Regulated Insertion-Deletion Systems

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Outline of my previous talk

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2. YES !! but with certain regulations on the contexts of application like
3. Semi-Conditional grammars
4. Simple Semi-Conditional grammars
5. Generalised Forbidding grammars
6. Matrix grammars (we did not discuss this)
7. Graph-Controlled grammars (we did not discuss this)

Insertion-Deletion Systems

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A counterpart of Rewriting Systems

Theoretical meaning of ins-del

- Insertion (Deletion) means appending (removing) a (sub)string to (from) a given string with specific contexts.
- This is not Rewriting and motivation comes from DNA.
- If a string α is inserted between two parts w_1 and w_2 of a string $w_1 w_2$ to get $w_1 \alpha w_2$, the operation is *insertion*.
- Notation: $\boxed{(w_1, \alpha, w_2)_{ins}}$: means $(w_1 w_2 \Longrightarrow w_1 \alpha w_2)$

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- If a substring β is deleted from a string $w_1\beta w_2$ to get w_1w_2, the operation is *deletion*.
- Notation: $\boxed{(w_1, \beta, w_2)_{del}}$: means $(w_1\beta w_2 \Longrightarrow w_1w_2\ )$
- Suffixes of w_1 and prefixes of w_2 are called the left and right context of α or β.
- Starting with axioms and iterating the ins-del operations, we get a set of terminal strings (language of ins-del system).

Definition

An insertion-deletion system is a construct $G = (V, T, A, R)$

- V is an alphabet, $T \subseteq V$, $A \subseteq V^*$
- R is a finite set of n rules of the form $\boxed{(u_i, \alpha_i, v_i)_t}$
  $t \in \{ins, del\}, 1 \leq i \leq n, \ \ u_i, v_i \in V^*, \ \alpha_i \in V^+.$

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Size of an Ins-Del (ID) system

Notation: $(\boxed{n, i', i''}; \boxed{m, j', j''})$ where

1. n = the maximal length of the insertion string
2. i' = maximal length of left contexts used in insertion rules
3. i'' = maximal length of right contexts used in insertion rules
4. m, j', j'' denote similar maximal lengths among deletion rules.

Ins-del systems for $\{a^n b^n \mid n \geq 1\}$

$G_1 =$
$(\{a, b\}, \{a, b\}, \{ab\}, R)$

- $r_1 : (a, ab, b)_{ins}$

Size $= (2, 1, 1; 0, 0, 0)$.

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$G_2 =$
$(\{a, X, b\}, \{a, b\}, \{ab\}, R)$

- $r_1 : (a, X, b)_{ins}$
- $r_2 : (X, ab, b)_{ins}$
- $r_3 : (\lambda, X, \lambda)_{del}$

Size $= (2, 1, 1; 1, 0, 0)$.

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$G_3 = (\{a, C, b\}, \{a, b\}, \{ab\}, R)$

- $r_1 : (a, aC, b)_{ins}$
- $r_2 : (a, b, C)_{ins}$
- $r_3 : (b, C, b)_{del}$

Size $= (2, 1, 1; 1, 1, 1)$.

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- $r_2 : (a, b, C)_{ins}$
- $r_3 : (b, C, b)_{del}$

Size $= (2, 1, 1; 1, 1, 1)$.

$G_4 = (\{a, \$, Y, b\}, \{a, b\}, \{ab\}, R)$

- $r_1 : (a, aY, b)_{ins}$
- $r_2 : (a, b\$, Y)_{ins}$
- $r_3 : (b, \$Y, b)_{del}$

Size $= (2, 1, 1; 2, 1, 1)$.

Ins-del systems for $\{a^n b^n \mid n \geq 1\}$

$G_1 = (\{a, b\}, \{a, b\}, \{ab\}, R)$

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- $r_3 : (\lambda, X, \lambda)_{del}$

Size $= (2, 1, 1; 1, 0, 0)$.

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Size $= (2, 1, 1; 1, 1, 1)$.

$G_4 = (\{a, \$, Y, b\}, \{a, b\}, \{ab\}, R)$

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- $r_2 : (a, b\$, Y)_{ins}$
- $r_3 : (b, \$Y, b)_{del}$

Size $= (2, 1, 1; 2, 1, 1)$.

$\{a^n b^n\} \in ID(2, 1, 1; 0, 0, 0)$.

Trivial yet important result

- If $L \in ID(s_1, s_2, s_3; s_4, s_5, s_6)$, then $L \in ID(t_1, t_2, t_3; t_4, t_5, t_6)$ for every $t_i \geq s_i$. Objective: Minimize the s_i's.
- If $L \in ID(s_1, s_2, s_3; s_4, s_5, s_6)$, then $L^r \in ID(s_1, s_3, s_2; s_4, s_6, s_5)$.
- If $\mathcal{L}$ is a language class that is closed under reversal and $\mathcal{L} = ID(s_1, s_2, s_3; s_4, s_5, s_6)$, then $\mathcal{L} = ID(s_1, s_3, s_2; s_4, s_6, s_5)$.
- Implication: If $RE = ID(1, 1, 0; 1, 0, 1)$ implies $RE = ID(1, 0, 1; 1, 1, 0)$.

With what sizes does an ID system (not known to) **characterize RE** ?

- $\boxed{(1, 1, 1; 1, 1, 1)}$
- $(1, 1, 1; 2, 0, 0)$
- $(2, 0, 0; 1, 1, 1)$
- $(2, 0, 0; 3, 0, 0)$
- $(3, 0, 0; 2, 0, 0)$

Classic Result 2017

For $i' + i'', j' + j'' \neq 0$,
ID$(2, i', i''; 2, j', j'')$=RE

$\boxed{\text{ID}(2, 0, 0; 2, 0, 0) \neq \mathrm{RE}}$

- $\boxed{(1, 1, 0; 1, 1, 1)}$
- $\boxed{(1, 1, 1; 1, 1, 0)}$
- $\boxed{(1, 1, 0; 1, 1, 0)}$
- $\boxed{(1, 1, 1; 1, 0, 0)}$
- $\boxed{(1, 0, 0; 1, 1, 1)}$
- $(1, 1, 0; 2, 0, 0)$
- $(2, 0, 0; 1, 1, 0)$
- and so on...

Variants of ins-del system

- Ins-del P systems by Krishna and Rama (2001)
- Tissue P systems with ins-del rules by Lakshmanan and Rama (2003)
- Graph-controlled ins-del systems by R Freund *et al* (2010).
- Matrix ins-del systems by Lakshmanan and Anand Mahendran (2011) and independently by I Petre and S Verlan (2012)
- Semi-conditional and Random Context ins-del systems by S Ivanov and S Verlan (2011)
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Common objective

To characterize recursively enumerable languages using any of the above regulated system with as minimal size/resource as possible.
To do so, we use Special Geffert Normal Form of type-0 grammars.

Special Geffert Normal Form (SGNF)

Definition

A type-0 grammar $G = (N, T, P, S)$ is in SGNF if

- N is partitioned into $N = N_1 \cup N_2$, where $N_2 = \{A, B, C, D\}$ and N_1 contains at least the two non-terminals S and S',
- The rules in P are of the form :

$\boxed{p : X \rightarrow bY,\ q : X \rightarrow Yb,\ h : S' \rightarrow \lambda,\ f : AB \rightarrow \lambda,\ g : CD \rightarrow \lambda.}$ where $X, Y \in N_1$, $X \neq Y$, $b \in T \cup N_2$ and p,q,h,f,g are labels.

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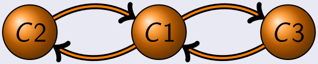
- In Phase I, the (linear-like) CF rules are applied and completed by applying $S' \to \lambda$.
- Adv. At any instant of string in the sentential form, there is only ONE variable from N_1 (No confusion of twins!).
- In Phase II, only $AB \to \lambda,\ CD \to \lambda$ rules are applied.

Graph-Controlled Insertion-Deletion (GCID)

Definition

- A GCID system is $\Pi = (k, V, T, A, H, i_0, i_f, R)$
- k is the number of components
- V is an alphabet, $T \subseteq V$, A is an axiom set, H is a label set.
- i_0 is the initial component and i_f is the final component.
- A rule in R is of the form $\ell : (i, (w_1, \alpha, w_2)_t, j)$, $t \in \{I, D\}$.
  - $\ell \in H$ is a label for the ins-del rule,
  - i: current component, j: target component

Starting with $\#\$$ we generate $\{ww \mid w \in \{a, b\}^*\} \notin CF$

$r_{11} : (1, (\#, a, \lambda)_{ins}, 2)$	$r_{21} : (2, (\$, a, \lambda)_{ins}, 1)$	Size is $(3; 1, 1, 0; 1, 0, 0)$
$r_{12} : (1, (\#, b, \lambda)_{ins}, 3)$	$r_{22} : (2, (\lambda, \#, \lambda)_{del}, 1)$	
$r_{13} : (1, (\lambda, \$, \lambda)_{del}, 2)$	$r_{31} : (3, (\$, b, \lambda)_{ins}, 1)$	

Size of GCID

The size of a GCID system is given by $(k; n, i', i''; m, j', j'')$ where

- $\boxed{k \text{ : Number of Components } (k \geq 1)}$
- n : Maximal length of the insertion string
- i' : Maximal length of the left context used in insertion rules
- i'' : Maximal length of the right context used in insertion rules
- m : Maximal length of the deletion string
- j' : Maximal length of the left context used in deletion rules
- j'' : Maximal length of the right context used in deletion rules

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- m : Maximal length of the deletion string
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Objective

1. With what size does a GCID system (with $n + m \in \{2, 3\}$) characterize RE?
2. Is the underlying control graph, a path?

