On Complexity of Offline Partial Dynamic Reconfiguration Scheduling

Jiří Matoušek

Brno University of Technology, Faculty of Information Technology
Božetěchova 2, 612 00 Brno, CZ
www.fit.vutbr.cz/∼imatousek

BRNO UNIVERSITY OF TECHNOLOGY
FIT
FACULTY OF INFORMATION TECHNOLOGY

Outline

Motivation

- topic of my Ph.D. thesis: The Use of Reconfigurable Architectures in Computer Networks

Partial Dynamic Reconfiguration (PDR)

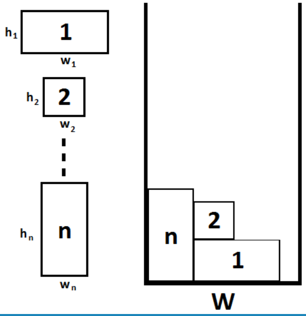
Process of changing configuration of some part of FPGA, while the rest of FPGA is untouched and still working.

- tasks can be time-multiplexed in order to share the same resources
  - we must schedule utilization of resources by tasks
- scheduling of tasks can be studied in terms of the **strip packing problem**

Definition [Lodi et al.]

- a set of *n* rectangular items, each having *width* w_i and *height* h_i, where $i \in 1, \ldots, n$
- a single container (called *strip*) having *width W* and unlimited *height*
- the task is to pack all items into the strip in such a way that they do not overlap and the height of used strip is minimal

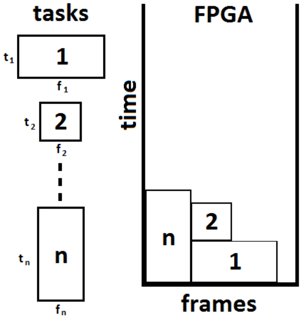
- each item has fixed orientation (i.e. items cannot be rotated)
- we consider an offline version (all parameters are known at the time of packing)
- the 2SP is **NP-hard** problem (often stated in the literature without proof)



Frame

- the smallest addressable part of FPGA configuration memory
- 1b column over the full height of FPGA
- frame´s properties reduce the problem dimension by one (2-D instead of 3-D strip packing problem)

- each **item** models one task
  - *width* – the number of used frames
  - *height* – processing time of the task)
- **strip** models available resources
  - *width* – the number of frames within FPGA
  - *height* – run time of the whole system



Definition of Terms

Decision Problem [Češka 2009]

- a decision problem P can be understood as a function f_P with the range $\{true, false\}$
- a decision problem is usually specified by
  - the set I_P of possible instances of the problem P
  - the subset $A_P \subseteq I_P$, $A_P = \{p \in I_P | f_P(p) = true\}$ of instances for which f_P evaluates to *true*

Optimization Problem [Černá 1998]

- an optimization problem P is specified by
  - the set I_P of possible instances of the problem P
  - the function F assigning each instance $x \in I_P$ a set of feasible solutions
  - the function v assigning each $r \in F(x)$ its value $v(r) \in \mathbb{Q}^+$
- we look for feasible solution with either **maximal** or **minimal** value

- each optimization problem has an associated decision problem

Definition of Auxiliary Problems

h-bin packing problem [Černá 1998]

- the h-bin packing problem is specified by
  - a set of n items, each having *weight* w_i, where $i \in 1, \ldots, n$
  - a set of k bins, each having *weight limit* W
- the question is whether it is possible to divide all items between bins in such a way, that no weight limit is violated
- h-bin packing problem is **NP-complete**

1-dimensional bin packing problem (1BP) [Lodi et al.]

- the 1BP is specified by
  - a set of n items, each having *weight* w_i, where $i \in 1, \ldots, n$
  - an infinite number of bins, each having *weight limit* W
- the task is to divide items into bins in such a way, that no weight limit is violated and the number of used bins is minimal

- the h-bin packing problem is a decision problem associated with the 1-dimensional bin packing problem

Lemma 1

- an optimization problem is NP-hard iff its associated decision problem is NP-complete
- for details see [Černá 1998], page 85

Proof of 2SP NP-hardness

- the h-bin packing problem is NP-complete and it is a decision problem associated with 1BP
- from previous and Lemma 1 we get NP-hardness of 1BP
- 1BP is a special case of 2SP
  - width of items are represented by their weights
  - height of items are set to $h_i = 1$, where $i \in 1, \dots, n$
  - width of the strip is represented by weight limit of bins
- if 1BP (special case) is NP-hard, then 2BP (general case) must also be NP-hard

□

Unresolved Issues

Organization of configuration memory in modern FPGAs

- the smallest addressable area is still frame, but it does not span the whole FPGA height anymore
- we should consider the 3-dimensional strip packing problem

Heterogenous FPGA structure

- except logic blocks, there are blocks of on-chip memory, multipliers, etc.
- heterogenous FPGA structure imposes additional restrictions on tasks' scheduling

Communicating and timing issues

- tasks must be placed at such positions in FPGA's structure, that they meet timing constraints and are able to communicate with the rest of the system
- in terms of formal model, this means introducing further restrictions

Conclusion

- scheduling the reconfiguration of tasks implemented on FPGAs can be studied in terms of the 2-dimensional strip packing problem
  - we have studied an offline version of 2SP, but sometimes an online verison could be the case
- it has been proven that 2-dimensional strip packing problem is NP-hard
  - within the proof we have used the h-bin packing problem and the 1BP
- limitations of presented formal model regarding to modern FPGAs have been discussed
  - it would be necessary to use 3-dimensional strip packing problem and impose far more restrictions on the model

Bibliography

- [Lodi et al.] A. Lodi, S. Martello and D. Vigo: Models and Bounds for Two-Dimensional Level Packing Problems, Journal of Combinatorial Optimization, Vol. 8, pp. 363-379
- [Češka 2009] M. Češka and T. Vojnar: Theoretical Computer Science TCS: Textbook, FIT BUT, January 2009
- [Černá 1998] I. Černá: Úvod do teórie zložitosti, FI MU, February 1998