Approximate Computing in Formal Languages

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- Usage

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- Solving NP-complete problem with DTM in polynomial time

Conclusion

Approximate Computing – motivation

Definition

- Tradeoff between quality of result and efficiency.
- A common characteristic: a perfect result is not necessary and an approximate or less-than-optimal result is sufficient

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Approximations are **natural**!

Approximate Computing – usage

- Signal and image processing
- Text search
- Clustering
- Data analysis
- Robotics
- Classification
- Neural networks
- Probabilistic computing
- Networking
- Hardware (cache)

The list is endless.

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Text search, Clustering, Data analysis: No golden result

Robotics, Classification, Neural networks, Probabilistic computing, Networking, Hardware (cache): Perfect result is not always possible

The list is endless.

Reference Language – L

Approximate Language – LA such as $|LA \cap L| \geq 1$

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Error Language

Lets have two languages L and LA.

$E = L\ xor\ LA = (L - LA) \cup (LA - L)$

Error language contains only unique strings.

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Because $L \ - LA = \ \emptyset$ then $|L| < |LA|$.

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Finite Languages

Creation of approximate **finite** language from the reference language *L* which is infinite.

A cover automaton for a **finite** language *L* is a FSM that accepts all words in *L* and possibly other words that are longer than any word in *L*.

$$L = \{a, b, aa, aaa, bab\}$$

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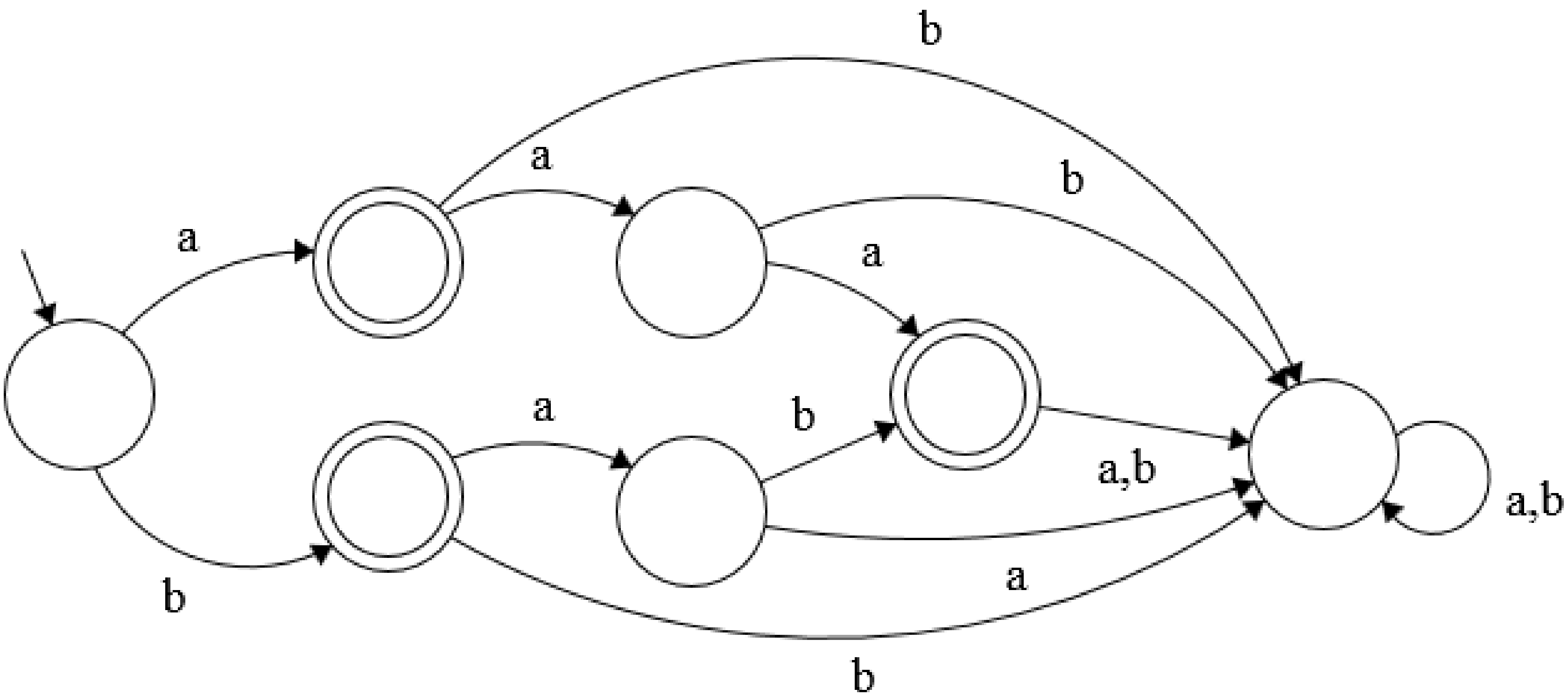