Computational completeness of *GCID* for $n = 1$, $m = 1$

No.	Size of the system $(k; 1, i', i''; 1, j', j'')$	No.of Comps	Control graph type
1.	$(k; 1, 0, 0; 1, 1, 1)$ or $(k; 1, 1, 1; 1, 0, 0)$	5	*path*
2.	$(k; 1, 1, 0; 1, 1, 0)$ or $(k; 1, 0, 1; 1, 0, 1)$	4 3 4	*Non − tree* *Non − tree* *path*
3.	$(k; 1, 1, 0; 1, 0, 1)$ or $(k; 1, 0, 1; 1, 1, 0)$	4 3 4	*Non − tree* *Non − tree* *path*
4.	$(k; 1, 1, 0; 1, 1, 1)$ or $(k; 1, 0, 1; 1, 1, 1)$	3	*path*
5.	$(k; 1, 1, 1; 1, 1, 0)$ or $(k; 1, 1, 1; 1, 0, 1)$	3	*path*
6.	$(k; 1, 1, 1; 1, 1, 1)$	1	Null

5

Computational completeness of $GCID$ for $n + m = 3$

No.	Size $(k; 1, i', i''; 2, j', j'')$	No. of Comps	Graph type
1.	$(k; 1, 0, 0; 2, 1, 1)$ or $(k; 2, 1, 1; 1, 0, 0)$	5	*path*
2.	$(k; 1, 1, 0; 2, 0, 0)$ or $(k; 1, 0, 1; 2, 0, 0)$ or $(k; 1, 1, 0; 2, 1, 0)$ or $(k; 1, 0, 1; 2, 0, 1)$ or $(k; 1, 1, 0; 2, 0, 1)$ or $(k; 1, 0, 1; 2, 1, 0)$	3 4	$Non - tree$ path
3.	$(k; 2, 0, 0; 1, 1, 0)$ or $(k; 2, 0, 0; 1, 0, 1)$	3 3	$Non - tree$ *path*
4.	$(k; 2, 1, 0; 1, 1, 0)$ or $(k; 2, 0, 1; 1, 0, 1)$ or $(k; 2, 1, 0; 1, 0, 1)$ or $(k; 2, 0, 1; 1, 1, 0)$ or $(k; 2, 1, 1; 1, 1, 0)$ or $(k; 2, 1, 1; 1, 0, 1)$ or $(k; 1, 1, 0; 2, 1, 1)$ or $(k; 1, 0, 1; 2, 1, 1)$	3	*path*
5.	$(k; 1, 1, 1; 2, 0, 0)$ or $(k; 1, 1, 1; 2, 1, 0)$ or $(k; 1, 1, 1; 2, 0, 1)$ or $(k; 1, 1, 1; 2, 1, 1)$ or $(k; 2, 0, 0; 1, 1, 1)$ or $(k; 2, 1, 0; 1, 1, 1)$ or $(k; 2, 0, 1; 1, 1, 1)$ or $(k; 2, 1, 1; 1, 1, 1)$	1	Null

RE $= \mathrm{GCID}_P(3;1,1,0;1,1,1)$ Axiom $= \kappa S \kappa'$

We simulate $r : X \to Y_1 Y_2$, $f : AB \to \lambda \mid CD \to \lambda$, $h : S' \to \lambda$ as:

Lesson learnt

- More contexts does not imply simple simulation

Component 1

$r1.1 : (1, (X, r, \lambda)_I, 2)$
$r1.2 : (1, (r, \Delta, \lambda)_I, 1)$
$r1.3 : (1, (r, Y_2, \lambda)_I, 2)$
$f1.1 : (1, (\lambda, f, \lambda)_I, 2)$
$h1.1 : (1, (\lambda, S', \lambda)_D, 1)$
$\kappa 1.1 : (1, (\lambda, \kappa, \lambda)_D, 1)$
$\kappa' 1.1 : (1, (\lambda, \kappa', \lambda)_D, 1)$

Component 2

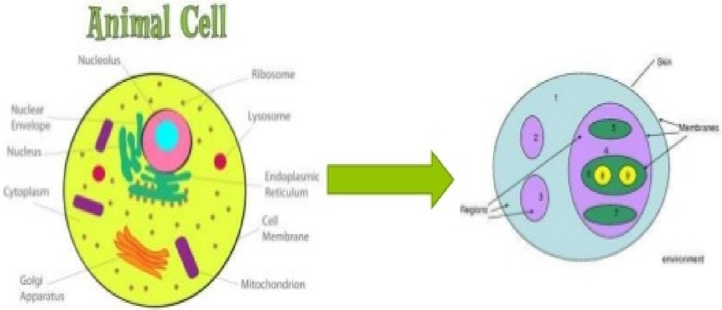
$r2.1 : (2, (\lambda, X, r)_D, 1)$
$r2.2c : (2, (Y_2, \Delta, c)_D, 3), c \neq \Delta$
$r2.3c' : (2, (c', r, Y_1)_D, 1)$
$f2.1 : (2, (f, A, B)_D, 3)$
$f2.2 : (2, (\lambda, f, \lambda)_D, 1)$

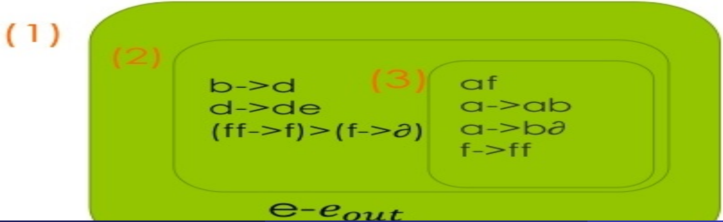
Component 3

$r3.1 : (3, (r, Y_1, \lambda)_I, 2)$
$r3.2 : (3, (f, B, \lambda)_D, 2)$

Why we prefer path?

It has applications in Membrane Computing.





Bridging the gap between LIN and CFL

- The systems GCID(k;1,1,0;1,0,0) and GCID(k;2,1,0;1,0,0) are not known to characterize RE (not even CFL) for any $k \geq 1$.
- However the systems GCID(k;1,1,0;1,0,0) and GCID(k;2,1,0;1,0,0) characterize LIN for $k \geq 3$.

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- However the systems GCID(k;1,1,0;1,0,0) and GCID(k;2,1,0;1,0,0) characterize LIN for $k \geq 3$.
- We aim to show that these systems characterize several classes between LIN and CFL for $k \geq 5$.
- To do so, we first introduce/look into some closure classes of LIN and we term them as super-linear languages.

Closure classes of linear Languages

Note: LIN is not closed under Kleene star and concatenation.

- $\mathcal{L}_{op}(LIN)$ = smallest class containing linear languages and is closed under the operation *op* (Kutrib, Malcher (2007))
- $MLIN := \mathcal{L}_{\circ}(LIN)$ (Metalinear languages)
- $SLIN := \mathcal{L}_{*}(LIN)$ (Starlinear languages)
- $SMLIN := \mathcal{L}_{*}(MLIN) = \mathcal{L}_{*}(\mathcal{L}_{\circ}(LIN))$ (containing MLIN...)
- $MSLIN := \mathcal{L}_{\circ}(SLIN) = \mathcal{L}_{\circ}(\mathcal{L}_{*}(LIN))$
- $SMSLIN := \mathcal{L}_{*}(MSLIN) = \mathcal{L}_{*}(\mathcal{L}_{\circ}(\mathcal{L}_{*}(LIN)))$
- $MSMLIN := \mathcal{L}_{\circ}(SMLIN) = \mathcal{L}_{\circ}(\mathcal{L}_{*}(\mathcal{L}_{\circ}(LIN)))$
- $RATLIN := \mathcal{L}_{\circ,*,\cup}(LIN)$
  The smallest class containing LIN and is closed under the 3 regular operations: concatenation, Kleene star and union.

Languages in closure classes

- $L \in MLIN$ iff $L = L_1 L_2 \dots L_k$ for some $k \geq 1$ and $L_i \in LIN$.
- $L \in SLIN$ iff $L = L_1^*$ for $L_1 \in LIN$.
- $L \in MSLIN$ iff $L = L_1^* L_2^* \dots L_k^*$ for some $k \geq 1$ and $L_i \in LIN$.
- $L \in SMLIN$ iff $L = (L_1 L_2 \dots L_k)^*$ for $k \geq 1$ and $L_i \in LIN$.
- $L \in SMSLIN$ iff $L = (M)^*$ for some $M = L_1^* \dots L_k^* \in MSLIN$.
- $L \in MSMLIN$ iff $L = M_1 M_2 \dots M_k$ for each $M_i \in SMLIN$,
  $M_i = (L_{i,1} L_{i,2} \dots L_{i,t_i})^*$ where $L_{i,j} \in LIN$.

Closure under reversal

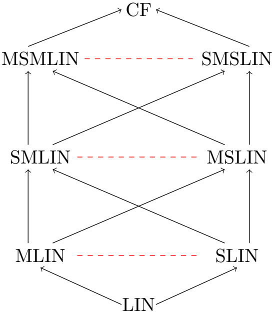
The classes MLIN, SLIN, MSLIN, SMLIN, MSMLIN and SMSLIN are all closed under reversal.

We use the fact that LIN is closed under reversal

- MLIN: $(L_1 L_2 \dots L_k)^R = L_k^R L_{k-1}^R \dots L_1^R$.
- SLIN: $(L_1^*)^R = (L_1^R)^*$.
- SMLIN: $((L_1 L_2 \dots L_k)^*)^R = ((L_1 \dots L_k)^R)^* = (L_k^R \dots L_2^R L_1^R)^*$.
- MSLIN: $(L_1^* L_2^* \dots L_k^*)^R = (L_k^R)^*(L_{k-1}^R)^* \dots (L_2^R)^*(L_1^R)^*$.

Similarly we can extend to MSMLIN and SMSLIN.

Inter-relationship



Solid arrow from A to B indicates $A \subseteq B$. Dashed line between A and B indicates A and B are incomparable.

1. $\mathrm{SLIN} \subseteq \mathrm{MSLIN} \cap \mathrm{SMLIN}$.
2. $\mathrm{MLIN} \subseteq \mathrm{MSLIN} \cap \mathrm{SMLIN}$.
3. $\mathrm{MSLIN} \subseteq \mathrm{MSMLIN} \cap \mathrm{SMSLIN}$.
4. $\mathrm{SMLIN} \subseteq \mathrm{MSMLIN} \cap \mathrm{SMSLIN}$.
5. Incomparable
   - MLIN and SLIN.
   - MSLIN and SMLIN.
   - MSMLIN and SMSLIN.

Sa(i)mple proofs

$\mathrm{MSLIN} \subseteq \mathrm{SMSLIN} \cap \mathrm{MSMLIN}$

- $MSLIN \subseteq SMSLIN$ and
  since $LIN \subseteq MLIN$, $MSLIN \subseteq MSMLIN$.

MSLIN and SMLIN are incomparable

- Let $L_1 = \{a^n b^n \mid n \geq 0\}$ and $L_2 = \{c^m d^m \mid m \geq 0\}$
- $(L_1 L_2)^* \in SMLIN \setminus MSLIN$
  1. $L = L_1 L_2 \in MLIN$ implies $L^* = (L_1 L_2)^* \in SMLIN$.
  2. $L = L_1 L_2 \notin LIN$ implies $L^* \notin SLIN$ and hence $L^* \notin MSLIN$.
- $L_1^* L_2^* \in MSLIN \setminus SMLIN$
- Important: $(L_1 L_2)^* \neq L_1^* L_2^*$ (check yourself!!)

Rewriting grammar for SLIN

Recall: $L \in SLIN$ iff $L = (L_1)^*$

1. Let $G_1 = (N_1, T, S_1, P_1)$ be linear grammar for L_1.
2. A language of SLIN is generated by a grammar $G = (N, T, S, P)$ where
   - $N = N_1 \cup \{S\}$
   - P includes the conventional LIN rules of P_1 and $X \rightarrow Ya$, $X \rightarrow aY$, $X \rightarrow \lambda$
   - The additional CF rules : $S \rightarrow SS_1 \mid \lambda$.

Rewriting grammar for MLIN and SMLIN

Recall: $L \in MLIN$ iff $L = L_1 L_2 \ldots L_k$

1. Let $G_i = (N_i, T, S_i, P_i)$ be linear grammar for L_i.
2. A language of MLIN is generated by a grammar $G = (N, T, S, P)$ where
   - $N = \bigcup_{i=1}^{k} N_i \cup \{S, S_2', S_3', \ldots S_{k+1}'\}$
   - P includes the conventional LIN rules of P_i and $X \to Ya$, $X \to aY$, $X \to \lambda$
   - The additional following CF rules.
     $S \to S_1 S_2'$
     $S_i' \to S_i S_{i+1}'$ for $2 \leq i \leq k$
     $S_{k+1}' \to \lambda$

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   - The additional following CF rules.
     $S \to S_1 S_2'$
     $S_i' \to S_i S_{i+1}'$ for $2 \le i \le k$
     $S_{k+1}' \to \lambda \quad | \ S_1 S_2'$ (Additional rule for **SMLIN**)

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     $X \to Ya$, $X \to aY$, $X \to \lambda$
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     $S_{k+1}' \to \lambda \quad | \; S_1 S_2'$ (Additional rule for **SMLIN**)

Sample derivation for MLIN is

$$S \Longrightarrow S_1 S_2' \Longrightarrow^* L_1 S_2' \Longrightarrow L_1 S_2 S_3' \Longrightarrow^* L_1 L_2 S_3' \Longrightarrow^* L_1 L_2 L_3 S_4'$$

Rewriting grammar for MSLIN

Recall: $L \in MSLIN$ iff $L = L_1^* L_2^* \dots L_k^*$

1. Let $G_i = (N_i, T, S_i, P_i)$ be linear grammar for L_i.
2. A language of MSLIN is generated by a grammar $G = (N, T, S, P)$ where
   - $N = \bigcup_{i=1}^{k} N_i \cup \{S, S_2', S_3', \dots S_{k+1}'\}$
   - P includes the conventional LIN rules of P_i and
     $X \to Ya$, $X \to aY$, $X \to \lambda$, $S_i \to \lambda$
   - The additional following CF rules.
     $S \to S_1 S_2'$
     $S_{i+1}' \to S_i S_{i+1}' \mid S_{i+1} S_{i+2}'$ for $1 \leq i \leq k-1$
     The first rule to stay in L_i and second rule to pass to L_{i+1}
     $S_{k+1}' \to \lambda$

Rewriting grammar for MSLIN

Recall: $L \in MSLIN$ iff $L = L_1^* L_2^* \dots L_k^*$

1. Let $G_i = (N_i, T, S_i, P_i)$ be linear grammar for L_i.
2. A language of MSLIN is generated by a grammar $G = (N, T, S, P)$ where
   - $N = \bigcup_{i=1}^{k} N_i \cup \{S, S_2', S_3', \dots S_{k+1}'\}$
   - P includes the conventional LIN rules of P_i and $X \to Ya$, $X \to aY$, $X \to \lambda$, $S_i \to \lambda$
   - The additional following CF rules.
     $S \to S_1 S_2'$
     $S_{i+1}' \to S_i S_{i+1}' \mid S_{i+1} S_{i+2}'$ for $1 \leq i \leq k-1$
     The first rule to stay in L_i and second rule to pass to L_{i+1}
     $S_{k+1}' \to \lambda \ \mid S_1 S_2'$, $S \to \lambda$ (Additional rule for **SMSLIN**)

Rewriting grammar for MSMLIN

Recall: $L \in MSMLIN$ iff $L = M_1 M_2 \ldots M_k$ for each $M_i \in SMLIN$.
$M_i = (L_{i,1} L_{i,2} \ldots L_{i,t_i})^*$ where $L_{i,j} \in LIN$.

1. Let $G_{i,j} = (N_{i,j}, T, S_{i,j}, P_{i,j})$ be linear grammar for $L_{i,j}$.
2. The grammar rules of MSMLIN include the conventional LIN rules of $P_{i,j}$ and P'.

Recalling SMLIN

$S \to S_1 S_2'$

for $2 \le j \le t$

$S_j' \to S_j S_{j+1}'$

$S_{t+1}' \to \lambda \mid S_1 S_2'$

Rules of P' for MSMLIN

$S \to S_{1,1} S_{1,2}'$

For $1 \le i \le k$ and $2 \le j \le t_i$

$S_{i,j}' \to S_{i,j} S_{i,j+1}'$

$S_{i,t_i+1}' \to S_{i,1} S_{i,2}' \mid \underbrace{S_{i+1,1} S_{i+1,2}'}_{for\ \ i \neq k} \mid \underbrace{\lambda}_{if\ \ i=k}$

$LIN \subsetneq GCID(3; 1, 1, 0; 1, 0, 0)$

We simulate the rules $p : X \to Ya$, $q : X \to aY$ and $h : X \to \lambda$ as:

Component 1

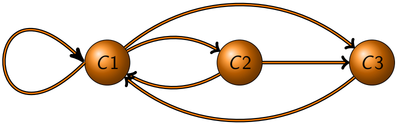
$p1.1 : (1, (X, p, \lambda)_{ins}, 3)$
p1.2: $(1, (p, a, \lambda)_{ins}, 2)$
p1.3: $(1, (p', Y, \lambda)_{ins}, 2)$
$q1.1 : (1, (X, q, \lambda)_{ins}, 3)$
$q1.2 : (1, (q, q', \lambda)_{ins}, 2)$
$q1.3 : (1, (q', Y, \lambda)_{ins}, 2)$
$h1.1 : (1, (\lambda, X, \lambda)_{ins}, 1)$

Component 2

$p2.1 : (2, (p, p', \lambda)_{ins}, 3)$
$p2.2 : (2, (\lambda, p', \lambda)_{del}, 1)$
$q2.1 : (2, (q, a, \lambda)_{ins}, 3)$
$q2.2 : (2, (\lambda, q', \lambda)_{del}, 1)$

Component 3

$p3.1 : (3, (\lambda, X, \lambda)_{del}, 1)$
$p3.2 : (3, (\lambda, p, \lambda)_{del}, 1)$
$q3.1 : (3, (\lambda, X, \lambda)_{del}, 1)$
$q3.2 : (3, (\lambda, q, \lambda)_{del}, 1)$



$LIN \subsetneq GCID(3; 2, 1, 0; 1, 0, 0)$

We simulate the rules $p : X \to aY$, $q : X \to Ya$, $h : X \to \lambda$ as:

Component 1

$p1.1 : (1, (X, p, \lambda)_{ins}, 2)$
p1.2: $(1, (p, aY, \lambda)_{ins}, 3)$
$q1.1 : (1, (X, q, \lambda)_{ins}, 2)$
$q1.2 : (1, (q, Ya, \lambda)_{ins}, 3)$
$h1.1 : (1, (\lambda, X, \lambda)_{del}, 1)$

Component 2

$p2.1 : (2, (\lambda, X, \lambda)_{del}, 1)$
$q2.1 : (2, (\lambda, X, \lambda)_{del}, 1)$

Component 3

$p3.1 : (3, (\lambda, p, \lambda)_{del}, 1)$
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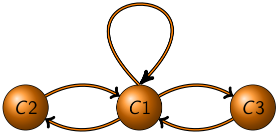

Simulating Transition rules of MLIN

Recall: $S'_{i+1} \rightarrow S_{i+1}S_{i+2}$ for $1 \leq i \leq k-1$ and $S'_{k+1} \rightarrow \lambda$
$MLIN \subseteq GCID(5;2,1,0;1,0,0)$. For each $1 \leq i \leq k$,

Component 1

$p_i 1.1 : (1, (X_i, p_i, \lambda)_{ins}, 2)$
$p_i 1.2 : (1, (p_i, aY_i, \lambda)_{ins}, 3)$
$q_i 1.1 : (1, (X_i, q_i, \lambda)_{ins}, 2)$
$q_i 1.2 : (1, (q_i, Y_i a, \lambda)_{ins}, 3)$
$h_i 1.1 : (1, (\lambda, X_i, \lambda)_{del}, 4)$

Component 2

$p_i 2.1 : (2, (\lambda, X_i, \lambda)_{del}, 1)$
$q_i 2.1 : (2, (\lambda, X_i, \lambda)_{del}, 1)$

Component 3

$p_i 3.1 : (3, (\lambda, p_i, \lambda)_{del}, 1)$
$q_i 3.1 : (3, (\lambda, q_i, \lambda)_{del}, 1)$

Component 4

For $i \neq k$
$r_i 4.1 : (4, (S'_{i+1}, S_{i+1}, \lambda)_{ins}, 5)$
$r_i 4.2 : (4, (S_{i+1}, S'_{i+2}, \lambda)_{ins}, 1)$
For $i = k$
$r_i 4.1 : (4, (\lambda, S'_{i+1}, \lambda)_{del}, 1)$

Component 5

For $i \neq k$
$r_i 5.1 : (5, (\lambda, S'_{i+1}, \lambda)_{del}, 4)$

Simulating Transition rules of MLIN

Recall: $S'_{i+1} \rightarrow S_{i+1}S_{i+2}$ for $1 \leq i \leq k-1$ and $S'_{k+1} \rightarrow \lambda$
$MLIN \subseteq GCID(5;2,1,0;1,0,0)$. For each $1 \leq i \leq k$,

Component 1

$p_i 1.1 : (1, (X_i, p_i, \lambda)_{ins}, 2)$
$p_i 1.2 : (1, (p_i, aY_i, \lambda)_{ins}, 3)$
$q_i 1.1 : (1, (X_i, q_i, \lambda)_{ins}, 2)$
$q_i 1.2 : (1, (q_i, Y_i a, \lambda)_{ins}, 3)$
$h_i 1.1 : (1, (\lambda, X_i, \lambda)_{del}, 4)$

Component 2

$p_i 2.1 : (2, (\lambda, X_i, \lambda)_{del}, 1)$
$q_i 2.1 : (2, (\lambda, X_i, \lambda)_{del}, 1)$

Component 3

$p_i 3.1 : (3, (\lambda, p_i, \lambda)_{del}, 1)$
$q_i 3.1 : (3, (\lambda, q_i, \lambda)_{del}, 1)$

Component 4

For $i \neq k$
$r_i 4.1 : (4, (S'_{i+1}, S_{i+1}, \lambda)_{ins}, 5)$
$r_i 4.2 : (4, (S_{i+1}, S'_{i+2}, \lambda)_{ins}, 1)$
For $i = k$
$r_i 4.1 : (4, (\lambda, S'_{i+1}, \lambda)_{del}, 1)$

Component 5

For $i \neq k$
$r_i 5.1 : (5, (\lambda, S'_{i+1}, \lambda)_{del}, 4)$

$$(S_1 S'_2)_1 \Longrightarrow^* (L_1 S'_2)_4 \Longrightarrow (L_1 S'_2 S_2)_5 \Longrightarrow (L_1 S_2)_4 \Longrightarrow (L_1 S_2 S'_3)_1$$

$MSLIN \subseteq GCID(5;2,1,0;1,0,0)$

Recall: $S'_{i+1} \to S_{i+1}S'_{i+1} \mid S_{i+1}S'_{i+2}$ for $1 \le i \le k-1$ and $S'_{k+1} \to \lambda$.
For each $1 \le i \le k$,

Component 1

$p_i1.1 : (1,(X_i,p_i,\lambda)_{ins},2)$
$p_i1.2 : (1,(p_i,aY_i,\lambda)_{ins},3)$
$q_i1.1 : (1,(X_i,q_i,\lambda)_{ins},2)$
$q_i1.2 : (1,(q_i,Y_ia,\lambda)_{ins},3)$
$h_i1.1 : (1,(\lambda,X_i,\lambda)_{del},4)$

Component 2

$p_i2.1 : (2,(\lambda,X_i,\lambda)_{del},1)$
$q_i2.1 : (2,(\lambda,X_i,\lambda)_{del},1)$

Component 3

$p_i3.1 : (3,(\lambda,p_i,\lambda)_{del},1)$
$q_i3.1 : (3,(\lambda,q_i,\lambda)_{del},1)$

Component 4

For $i \neq k$
$r_i4.1 : (4,(S'_{i+1},S_{i+1},\lambda)_{ins},5)$
$r_i4.2 : (4,(S_{i+1},S'_{i+2},\lambda)_{ins},1)$
$r_i4.3 : (4,(S_{i+1},S'_{i+1},\lambda)_{ins},1)$
For $i = k$
$r_i4.1 : (4,(\lambda,S'_{i+1},\lambda)_{del},1)$

Component 5

For $i \neq k$
$r_i5.1 : (5,(\lambda,S'_{i+1},\lambda)_{del},4)$

Summary of the results

Each of SLIN, MLIN, SMLIN, MSLIN, SMSLIN, MSMLIN is a subset of each of the following.

- GCID(5;2,1,0;1,0,0) with tree as a control graph
- GCID(5;1,1,0;1,0,0) with non-tree as a control graph

Summary of the results

Each of SLIN, MLIN, SMLIN, MSLIN, SMSLIN, MSMLIN is a subset of each of the following.

- GCID(5;2,1,0;1,0,0) with tree as a control graph
- GCID(5;1,1,0;1,0,0) with non-tree as a control graph

The obtained results can be stated as a general theorem.

Generic Theorem

For integers $t, n, m \geq 1$ and $i', i'', j', j'' \geq 0$ with $i' + i'' \geq 1$ and $X \in \{NTr, Tr\}$, if $\mathrm{LIN} \subseteq \mathrm{GCID}_X(t; n, i', i''; m, j', j'')$, then $\mathrm{F} \subseteq \mathrm{GCID}_X(t+2; n, i', i''; m, j', j'')$ where F $\in$ {SLIN, MLIN, SMLIN, MSLIN, SMSLIN, MSMLIN}.

Extending the results

RATLIN: smallest family containing LIN and closed under union, concatenation and Kleene star.

- Let $L = (L_1 L_2)^* L_3^* L_4 L_5^*$
- Continuation points

$i =$	1	2	3	4	5
$\mathrm{cont}(i)$	2	$1, 3, 4$	$3, 4$	5	$5, 6$

- Assumption: $i + 1 \in cont(i)$

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- Assumption: $i + 1 \in cont(i)$

Transition rules: Axiom $= S'_1$

$S'_i \rightarrow S_i S'_c$ for all $c \in cont(i)$ and $1 \leq i \leq k$

$S'_{k+1} \rightarrow \lambda$

Matrix Ins-del system

Definition

A matrix insertion-deletion system is a construct $\Gamma = (V, T, A, R)$

- V is an alphabet, $T \subseteq V$, A is a finite language over V
- R is a finite set of matrices $\{m_1, m_2, \ldots m_l\}$
- $\boxed{m_i = [(u_1, \alpha_1, v_1)_{t_1}, (u_2, \alpha_2, v_2)_{t_2}, \ldots, (u_k, \alpha_k, v_k)_{t_k}]}$

Notes to remember:

- On choosing a matrix m_i, all rules in m_i are applied in order.
- If a rule in m_i cannot be applied, then m_i itself is not applied.

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Notes to remember:

- On choosing a matrix m_i, all rules in m_i are applied in order.
- If a rule in m_i cannot be applied, then m_i itself is not applied.

Size

Size of a matrix ins-del system is $(k; n, i', i''; m, j', j'')$ where

$\boxed{k : \text{Maximum number of ins-del rules in a matrix}}$

$n, i', i''; m, j', j''$ are same as in ID size.

Examples

Language generated by the following matrix ins-del systems?

Axiom: $\#\$$

r1 $= [(\#, a, \lambda)_{ins}, (\$, a, \lambda)_{ins}]$
r2 $= [(\#, b, \lambda)_{ins}, (\$, b, \lambda)_{ins}]$
r3 $= [(\lambda, \#, \lambda)_{del}, (\lambda, \$, \lambda)_{del}]$

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Language $= \{ww \mid w \in \{a, b\}^*\}$
Size of the system is
$(2; 1, 1, 0; 1, 0, 0)$.

Examples

Language generated by the following matrix ins-del systems?

Axiom: $\#\$$

$r1 = [(\#, a, \lambda)_{ins}, (\$, a, \lambda)_{ins}]$
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Examples

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Language $= \{ww \mid w \in \{a, b\}^*\}$
Size of the system is $(2; 1, 1, 0; 1, 0, 0)$.

Axiom: $\#$

r1 $= [(\lambda, a, \#)_{ins}, (\#, b, \lambda)_{ins}]$
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Language $= \{a^n b^n \mid n \geq 0\}$
Size of the system is $(2; 1, 1, 1; 1, 0, 0)$.

Examples

Language generated by the following matrix ins-del systems?

Axiom: $\#\$$

$r1 = [(\#, a, \lambda)_{ins}, (\$, a, \lambda)_{ins}]$
$r2 = [(\#, b, \lambda)_{ins}, (\$, b, \lambda)_{ins}]$
$r3 = [(\lambda, \#, \lambda)_{del}, (\lambda, \$, \lambda)_{del}]$

Language $= \{ww \mid w \in \{a, b\}^*\}$
Size of the system is $(2; 1, 1, 0; 1, 0, 0)$.

Axiom: $\#$

$r1 = [(\lambda, a, \#)_{ins}, (\#, b, \lambda)_{ins}]$
$r2 = [(\lambda, \#, \lambda)_{del}]$

Language $= \{a^n b^n \mid n \geq 0\}$
Size of the system is $(2; 1, 1, 1; 1, 0, 0)$.

Helpful Results

- $\boxed{MAT(k; n, i', i''; m, j', j'') = [MAT(k; n, i'', i'; m, j'', j')]^R}$
- Since RE is closed under reversal,
  $MAT(k; n, i', i''; m, j', j'') = RE = MAT(k; n, i'', i'; m, j'', j')$.

Exhaustive Analysis for $n = |Ins| = 1$, $m = |Del| = 1$

Size $(k; 1, i', i''; 1, j', j'')$; $i', i'', j', j'' \in \{0, 1\}$	Reference	k	Language Family Relation
$(k; 1, 0, 0; 1, 0, 0)$	S.Verlan 2007	1	$\subset REG$
$(k; 1, 0, 0; 1, 1, 0), (k; 1, 0, 0; 1, 0, 1)$		≥ 1	OPEN
$(k; 1, 0, 0; 1, 1, 1)$	HLI 2018 HLI 2019	3 2	$= \mathrm{RE}$ $= \mathrm{RE}$
$(k; 1, 1, 0; 1, 0, 0)$, $(k; 1, 0, 0; 1, 0, 0)$	HLI 2019	3	$\supset \mathcal{L}_{reg}(\mathrm{LIN})$
$(k; 1, 1, 1; 1, 0, 0)$	HLI 2018 HLI 2019	3 2	$= \mathrm{RE}$ $\supset \mathcal{L}_{reg}(\mathrm{LIN})$
$(k; 1, 1, 0; 1, 1, 0)$, $(k; 1, 1, 0; 1, 0, 1)$ $(k; 1, 0, 1; 1, 0, 1)$, $(k; 1, 0, 1; 1, 1, 0)$	S.Verlan 2012 HLI 2019	3 2	$= \mathrm{RE}$ $= \mathrm{RE}$
$(k; 1, 1, 0; 1, 1, 1)$, $(k; 1, 0, 1; 1, 1, 1)$	HLI 2018	2	$= \mathrm{RE}$
$(k; 1, 1, 1; 1, 1, 0)$, $(k; 1, 1, 1; 1, 0, 1)$	HLI 2018	2	$= \mathrm{RE}$
$(k; 1, 1, 1; 1, 1, 1)$	Takahari 2003	1	$= \mathrm{RE}$

Power of MID systems of size $(k; 1, i', i''; 1, j', j'')$

HLI 2018: H Fernau, Lakshmanan, Indhumathi, Investigations on the Power of Matrix Insertion-Deletion Systems of Small Sizes, Natural Computing, 2018, 17(2), 249 - 269.

HLI 2019: -do-, On Matrix Ins-Del Systems of Small Sum-Norm, SOFSEM 2019, LNCS 11376, 192-205.

Exhaustive Analysis for $n + m = 3$

Size $(k; 1, i', i''; 2, j', j'')$; $i', i'', j', j'' \in \{0, 1\}$ or $(k; 2, i', i''; 1, j', j'')$; $i', i'', j', j'' \in \{0, 1\}$	Reference	k	Language Family Relation
$(k; 1, 0, 0; 2, 0, 0)$, $(k; 2, 0, 0; 1, 0, 0)$	Verlan 2007	1	$\subset REG$
$(k; 1, 0, 0; 2, 1, 0)$, $(k; 1, 0, 0; 2, 0, 1)$		≥ 1	OPEN
$(k; 1, 1, 0; 2, 0, 0)$, $(k; 1, 1, 0; 2, 1, 0)$, $(k; 1, 1, 0; 2, 0, 1)$ $(k; 2, 0, 0; 1, 1, 0)$, $(k; 2, 1, 0; 1, 1, 0)$, $(k; 2, 0, 1; 1, 1, 0)$	Verlan 2012	2	$= \mathrm{RE}$
$(k; 1, 0, 0; 2, 1, 1), (k; 2, 1, 1; 1, 0, 0)$	HLI 2018	3	$= \mathrm{RE}$
$(k; 1, 1, 0; 2, 1, 1)$, $(k; 1, 0, 1; 2, 0, 0)$, $(k; 1, 0, 1; 2, 1, 1)$ $(k; 1, 0, 1; 2, 1, 0)$, $(k; 1, 0, 1; 2, 0, 1)$	HLI 2018	2	$= \mathrm{RE}$
$(k; 2, 0, 0; 1, 0, 1)$, $(k; 2, 1, 0; 1, 0, 1)$, $(k; 2, 0, 1; 1, 0, 1)$ $(k; 2, 1, 1; 1, 1, 0)$, $(k; 2, 1, 1; 1, 0, 1)$	HLI 2018	2	$= \mathrm{RE}$
$(k; 2, 1, 0; 1, 0, 0)$, $(k; 2, 0, 1; 1, 0, 0)$	HLI 2019	2	$\supset \mathcal{L}_{reg}(\mathrm{LIN})$
$(k; 2, 0, 0; 1, 1, 1)$, $(k; 2, 1, 0; 1, 1, 1)$, $(k; 2, 0, 1; 1, 1, 1)$	Krassovitskiy 2008	1	$= \mathrm{RE}$
$(k; 1, 1, 1; 2, 0, 0)$, $(k; 1, 1, 1; 2, 1, 0)$, $(k; 1, 1, 1; 2, 0, 1)$	Paun 1998	1	$= \mathrm{RE}$
$(k; 1, 1, 1; 2, 1, 1)$, $(k; 2, 1, 1; 1, 1, 1)$	Takahari 2003	1	$= \mathrm{RE}$

Power of MID systems of size $(k; 1, i', i''; 2, j', j'')$ or $(k; 2, i', i''; 1, j', j'')$

MAT(3;1,0,0;1,1,1) = RE

Consider a type-0 grammar $G = (N, T, P, S)$ in SGNF.

Simulating p: $X \rightarrow bY$

$p1 = [(\lambda, p, \lambda)_{ins},\ (\lambda, p', \lambda)_{ins}, (p', X, p)_{del}]$
$p2 = [(\lambda, b, \lambda)_{ins}, (\lambda, Y, \lambda)_{ins},\ (b, p, Y)_{del}]$
$p3 = [(\lambda, p', b)_{del}]$ (right context is required to ensure p3 is applied after p2)

MAT(3;1,0,0;1,1,1) = RE

Consider a type-0 grammar $G = (N, T, P, S)$ in SGNF.

Simulating p: $X \to bY$

$p1 = [(\lambda, p, \lambda)_{ins},\ (\lambda, p', \lambda)_{ins}, (p', X, p)_{del}]$
$p2 = [(\lambda, b, \lambda)_{ins}, (\lambda, Y, \lambda)_{ins},\ (b, p, Y)_{del}]$
$p3 = [(\lambda, p', b)_{del}]$ (right context is required to ensure p3 is applied after p2)

Simulating q: $X \to Yb$

$q1 = [(\lambda, q, \lambda)_{ins},\ (\lambda, q', \lambda)_{ins}, (q', X, q)_{del}]$
$q2 = [(\lambda, b, \lambda)_{ins}, (\lambda, Y, \lambda)_{ins},\ (Y, q', b)_{del}]$
$q3 = [(b, q, \lambda)_{del}]$ (left context is required to ensure p3 is applied after p2)

MAT(3;1,0,0;1,1,1) = RE

Consider a type-0 grammar $G = (N, T, P, S)$ in SGNF.

Simulating p: $X \rightarrow bY$

$p1 = [(\lambda, p, \lambda)_{ins},\ (\lambda, p', \lambda)_{ins}, (p', X, p)_{del}]$
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$q3 = [(b, q, \lambda)_{del}]$ (left context is required to ensure p3 is applied after p2)

Simulating f: $AB \rightarrow \lambda$

$f1 = [(\lambda, f, \lambda)_{ins}, (\lambda, f', \lambda)_{ins}, (f, A, B)_{del}]$
$f2 = [(f, B, f')_{del}, (\lambda, f', \lambda)_{del}, (\lambda, f, \lambda)_{del}]$

MAT(2;1,1,0;1,1,1) = RE

Simulating p: $X \to bY$: Axiom$= S\#\$$

$$
\begin{aligned}
p1 &= [(X,p,\lambda)_{ins},\ (\#,p',\lambda)_{ins}] & p4 &= [(p,b,\lambda)_{ins},\ (p''',p'',p')_{del}]\\
p2 &= [(\lambda,X,p)_{del},\ (\#,p'',\lambda)_{ins}] & p5 &= [(\lambda,p,b)_{del},\ (p''',p',\$)_{del}]\\
p3 &= [(p,Y,\lambda)_{ins},\ (\#,p''',\lambda)_{ins}] & p6 &= [(\#,p''',\$)_{del}]
\end{aligned}
$$

Simulating f: $AB \to \lambda$

$$
\begin{aligned}
f1 &= [(B,f,\lambda)_{ins},\ (\#,f',\lambda)_{ins}] & f1' &= [(B,f,\lambda)_{ins}]\\
f2 &= [(\lambda,B,f)_{del},\ (\lambda,A,f)_{del}] & f2' &= [(\lambda,B,f)_{del},\ (\lambda,A,f)_{del}]\\
f3 &= [(\lambda,f,\lambda)_{del},\ (\#,f',\$)_{del}] & f3' &= [(\lambda,f,\lambda)_{del}]
\end{aligned}
$$

Malicious derivation for $f : AB \to \lambda$

$$
AAB\delta B\#\$ \Rightarrow^2_{f1'} AABf\delta Bf\#\$ \Rightarrow^2_{f2'}
$$

$$
\underline{AAB}f\delta\underline{B}f\#f'f'\$ = f\delta f\#\$ \Rightarrow_{f3'} \delta\#\$
$$

Note: $[(\lambda,\#,\lambda),(\lambda,\$,\lambda)]$ is applied at the end of the derivation.

MAT(2;1,1,0;1,1,0) = RE

Simulating p: $X \rightarrow bY$

$$p1 = [(X, p, \lambda)_{ins},\ (\lambda, p', \lambda)_{ins}]$$
$$p2 = [(p', X, \lambda)_{del},\ (p', p'', \lambda)_{ins}]$$
$$p3 = [(p'', p, \lambda)_{del},\ (p'', Y, \lambda)_{ins}]$$
$$p4 = [(p', b, \lambda)_{ins},\ (b, p'', \lambda)_{del}]$$
$$p5 = [(\lambda, p', \lambda)_{del}]$$

MAT(2;1,1,0;1,1,0) = RE

Simulating p: $X \to bY$

$$
\begin{aligned}
p1 &= [(X, p, \lambda)_{ins},\ (\lambda, p', \lambda)_{ins}] \\
p2 &= [(p', X, \lambda)_{del},\ (p', p'', \lambda)_{ins}] \\
p3 &= [(p'', p, \lambda)_{del},\ (p'', Y, \lambda)_{ins}]
\end{aligned}
\qquad
\begin{aligned}
p4 &= [(p', b, \lambda)_{ins},\ (b, p'', \lambda)_{del}] \\
p5 &= [(\lambda, p', \lambda)_{del}]
\end{aligned}
$$

Applying p1 twice??

$X \Rightarrow_{p1} p'Xpp\ldots p' \Rightarrow_{p2} p'p''pp\ldots p' \Rightarrow_{p3} p'p''Yp\ldots p' \Rightarrow_{p4}$
$p'bYp\ldots p' \Rightarrow^2_{p5} bYp$. Cannot reapply p3 to get rid of the second p.

Simulating f: $AB \to \lambda$

A new idea of moving in a Z.

$$
\begin{aligned}
h1 &= [(\lambda, S', \lambda)_{del},\ (\lambda, Z, \lambda)_{ins}] \\
f1 &= [(Z, A, \lambda)_{del},\ (Z, B, \lambda)_{del}]
\end{aligned}
\qquad
\begin{aligned}
moveZ &= [(\lambda, Z, \lambda)_{del},\ (\lambda, Z, \lambda)_{ins}] \\
delZ &= [(\lambda, Z, \lambda)_{del}]
\end{aligned}
$$

MAT rules for Super-linear grammars

Each of SLIN, MLIN, SMLIN, MSLIN, SMSLIN, MSMLIN is a subset of each of the following.

- MAT(3;1,1,0;1,0,0)
- MAT(2;2,1,0;1,0,0)
- MAT(2;1,1,1;1,0,0)

MAT rules for Super-linear grammars

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- MAT(2;2,1,0;1,0,0)
- MAT(2;1,1,1;1,0,0)

Generic Theorem

For integers $t, n, m \geq 1$ and $i', i'', j', j'' \geq 0$ with $i' + i'' \geq 1$, if $\mathrm{LIN} \subseteq \mathrm{MAT}(t; n, i', i''; m, j', j'')$, then $\mathrm{F} \subseteq \mathrm{MAT}(t; n, i', i''; m, j', j'')$ where F $\in$ {SLIN, MLIN, SMLIN, MSLIN, SMSLIN, MSMLIN, RATLIN}.

