Parallel Rewriting Over Word Monoids

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Overview

- **Motivation**
  - Regulation of the grammatical parallelism
- **Introduction**
  - Classification
- **E0L Systems**
  - EP0L Systems
  - WME0L grammar
  - Generative Power

Motivation

- regulation of the grammatical parallelism
- grammar alphabet always strictly taken as finite set of letters
- parallel rewriting **over word** monoids easily **increase generative power**

word = context

Introduction

- parallelism is represented with **E0L** grammar systems
- **WME(P)0L** grammars - E0L grammars over word monoids
- SE(P)0L as **WME(P)0L(2)** - WME(P)0L grammars with words of length 2 - **symbiotic E0L grammars**

EOL System

$$G = (V, T, P, S)$$

- V is the total alphabet,
- T is a finite set of terminals, $T \subseteq V$,
- P is a finite set of productions in the form

  $$a \longrightarrow w$$

  with $a \in V$, and $w \in V^*$,
- S is the axiom, $S \in V$.

EP0L Systems

EP0L System

$$G = (V, T, P, S)$$

- V is the total alphabet,
- T is a finite set of terminals, $T \subseteq V$,
- P is a finite set of productions in the form

  $$a \longrightarrow w$$

  with $a \in V$, and $w \in V^+$,
- S is the axiom, $S \in V$.

Derivation Step

Let $u = a_1 a_2 \ldots a_n$, and $v = w_1 w_2 \ldots w_n$. If there exists a production rule $a_i \longrightarrow w_i \in P$ for all $1 \leq i \leq n$, then

$$u \Longrightarrow v.$$

Generated Language

$$L(G) = \{w \in T^* \mid S \Longrightarrow^* w\}$$

Generative Power

$$\mathcal{L}(REG) \subset \mathcal{L}(CF) \subset \mathcal{L}(E0L) = \mathcal{L}(EP0L) \subset \mathcal{L}(CS) \subset \mathcal{L}(RE)$$

WMEOL grammar

EOL grammar on word monoid - WMEOL grammar

$WMEOL(i)$ grammar is a pair

$$(G, W)$$

where

$$G = (V, T, P, S)$$

- G is an EOL grammar,
- W is the set of generators, finite language over V
- i is a grammar degree, if every $y \in W$ satisfies $|y| \leq i$.

Derivation Step

Let $x, y \in W^*$:

- $x = a_1 a_2 \cdots a_n$, $y = y_1 y_2 \cdots y_n$
- $a_i \in V$, $y_i \in V^*$
- $1 \leq i \leq n$, and $n \geq 0$

If $a_i y_i \in P$ for all $i = 1 \ldots n$, then x directly derives y according to rules $a_1 \to y_1$, $a_2 \to y_2, \ldots, a_n \to y_n$, symbolically written as

$$x \Rightarrow_{(G,W)} y[a_1 \to y_1, \ldots, a_n \to y_n]$$

Generated Language

$$L(G, W) = \left\{ w \in T^* | S \Rightarrow^*_{(G,W)} w \right\}$$

Generative Power

$$\mathcal{L}(REG) \subset \mathcal{L}(CF) \subset \mathcal{L}(E0L) = \mathcal{L}(EP0L) \subset \mathcal{L}(WME0L) = \mathcal{L}(WMEP0L) \subset \mathcal{L}(CS) = \mathcal{L}(WMEP0L(2)) \subset \mathcal{L}(RE) = \mathcal{L}(WME0L(2))$$

Generative Power

$$\mathbf{CF}$$
$$\subset$$
$$\mathbf{WMEPOL}(1) = \mathbf{WMEOL}(1) = \mathbf{EPOL} = \mathbf{EOL}$$
$$\subset$$
$$\mathbf{WMEPOL}(2) = \mathbf{CS}$$
$$\subset$$
$$\mathbf{WMEOL}(2) = \mathbf{RE}$$

$$\mathbf{WMEPOL}(0) = \mathbf{WMEOL}(0) = \emptyset$$

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Thank You For Your Attention !