_G I_{start} \subseteq {}_G I$
- ${}_G I_{end}$ – set of end items, ${}_G I_{end} \subseteq {}_G I$
- ${}_G\Omega = 2^{{}_G I}$ – state space

Construction of ${}_G\Theta$ – Idea 1 of 3

1. Change the start symbol S to a new start symbol Z in G, and add a dummy rule $Z \rightarrow S$
   - Every derivation in G now starts by applying $Z \rightarrow S$
2. Initially, set ${}_G\Theta = \emptyset$, ${}_GW = \{\{Z \rightarrow \diamond S\}\}$
   - ${}_GW$ – auxiliary subset of item power set
3. Repeat extensions I and II until no new item set can be included in ${}_GW$

Extension I

- Let $I \in {}_GW$. Suppose that u appears on the *pd* top, and let $A \to uBv \in P$
- Observe: if $A \to u\diamond Bv \in I$ and $B \to \diamond z \in {}_GI_{start}$, then by using $B \to z$, the parser can reduce z to B
  - Does not affect u on the *pd* top because $B \to \diamond z$ is a start item
- Thus, add $B \to \diamond z$ into I
- Repeat until I can no longer be extended in this way
- Add the resulting I to ${}_G\Theta$

repeat
  if $A \to u\diamond Bv \in I$ **and** $B \to z \in P$ **then**
    include $B \to \diamond z$ into I
  end if
until no change
include I into ${}_G\Theta$

Extension II

- Based upon a relation ${}_G\circlearrowleft$ from ${}_G\Omega \times (N \cup T)$ to ${}_G\Omega$:

$$ {}_G\circlearrowleft(I, X) = \{A \to uX\diamond v | A \to u\diamond Xv \in I, A \in N, u, v \in (N \cup T)^*\} $$

- Let $I \in {}_GW$ and $A \to uX\diamond v \in I$
- Consider a part of rightmost derivation in G in reverse order, during which a portion of the input string is reduced to X – simulating this part, the parser obtains X on the pushdown
- Thus, for every $I \in {}_GW$ and $X \in N \cup T$, extend ${}_GW$ by ${}_G\circlearrowleft(I, X)$ unless ${}_G\circlearrowleft(I, X)$ is empty

  for all $X \in N \cup T$ with ${}_G\circlearrowleft(I, X) \neq \emptyset$ **do**
    include ${}_G\circlearrowleft(I, X)$ into ${}_GW$
  end for

Algorithm 2.1: Construction of ${}_G\Theta$

- **Input:** An LR grammar, $G = (N, T, P, S)$, extended by the dummy rule $Z \to S$, where Z is the new start symbol.
- **Output:** ${}_G\Theta$.
- **Note:** An auxiliary set ${}_GW \subseteq {}_G\Omega$ is used.

Method

set ${}_GW = \{\{Z \to \diamond S\}\}$
set ${}_G\Theta = \emptyset$
repeat
  for all $I \in {}_GW$ **do**
    repeat {start of extension I}
      if $A \to u\diamond Bv \in I$ **and** $B \to z \in P$ **then**
        include $B \to \diamond z$ into I
      end if
    until no change
    include I into ${}_G\Theta$
    for all $X \in N \cup T$ with ${}_G\circlearrowleft(I, X) \neq \emptyset$ **do** {start of extension II}
      include ${}_G\circlearrowleft(I, X)$ into ${}_GW$
    end for
  end for
until no change

Example

- Consider ${}_{cond}G$. Add a dummy rule $Z \rightarrow S$ and define Z as the start symbol

$$0: Z \rightarrow S \quad 1: S \rightarrow S \vee A \quad 2: S \rightarrow A \quad 3: A \rightarrow A \wedge B$$
$$4: A \rightarrow B \quad 5: B \rightarrow (S) \quad 6: B \rightarrow i$$

- Apply Algorithm 2.1. First, set ${}_{cond\,G}\Theta = \emptyset$, ${}_GW = \{\{Z \rightarrow \diamond S\}\}$
- By extension I, extend $\{Z \rightarrow \diamond S\} \in {}_GW$ to: $\{Z \rightarrow \diamond S, S \rightarrow \diamond S \vee A, S \rightarrow \diamond A, A \rightarrow \diamond A \wedge B, A \rightarrow \diamond B, B \rightarrow \diamond(S), B \rightarrow \diamond i\}$
- For $I = \{Z \rightarrow \diamond S, S \rightarrow S \vee A\}$, we have ${}_G\circlearrowleft(I, S) = \{Z \rightarrow S\diamond, S \rightarrow S\diamond \vee A\}$
- Thus, by extension II, include $\{Z \rightarrow S\diamond, S \rightarrow S\diamond \vee A\}$ into ${}_GW$
- Perform second iteration of I and II, and so on

Rules

$0: Z \to S$ $1: S \to S \vee A$ $2: S \to A$ $3: A \to A \wedge B$
$4: A \to B$ $5: B \to (S)$ $6: B \to i$

${}_{cond}\,{}_G\Theta$	**Item Sets**
θ_1	$\{Z \to \diamond S,$

Construction of $_G\Theta$ – Example

Rules

$0: Z \rightarrow S$ $1: S \rightarrow S \vee A$ $2: S \rightarrow A$ $3: A \rightarrow A \wedge B$
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θ_1	$\{Z \rightarrow \diamond S, S \rightarrow \diamond S \vee A,$

Rules

$0: Z \to S$ $\quad 1: S \to S \vee A$ $\quad 2: S \to A$ $\quad 3: A \to A \wedge B$

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Construction of $_G\Theta$ – Example

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0: $Z \rightarrow S$ 1: $S \rightarrow S \vee A$ 2: $S \rightarrow A$ 3: $A \rightarrow A \wedge B$
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$0: Z \rightarrow S$ $\quad 1: S \rightarrow S \vee A$ $\quad 2: S \rightarrow A$ $\quad 3: A \rightarrow A \wedge B$

$4: A \rightarrow B$ $\quad 5: B \rightarrow (S)$ $\quad 6: B \rightarrow i$

${}_{cond}\, G^\Theta$	**Item Sets**
θ_1	$\{Z \rightarrow \diamond S, S \rightarrow \diamond S \vee A, S \rightarrow \diamond A, A \rightarrow \diamond A \wedge B, A \rightarrow \diamond B,$ $B \rightarrow \diamond (S),$

Construction of ${}_G\Theta$ – Example

Rules

$0: Z \to S$ $\quad 1: S \to S \vee A$ $\quad 2: S \to A$ $\quad 3: A \to A \wedge B$
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Construction of ${}_G\Theta$ – Example

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$0\colon Z \to S \quad 1\colon S \to S \vee A \quad 2\colon S \to A \quad 3\colon A \to A \wedge B$
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θ_2	$\{Z \to S\diamond,$

Construction of ${}_G\Theta$ – Example

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θ_1	$\{Z \to \diamond S, S \to \diamond S \vee A, S \to \diamond A, A \to \diamond A \wedge B, A \to \diamond B, B \to \diamond(S), B \to \diamond i\}$
θ_2	$\{Z \to S\diamond, S \to S\diamond \vee A\}$

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${}_{cond}\, G^{\Theta}$	**Item Sets**
θ_1	$\{Z \to \diamond S, S \to \diamond S \vee A, S \to \diamond A, A \to \diamond A \wedge B, A \to \diamond B,$ $B \to \diamond(S), B \to \diamond i\}$
θ_2	$\{Z \to S\diamond, S \to S\diamond \vee A\}$
θ_3	$\{S \to A\diamond,$

Rules

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θ_4	$\{A \to B\diamond\}$

Construction of $_G\Theta$ – Example

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θ_4	$\{A \rightarrow B \diamond\}$
θ_5	$\{B \rightarrow (\diamond S),$

Construction of ${}_G\Theta$ – Example

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$_{cond}\, G^{\Theta}$	Item Sets
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θ_4	$\{A \to B\diamond\}$
θ_5	$\{B \to (\diamond S), S \to \diamond S \vee A, S \to \diamond A, A \to \diamond A \wedge B, A \to \diamond B,$ $B \to \diamond(S),$

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θ_4	$\{A \to B\diamond\}$
θ_5	$\{B \to (\diamond S), S \to \diamond S \vee A, S \to \diamond A, A \to \diamond A \wedge B, A \to \diamond B, B \to \diamond(S), B \to \diamond i\}$
θ_6	$\{B \to i\diamond\}$

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θ_3	$\{S \rightarrow A\diamond, A \rightarrow A\diamond \wedge B\}$
θ_4	$\{A \rightarrow B\diamond\}$
θ_5	$\{B \rightarrow (\diamond S), S \rightarrow \diamond S \vee A, S \rightarrow \diamond A, A \rightarrow \diamond A \wedge B, A \rightarrow \diamond B,$ $B \rightarrow \diamond(S), B \rightarrow \diamond i\}$
θ_6	$\{B \rightarrow i\diamond\}$
θ_7	$\{S \rightarrow S \vee \diamond A, A \rightarrow \diamond A \wedge B, A \rightarrow \diamond B, B \rightarrow \diamond(S), B \rightarrow \diamond i\}$

Construction of ${}_{G}\Theta$ – Example

Rules

$0: Z \to S \quad 1: S \to S \vee A \quad 2: S \to A \quad 3: A \to A \wedge B$

$4: A \to B \quad 5: B \to (S) \quad 6: B \to i$

${}_{cond}G^{\Theta}$	**Item Sets**
θ_1	$\{Z \to \diamond S, S \to \diamond S \vee A, S \to \diamond A, A \to \diamond A \wedge B, A \to \diamond B, B \to \diamond(S), B \to \diamond i\}$
θ_2	$\{Z \to S\diamond, S \to S\diamond \vee A\}$
θ_3	$\{S \to A\diamond, A \to A\diamond \wedge B\}$
θ_4	$\{A \to B\diamond\}$
θ_5	$\{B \to (\diamond S), S \to \diamond S \vee A, S \to \diamond A, A \to \diamond A \wedge B, A \to \diamond B, B \to \diamond(S), B \to \diamond i\}$
θ_6	$\{B \to i\diamond\}$
θ_7	$\{S \to S \vee \diamond A, A \to \diamond A \wedge B, A \to \diamond B, B \to \diamond(S), B \to \diamond i\}$
θ_8	$\{A \to A \wedge \diamond B, B \to \diamond(S), B \to \diamond i\}$

Rules

$0: Z \to S$ $\quad 1: S \to S \vee A$ $\quad 2: S \to A$ $\quad 3: A \to A \wedge B$

$4: A \to B$ $\quad 5: B \to (S)$ $\quad 6: B \to i$

$_{cond}\,{}_G\Theta$	**Item Sets**
θ_1	$\{Z \to \diamond S, S \to \diamond S \vee A, S \to \diamond A, A \to \diamond A \wedge B, A \to \diamond B,$ $B \to \diamond(S), B \to \diamond i\}$
θ_2	$\{Z \to S\diamond, S \to S\diamond \vee A\}$
θ_3	$\{S \to A\diamond, A \to A\diamond \wedge B\}$
θ_4	$\{A \to B\diamond\}$
θ_5	$\{B \to (\diamond S), S \to \diamond S \vee A, S \to \diamond A, A \to \diamond A \wedge B, A \to \diamond B,$ $B \to \diamond(S), B \to \diamond i\}$
θ_6	$\{B \to i\diamond\}$
θ_7	$\{S \to S \vee \diamond A, A \to \diamond A \wedge B, A \to \diamond B, B \to \diamond(S), B \to \diamond i\}$
θ_8	$\{A \to A \wedge \diamond B, B \to \diamond(S), B \to \diamond i\}$
θ_9	$\{B \to (S\diamond), S \to S\diamond \vee A\}$

Rules

$0: Z \rightarrow S$ $\quad 1: S \rightarrow S \vee A$ $\quad 2: S \rightarrow A$ $\quad 3: A \rightarrow A \wedge B$

$4: A \rightarrow B$ $\quad 5: B \rightarrow (S)$ $\quad 6: B \rightarrow i$

$_{cond}G\Theta$	Item Sets
θ_1	$\{Z \rightarrow \diamond S, S \rightarrow \diamond S \vee A, S \rightarrow \diamond A, A \rightarrow \diamond A \wedge B, A \rightarrow \diamond B, B \rightarrow \diamond(S), B \rightarrow \diamond i\}$
θ_2	$\{Z \rightarrow S\diamond, S \rightarrow S\diamond \vee A\}$
θ_3	$\{S \rightarrow A\diamond, A \rightarrow A\diamond \wedge B\}$
θ_4	$\{A \rightarrow B\diamond\}$
θ_5	$\{B \rightarrow (\diamond S), S \rightarrow \diamond S \vee A, S \rightarrow \diamond A, A \rightarrow \diamond A \wedge B, A \rightarrow \diamond B, B \rightarrow \diamond(S), B \rightarrow \diamond i\}$
θ_6	$\{B \rightarrow i\diamond\}$
θ_7	$\{S \rightarrow S \vee \diamond A, A \rightarrow \diamond A \wedge B, A \rightarrow \diamond B, B \rightarrow \diamond(S), B \rightarrow \diamond i\}$
θ_8	$\{A \rightarrow A \wedge \diamond B, B \rightarrow \diamond(S), B \rightarrow \diamond i\}$
θ_9	$\{B \rightarrow (S\diamond), S \rightarrow S\diamond \vee A\}$
θ_{10}	$\{S \rightarrow S \vee A\diamond, A \rightarrow A\diamond \wedge B\}$

Rules

$0: Z \to S$ $\quad$ $1: S \to S \vee A$ $\quad$ $2: S \to A$ $\quad$ $3: A \to A \wedge B$

$4: A \to B$ $\quad$ $5: B \to (S)$ $\quad$ $6: B \to i$

${}_{cond}G\Theta$	**Item Sets**
θ_1	$\{Z \to \diamond S, S \to \diamond S \vee A, S \to \diamond A, A \to \diamond A \wedge B, A \to \diamond B, B \to \diamond(S), B \to \diamond i\}$
θ_2	$\{Z \to S\diamond, S \to S\diamond \vee A\}$
θ_3	$\{S \to A\diamond, A \to A\diamond \wedge B\}$
θ_4	$\{A \to B\diamond\}$
θ_5	$\{B \to (\diamond S), S \to \diamond S \vee A, S \to \diamond A, A \to \diamond A \wedge B, A \to \diamond B, B \to \diamond(S), B \to \diamond i\}$
θ_6	$\{B \to i\diamond\}$
θ_7	$\{S \to S \vee \diamond A, A \to \diamond A \wedge B, A \to \diamond B, B \to \diamond(S), B \to \diamond i\}$
θ_8	$\{A \to A \wedge \diamond B, B \to \diamond(S), B \to \diamond i\}$
θ_9	$\{B \to (S\diamond), S \to S\diamond \vee A\}$
θ_{10}	$\{S \to S \vee A\diamond, A \to A\diamond \wedge B\}$
θ_{11}	$\{A \to A \wedge B\diamond\}$

Rules

$0: Z \to S$ $\quad 1: S \to S \vee A$ $\quad 2: S \to A$ $\quad 3: A \to A \wedge B$

$4: A \to B$ $\quad 5: B \to (S)$ $\quad 6: B \to i$

$_{cond}G\Theta$	**Item Sets**
θ_1	$\{Z \to \diamond S, S \to \diamond S \vee A, S \to \diamond A, A \to \diamond A \wedge B, A \to \diamond B, B \to \diamond(S), B \to \diamond i\}$
θ_2	$\{Z \to S\diamond, S \to S\diamond \vee A\}$
θ_3	$\{S \to A\diamond, A \to A\diamond \wedge B\}$
θ_4	$\{A \to B\diamond\}$
θ_5	$\{B \to (\diamond S), S \to \diamond S \vee A, S \to \diamond A, A \to \diamond A \wedge B, A \to \diamond B, B \to \diamond(S), B \to \diamond i\}$
θ_6	$\{B \to i\diamond\}$
θ_7	$\{S \to S \vee \diamond A, A \to \diamond A \wedge B, A \to \diamond B, B \to \diamond(S), B \to \diamond i\}$
θ_8	$\{A \to A \wedge \diamond B, B \to \diamond(S), B \to \diamond i\}$
θ_9	$\{B \to (S\diamond), S \to S\diamond \vee A\}$
θ_{10}	$\{S \to S \vee A\diamond, A \to A\diamond \wedge B\}$
θ_{11}	$\{A \to A \wedge B\diamond\}$
θ_{12}	$\{B \to (S)\diamond\}$

Construction of LR Table

I. *goto* part

- Consider item $A \rightarrow u\diamond Bv \in \theta$, where $\theta \in {}_G\Theta$, $A, B \in N$ and $u, v \in (N \cup T)^*$
- After reducing portion of the input string to B, parser extends the prefix u by B, so uB occurs on the *pd* top

if ${}_G\circlearrowleft(\theta_i, B) = \theta_j - {}_G I_{start}$, where $B \in N$ **then**
  $goto[\theta_i, B] = \theta_j$
end if

II. *action* part – shift

- By analogy with I

if ${}_G\circlearrowleft(\theta_i, b) = \theta_j - {}_G I_{start}$, where $b \in T$ **then**
  $action[\theta_i, b] = \theta_j$
end if

III. *action* part – reduction

- Consider a rule $p\colon A \to u \in P$ and $A \to u\diamond \in {}_G I_{end}$
  - A complete handle *u* on *pd* top
- Parser reduces *u* to *A* provided that after the reduction, *A* is followed by terminal *a* that may legally occur after *A* in a sentential form

  if $A \to u\diamond \in \theta_i, a \in follow(A), p\colon A \to u \in P$ **then**
  $action[\theta_i, a] = p$
  end if
- Note that:
  - Every derivation starts with $0\colon Z \to S$
  - LR parser simulates rightmost derivations in reverse
  - Input symbol $\triangleleft$ – all the input has been read
- Thus, if $Z \to S\diamond \in \theta_i$, set $action[\theta_i, \triangleleft] = \smiley$ (parsing completed successfully)

  if $Z \to S\diamond \in \theta_i$ **then**
  $action[\theta_i, \triangleleft] = \smiley$
  end if

Algorithm 2.2: LR Table (1/2)

- **Input:** An LR grammar $G = (N, T, P, S)$, in which Z and $0 : Z \rightarrow S$ have the same meaning as in Algoritm 2.1, and ${}_G\Theta$ constructed by Algorithm 2.1.
- **Output:** A G-based LR table, consisting of the *action* and *goto* parts.
- **Note:** We suppose that $A, B \in N$, $b \in T$ and $u, v \in (N \cup T)^*$ in this algorithm.

Method

denote the rows of *action* and *goto* with the members of ${}_G\Theta$
denote the columns of *action* and *goto* with the members of T and N, respectively

{continued on next slide}

Method (cont.)

repeat
  for all $\theta_i, \theta_j \in {}_G\Theta$ **do**
    if ${}_G\circlearrowleft(\theta_i, B) = \theta_j - {}_G I_{start}$, where $B \in N$ **then**
      $goto[\theta_i, B] = \theta_j$
    end if
    if ${}_G\circlearrowleft(\theta_i, b) = \theta_j - {}_G I_{start}$, where $b \in T$ **then**
      $action[\theta_i, b] = \theta_j$
    end if
    if $A \to u\diamond \in \theta_i \cap {}_G I_{end}, a \in follow(A), i : A \to u \in P$ **then**
      $action[\theta_i, a] = i$
    end if
  end for
until no change
if $Z \to S\diamond \in \theta_i$ **then**
  $action[\theta_i, \lhd] = ☺$ {success}
  {all the other entries remain blank and, thereby, signalize a syntax error}
end if

Example

- Consider again $_{cond}G$

$$0: Z \to S \quad 1: S \to S \vee A \quad 2: S \to A \quad 3: A \to A \wedge B$$
$$4: A \to B \quad 5: B \to (S) \quad 6: B \to i$$

- Consider $_{cond}G\Theta = \{\theta_1, \theta_2, \ldots, \theta_{12}\}$ (obtained in previous example)
- According to the first **if** statement in Algorithm 2.2, $goto[\theta_1, S] = \theta_2$ because $S \to \diamond S \vee A \in \theta_1$ and $S \to S\diamond \vee A \in \theta_2$
- Second **if** statement: $action[\theta_2, \vee] = \theta_7$ because $S \to S\diamond \vee A \in \theta_2$ and $S \to S \vee \diamond A \in \theta_7$
- Third **if** statement: $action[\theta_{10}, \vee] = 2$ because $2: S \to A\diamond \in \theta_{10}$ and $\vee \in follow(A)$
- Repeat until there is no change
- Set $action[\theta_2, \lhd] = ☺$ because θ_2 contains $Z \to S\diamond$

Construction of LR Table – Example

Table: *G*-based LR table example

	$\wedge$	$\vee$	i	(	)	$\lhd$	S	A	B
θ_1			θ_6	θ_5			θ_2	θ_3	θ_4
θ_2		θ_7				☺			
θ_3	θ_8	2			2	2			
θ_4	4	4			4	4			
θ_5			θ_6	θ_5			θ_9	θ_3	θ_4
θ_6	6	6			6	6			
θ_7			θ_6	θ_5				θ_{10}	θ_4
θ_8			θ_6	θ_5					θ_{11}
θ_9		θ_7			θ_{12}				
θ_{10}	θ_7	1			1	1			
θ_{11}	3	3			3	3			
θ_{12}	5	5			5	5			
	action part						goto part		

Topic

- LR Parsing Algorithm
- Construction of LR Table
- **Handling Errors in LR Parsing**

LR Parsing: Handling Errors

Error detection

No valid continuation for the portion of the input thus far scanned

- More exact than in precedence parsing
- Detection of all possible errors by using *action part*
    - We can reduce the size of *goto part* by removing unneeded blank entries

LR error recovery methods

- Panic-mode LR Error Recovery
- Ad-hoc Recovery

Panic-mode LR Error Recovery

Method

- Try to isolate **the shortest** possible erroneous substring,
- skip this substring, and
- resume parsing process

- Basic idea of this method: we have selected set of nonterminals ${}_G O$ representing major pieces of program such as expressions or statements
- Find the shortest string uv, where:
  - $u \in (N \cup T)^*$ is obtained from the current pushdown top $x \in ((N \cup T)_G\Theta)^*$ by deletion of all symbols from ${}_G\Theta$
  - v is the shortest input prefix followed by input symbol a from $follow(A)$, where $A \in O$ and $A_{rm} \Rightarrow^* uv$
- Let x be preceded by $o \in {}_G\Theta$ and $goto[o, A] = \theta$
- To recover, this method replaces x with $A\theta$ on the pushdown and skips the input prefix v
- After this it resumes the parsing process from $action[\theta, a]$

Ad-hoc Recovery

- Resembles the way the precedence parser handles the table-detected errors
- This method considers **each** blank *action* entry, which signalize error
- We decide the most probable mistake that led to particular error and according to this we design recovery procedure
- Typical recovery routines: modify the pushdown or input by *changing, inserting* or *deleting* some symbols
- Modification **has to** avoid infinite loops
- Each blank entry is filled with the reference to the corresponding recovery routine

Ad-hoc Recovery Example

- Consider again the grammar G:

$$1: S \to S \vee A \quad 2: S \to A \quad 3: A \to A \wedge B$$
$$4: A \to B \quad 5: B \to (S) \quad 6: B \to i$$

where S is the start symbol, $T = \{\vee, \wedge, (,), i\}$ and $N = \{S, A, B\}$

- As an expression we take

$$i\vee)$$

- The parsing process for this input is interrupted after six steps $\Rightarrow$ **RECOVERY**
- We update the *action* part of table by filling the blank entries by recovery routines, the *goto* part of LR table stays the same
- The construction of recovery procedures needs sophisticated approach

Ad-hoc Recovery Example

Table: Example of *action* part of *G*-based LR table with ad-hoc recovery

	$\wedge$	$\vee$	i	(	)	$\triangleleft$
θ_1	①	①	θ_6	θ_5	②	①
θ_2	①	θ_7	③	③	②	☺
θ_3	θ_8	2	③	③	2	2
θ_4	4	4	③	③	4	4
θ_5	①	①	θ_6	θ_5	②	①
θ_6	6	6	③	③	6	6
θ_7	①	①	θ_6	θ_5	②	①
θ_8	①	①	θ_6	θ_5	②	①
θ_9	①	θ_7	③	③	θ_{12}	①
θ_{10}	θ_7	1	③	③	1	1
θ_{11}	3	3	③	③	3	3
θ_{12}	5	5	③	③	5	5

Ad-hoc Recovery Example

- The description of recovery procedures ① through ④
- Consider string $i \vee ($ as an input

① **diagnostic**: missing i or (, **recovery**: insert $i\theta_6$ onto the pushdown
② **diagnostic**: unbalanced), **recovery**: delete the input)
③ **diagnostic**: missing operator, **recovery**: insert $\vee\theta_5$ onto the pushdown
④ **diagnostic**: missing), **recovery**: insert $)\theta_6$ onto the pushdown

Then we can make LR parse. After the input is finaly accepted there are saved error reports with the information about used recovery processes.

Thank you for your attention!

End