Simulation of MLIN

Recall: Apart from the usual LIN rules, the transition rules in MLIN are $S'_{i+1} \to S_{i+1}S_{i+2}$ for $1 \leq i \leq k-1$ and $S'_{k+1} \to \lambda$, for each $1 \leq i \leq k$,

MLIN $\subseteq$ MAT$(3; 1, 1, 0; 1, 0, 0)$: Axiom $= S_1 S'_2$

$p1 = [(X_i, p_i, \lambda)_{ins},\ (p_i, p'_i, \lambda)_{ins}, (\lambda, X_i, \lambda)_{del}]$
$p2 = [(p_i, a_i, \lambda)_{ins}, (p'_i, Y_i, \lambda)_{ins},\ (\lambda, p_i, \lambda)_{del}]$
$p3 = [(\lambda, p'_i, \lambda)_{del}]$
$p4 = [(S'_{i+1}, S'_{i+2}, \lambda)_{ins}, (S'_{i+1}, S_{i+1}, \lambda)_{ins},\ (\lambda, S'_{i+1}, \lambda)_{del}]$ (for each $1 \leq i \leq k-1$)
$p5 = [(\lambda, S'_{k+1}, \lambda)_{del}]$

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$p4 = [(S'_{i+1}, S'_{i+2}, \lambda)_{ins}, (S'_{i+1}, S_{i+1}, \lambda)_{ins},\ (\lambda, S'_{i+1}, \lambda)_{del}]$ (for each $1 \le i \le k-1$)
$p5 = [(\lambda, S'_{k+1}, \lambda)_{del}]$

MLIN $\subseteq$ MAT$(2; 1, 1, 1; 1, 0, 0)$: Axiom $= S_1 S'_2$

$p1 = [(X_i, p_i, \lambda)_{ins},\ (\lambda, X_i, \lambda)_{del}]$
$p2 = [(p_i, p'_i, \lambda)_{ins},\ (p_i, a_i, p'_i)_{ins}]$ (cannot reuse due to second rule)
$p3 = [(a_i, Y_i, p'_i)_{ins},\ (\lambda, p_i, \lambda)_{del}]$
$p4 = [(\lambda, p'_i, \lambda)_{del}]$
$p5 = [(S'_{i+1}, S'_{i+2}, \lambda)_{ins}, (S'_{i+1}, S_{i+1}, S'_{i+2})_{ins}]$ (for each $1 \le i \le k-1$)
$p5 = [(\lambda, S'_{i+1}, \lambda)_{del}]$(for each $1 \le i \le k$)

Semi-conditional ins-del system

Definition

A semi-conditional ins-del system (SCID) of degree (i, j) is $G = (V, T, A, P)$, where P is a finite set of rules of the form $((u, x, v)_t, \alpha, \beta)$, where

- $(u, x, v)_t$ is an ins-del rule, $t \in \{ins, del\}$,
- $\alpha, \beta = \phi$ or $\alpha, \beta \subset (N \cup T)^*$ (finite languages) and
- $|\alpha_r| \leq i$ for $\alpha_r \in \alpha$, and $|\beta_s| \leq j$ for $\beta_s \in \beta$.

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- $|\alpha_r| \leq i$ for $\alpha_r \in \alpha$, and $|\beta_s| \leq j$ for $\beta_s \in \beta$.

Rule application in derivation

$((u, x, v)_t, \alpha, \beta)$ is applied to a string w iff every string in

- [Permitting set] α (when $\alpha \neq \phi$) is a substring of w and
- [Forbidding set] β (when $\beta \neq \phi$) is not a substring of w.
- If $\alpha = \phi$, $\beta = \phi$, the rule is applied without any restriction.

SSCID and an Example

Variants

A semi-conditional grammar is called

- Random Context: if degree $(i, j) = (1, 1)$.
- Simple: If either $\alpha = \phi$ or $\beta = \phi$ in every rule of P.

Example: $L_1 = \{a^n b^n c^n \mid n \geq 1\} \notin CF$

Consider $G_1 = (\{a, b, c, A, B\}, \{a, b, c\}, abc, R)$ where R is

- $[(a, aAb, b)_{ins}, \emptyset, B]$
- $[(b, Bc, c)_{ins}, A, \emptyset]$
- $[(\lambda, A, \lambda)_{del}, B, \emptyset]$
- $[(\lambda, B, \lambda)_{del}, \emptyset, A]$

SSCID and an Example

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- Random Context: if degree $(i, j) = (1, 1)$.
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- $[(a, aAb, b)_{ins}, \emptyset, B]$
- $[(b, Bc, c)_{ins}, A, \emptyset]$
- $[(\lambda, A, \lambda)_{del}, B, \emptyset]$
- $[(\lambda, B, \lambda)_{del}, \emptyset, A]$
- Simple and Random Context
- Size $= (3, 1, 1; 1, 0, 0)$
- Degree $= (1, 1)$

Existing vs Our Results

Semi-conditional Ins-del systems of following sizes (do not) describe the class of RE languages

Existing Results (S.Ivanov, S.Verlan, Fund.Inf., 2012)

- $SCID_{2,2}(1,0,0;1,0,0)$
- $SCID_{1,1}(2,0,0;1,1,0)$
- $SCID_{1,1}(1,1,0;1,1,1)$
- $SCID_{1,1}(1,1,0;2,0,0)$
- None is simple

Results of UCNC 2018

- $\boxed{SSCID_{2,1}(2,0,0;2,0,0)}$
- $SSCID_{3,1}(1,1,0;1,1,0)$
- $SSCID_{2,1}(1,1,0;1,1,1)$
- $SSCID_{2,1}(1,1,0;2,0,0)$
- All are simple

$\text{SSCID}_{2,1}(2,0,0;2,0,0)$=RE

Simulation of $\boxed{f : AB \to \lambda \text{ by } (\lambda, AB, \lambda, \phi, \phi)}$ is direct.

$\text{SSCID}_{2,1}(2,0,0;2,0,0)$=RE

Simulation of $\boxed{f : AB \to \lambda \text{ by } (\lambda, AB, \lambda, \phi, \phi)}$ is direct.

Simulations of $\boxed{p : X \to bY}$ and $\boxed{q : X \to Yb}$ are similar.

Simulating $q : X \to Yb$

$q1 : [(\lambda, qq', \lambda)_{ins}, \emptyset, \{q, q', q'', q'''\}]$
$q2 : [(\lambda, q'X, \lambda)_{del}, \{qq'\}, \emptyset]$
$q3 : [(\lambda, q''b, \lambda)_{ins}, \emptyset, \{q', q'', q'''\}]$
$q4 : [(\lambda, q'''Y, \lambda)_{ins}, \emptyset, N' \cup \{q', q'''\}]$
$q5 : [(\lambda, q''', \lambda)_{del}, \{q''q'''\}, \emptyset]$
$q6 : [(\lambda, qq'', \lambda)_{del}, \{Yb\}, \emptyset]$

SSCID$_{2,1}(2,0,0;2,0,0)$=RE

Simulation of $\boxed{f : AB \to \lambda \text{ by } (\lambda, AB, \lambda, \phi, \phi)}$ is direct.

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$q4 : [(\lambda, q'''Y, \lambda)_{ins}, \emptyset, N' \cup \{q', q'''\}]$
$q5 : [(\lambda, q''', \lambda)_{del}, \{q''q'''\}, \emptyset]$
$q6 : [(\lambda, qq'', \lambda)_{del}, \{Yb\}, \emptyset]$

Another simulation?

$q1 : [(\lambda, qq', \lambda)_{ins}, \emptyset, \{q, q'', q'''\}]$
$q2 : [(\lambda, q'X, \lambda)_{del}, \{qq'\}, \emptyset]$
$\hat{q3} : [(\lambda, Yq'', \lambda)_{ins}, \emptyset, N' \cup \{q'', q'''\}]$
$\hat{q4} : [(\lambda, bq''', \lambda)_{ins}, \emptyset, N' \cup \{q', q'''\}]$
$\hat{q5} : [(\lambda, q''q, \lambda)_{del}, \{q'''q''\}, \emptyset]$
$\hat{q6} : [(\lambda, q''', \lambda)_{del}, \emptyset, \{q, q''\}]$

$\mathrm{SSCID}_{2,1}(2,0,0;2,0,0)$=RE

Simulation of $\boxed{f : AB \to \lambda \text{ by } (\lambda, AB, \lambda, \phi, \phi)}$ is direct.

Simulations of $\boxed{p : X \to bY}$ and $\boxed{q : X \to Yb}$ are similar.

Simulating $q : X \to Yb$	Another simulation?
$q1 : [(\lambda, qq', \lambda)_{ins}, \emptyset, \{q, q', q'', q'''\}]$	$q1 : [(\lambda, qq', \lambda)_{ins}, \emptyset, \{q, q'', q'''\}]$
$q2 : [(\lambda, q'X, \lambda)_{del}, \{qq'\}, \emptyset]$	$q2 : [(\lambda, q'X, \lambda)_{del}, \{qq'\}, \emptyset]$
$q3 : [(\lambda, q''b, \lambda)_{ins}, \emptyset, \{q', q'', q'''\}]$	$\hat{q}3 : [(\lambda, Yq'', \lambda)_{ins}, \emptyset, N' \cup \{q'', q'''\}]$
$q4 : [(\lambda, q'''Y, \lambda)_{ins}, \emptyset, N' \cup \{q', q'''\}]$	$\hat{q}4 : [(\lambda, bq''', \lambda)_{ins}, \emptyset, N' \cup \{q', q'''\}]$
$q5 : [(\lambda, q''', \lambda)_{del}, \{q''q'''\}, \emptyset]$	$\hat{q}5 : [(\lambda, q''q, \lambda)_{del}, \{q'''q''\}, \emptyset]$
$q6 : [(\lambda, qq'', \lambda)_{del}, \{Yb\}, \emptyset]$	$\hat{q}6 : [(\lambda, q''', \lambda)_{del}, \emptyset, \{q, q''\}]$

Suppose we have a terminal string α,
$\alpha \Rightarrow_{\hat{q}4} \alpha bq''' \Rightarrow_{\hat{q}6} \alpha b = \alpha' \in T^*$
We get another terminal string without any reason.
(ERROR with right side rules!!)