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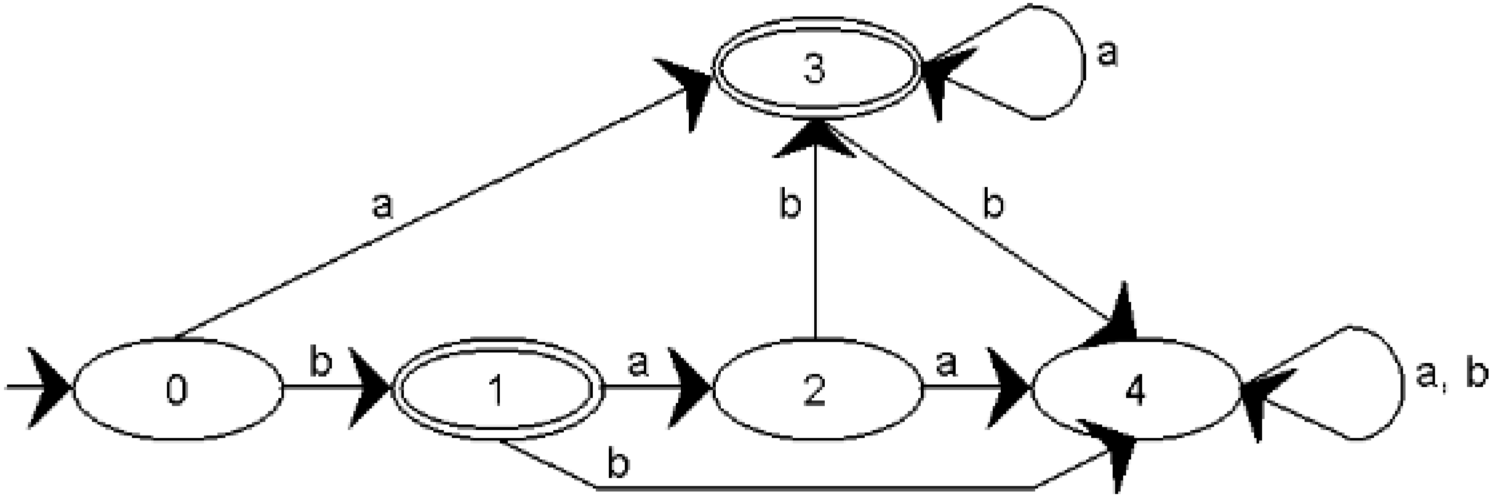
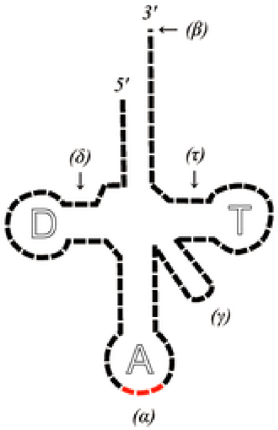
$$LA = \{aa^*, b, baba^*\}$$



Task Find all the tRNA genes in DNA of bacterium e. coli.



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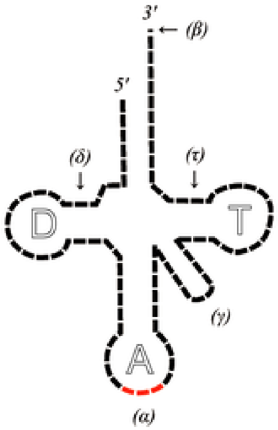
substrings string



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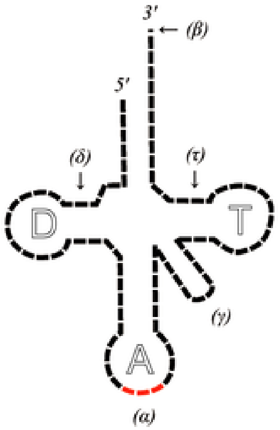

Substrings can be found by the RE:

(.{14}A[AG].{1,3}G.{11,14}[ATC]T(...)[AG].{11,31}GTTC[AG]A.[TC]C.{12}CCA)
|(TGG.{12}G[AG].T[TC]GAAC.{11,31}[TC](...)A[ATG].{11,14}C.{1,3}[TC]T.{14})

Regular expression approximation

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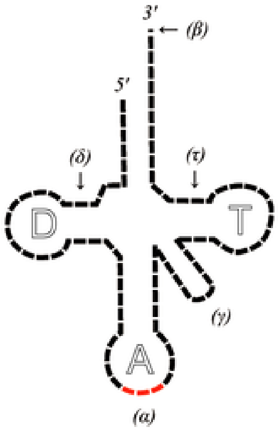

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Experiment	*Found*	*Filtered*
a) Fully functional RE	94	85

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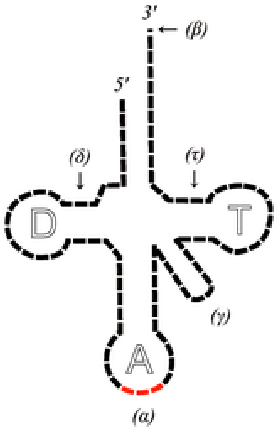

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b) