Automatic Polynomial Transformation of Differential Equations and Derivation Closure

Václav Vopěnka

October 19, 2011

Abstract

In this paper we will focus on derivations of one variable mathematical functions in form $\mathbb{R} \to \mathbb{R}$. Retrieving derivations of mathematical functions and polynomials composed of functions is essential for generating Taylor polynomial approximation of differential equations, particularly initial problems. If the derivation rules of each function are included into the set of differential equations using Moore's formulas [2] and such functions are substituted by new variables, high speed-up in evaluation of coefficients of Taylor series can be achieved. Evaluation of high order derivations can be broken into evaluation of multidimensional Pascal triangles and permutations on transformation table.

In this paper we will study, how mathematical derivations can be described using grammars from programming language theory and how this grammars can be used in numerical mathematics.

We will also define a derivation closure of a function, which is a set of functions generated by an infinite amount of derivations applied to a given function. We will present an algorithm for automatic generation of this set and we will show how it can be used to transform differential equations in general form into set of differential equations with polynomials on right hand side.

Finally we will present equations for retrieving n-th derivation of expressions $y_1 y_2 \cdots y_n$, y^n and y^{-n}. We will show how these equations can be derived from multidimensional Pascal triangles published in [1]. We will also show how can permutations be used to quickly retrieve high order derivations of simple differential equations.

References

[1] Edward A. Eaton. A multidimensional extension of pascal's triangle. *SIGSMALL/PC Notes*, 16:34–37, May 1990.

[2] R. E. Moore, R. B. Kearfott, and M. J. Cloud. *Introduction to Interval Analysis*. SIAM, 2009.