Forbidding Ins-del systems

Definition

A forbidding ins-del system (FID) of degree k is $G = (V, T, A, P)$, where P is a finite set of rules of the form $((u, x, v)_t, F)$, where

- $(u, x, v)_t$ is an ins-del rule, $t \in \{ins, del\}$,
- $F = \phi$ or $F \subset (N \cup T)^*$ (finite languages) and
- $|f_r| \leq k$ for $f_r \in F$.

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- $(u, x, v)_t$ is an ins-del rule, $t \in \{ins, del\}$,
- $F = \phi$ or $F \subset (N \cup T)^*$ (finite languages) and
- $|f_r| \leq k$ for $f_r \in F$.

Points to note

- $((u, x, v)_t, F)$ is applied to a string w iff every string in [Forbidding set] F $(\neq \phi)$ is not a substring of w.
- If $F = \phi$, then the rule $((u, x, v)_t, \phi)$ can be applied without any restriction.
- (S)SCID$_{0,k}(s)$=FID$_k(s)$.

Computational Completeness

Recall: (S)SCID results (= RE)

- $SSCID_{2,1}(2,0,0;2,0,0)$
- $SSCID_{2,1}(1,1,0;2,0,0)$
- $SSCID_{2,1}(1,1,0;1,1,1)$
- $SSCID_{3,1}(1,1,0;1,1,0)$
- $SSCID_{3,1}(1,1,0;1,0,1)$

Following systems = RE

- $FID_2(2,0,0;2,0,0)$
- $FID_2(1,1,0;2,0,0)$, $FID_2(1,0,1;2,0,0)$
- $FID_2(2,0,0;1,1,0)$, $FID_2(2,0,0;1,0,1)$
- $FID_2(1,1,0;1,1,0)$, $FID_2(1,0,1;1,0,1)$
- $FID_2(1,1,0;1,0,1)$, $FID_2(1,0,1;1,1,0)$

$\mathrm{FID}_2(2,0,0;2,0,0)$=RE

Simulating $X \to Yb$ by $\mathrm{FID}_2(2,0,0;2,0,0)$

$q1 = [(qq')_{ins}, \{\mathcal{M} \cup (N' \setminus \{X\})\}]$
$q2 = [(q'X)_{del}, \{\mathcal{M} \setminus \{q, q'\} \cup (N' \setminus \{X\})\}]$
$q3 = [(q''b)_{ins}, \{\mathcal{M} \setminus \{q\} \cup N'\}]$
$q4 = [(q'''Y)_{ins}, (\mathcal{M} \setminus \{q, q''\}) \cup N' \cup \{Zq'' \mid Z \neq q\} \cup \{qZ \mid Z \neq q''\}]$
$q5 = [(q^{iv}q^{v})_{ins}, (\mathcal{M} \setminus \{q, q'', q'''\}) \cup (N' \setminus \{Y\}) \cup \{q''b\} \cup \{qZ \mid Z \neq q''\}]$
$q6 = [(q'''q^{iv})_{del}, \{\mathcal{M}\{q, q'', q''', q^{iv}, q^{v}\} \cup (N' \setminus \{Y\}) \cup \{q'''Y, q''b\} \cup \{Zq''' \mid Z \neq q''\} \cup \{qZ \mid Z \neq q''\}]$
$q7 = [(q'')_{del}, \{\mathcal{M}\{q, q'', q^{v}\} \cup (N' \setminus \{Y\}) \cup \{q'''Y, q''b\} \cup \{q''Z \mid Z \neq q^{v}\}]$
$q8 = [(qq^{v})_{del}, \{\mathcal{M}\{q, q^{v}\} \cup (N' \setminus \{Y\})]$

$\mathrm{FID}_2(2,0,0;2,0,0)$=RE

Simulating $X \to Yb$ by $\mathrm{FID}_2(2,0,0;2,0,0)$

$q1 = [(qq')_{ins}, \{\mathcal{M} \cup (N' \setminus \{X\})\}]$
$q2 = [(q'X)_{del}, \{\mathcal{M} \setminus \{q, q'\} \cup (N' \setminus \{X\})\}]$
$q3 = [(q''b)_{ins}, \{\mathcal{M} \setminus \{q\} \cup N'\}]$
$q4 = [(q'''Y)_{ins}, (\mathcal{M} \setminus \{q, q''\}) \cup N' \cup \{Zq'' \mid Z \neq q\} \cup \{qZ \mid Z \neq q''\}]$
$q5 = [(q^{iv}q^{v})_{ins}, (\mathcal{M} \setminus \{q, q'', q'''\}) \cup (N' \setminus \{Y\}) \cup \{q''b\} \cup \{qZ \mid Z \neq q''\}]$
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$q8 = [(qq^{v})_{del}, \{\mathcal{M}\{q, q^{v}\} \cup (N' \setminus \{Y\})]$

$X \Rightarrow_{q1} qq'X \Rightarrow_{q2} q \Rightarrow_{q3} qq''b \Rightarrow_{q4} qq''q'''Yb \Rightarrow_{q5} qq''q'''q^{iv}q^{v}Yb \Rightarrow_{q6} qq''q^{v}Yb \Rightarrow_{q7} qq^{v}Yb \Rightarrow_{q8} Yb$

$FID_2(1,1,0;1,0,1) = RE$

Simulating $X \rightarrow bY$ by $FID_2(1,1,0;1,0,1)$

$p1 = [(X, p, \lambda)_{ins}, \mathcal{M} \cup (N' \setminus \{X\})]$
$p2 = [(\lambda, X, p)_{del}, (\mathcal{M} \setminus \{p\}) \cup (N' \setminus \{X\})]$
$p3 = [(p, Y, \lambda)_{ins}, (\mathcal{M} \setminus \{p\}) \cup N']$
$p4 = [(p, p', \lambda)_{ins}, (\mathcal{M} \setminus \{p\}) \cup (N' \setminus \{Y\}) \cup (\{pZ \mid Z \neq Y\})]$
$p5 = [(p', b, \lambda)_{ins}, (\mathcal{M} \setminus \{p, p'\}) \cup (N' \setminus \{Y\}) \cup (\{p'Z \mid Z \neq Y\})]$
$p6 = [(\lambda, p, p')_{del}, \{p'Y\}]$
$p7 = [(\lambda, p', \lambda)_{del}, \{p\}]$

$FID_2(1,1,0;1,0,1) = RE$

Simulating $X \to bY$ by $FID_2(1,1,0;1,0,1)$

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$p2 = [(\lambda, X, p)_{del}, (\mathcal{M} \setminus \{p\}) \cup (N' \setminus \{X\})]$
$p3 = [(p, Y, \lambda)_{ins}, (\mathcal{M} \setminus \{p\}) \cup N']$
$p4 = [(p, p', \lambda)_{ins}, (\mathcal{M} \setminus \{p\}) \cup (N' \setminus \{Y\}) \cup (\{pZ \mid Z \neq Y\})]$
$p5 = [(p', b, \lambda)_{ins}, (\mathcal{M} \setminus \{p, p'\}) \cup (N' \setminus \{Y\}) \cup (\{p'Z \mid Z \neq Y\})]$
$p6 = [(\lambda, p, p')_{del}, \{p'Y\}]$
$p7 = [(\lambda, p', \lambda)_{del}, \{p\}]$

Optimizing the rules - Does the following work?? WHY??

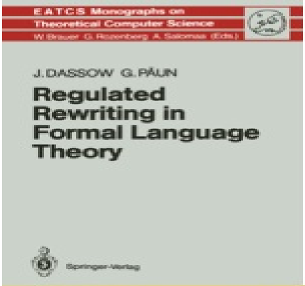
$p1 = [(X, p, \lambda)_{ins}, \mathcal{M} \cup (N' \setminus \{X\})]$
$p2 = [(\lambda, X, p)_{del}, (\mathcal{M} \setminus \{p\}) \cup (N' \setminus \{X\})]$
$p3 = [(p, Y, \lambda)_{ins}, (\mathcal{M} \setminus \{p\}) \cup N']$
$p5 = [(p, b, \lambda)_{ins}, (\mathcal{M} \setminus \{p\}) \cup (N' \setminus \{Y\}) \cup (\{pZ \mid Z \neq Y\})]$
$p7 = [(\lambda, p, \lambda)_{del}, \{\alpha Y \mid \alpha \neq b\}]$

Summary

Outcome of the talk

- Defined Insertion-Deletion systems
- Variants of Ins-del systems
  1. Matrix
  2. Graph-Controlled
  3. (Simple) Semi-conditional
  4. Forbidding
- Showed how these systems can simulate RE with certain sizes.

References



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THANK YOU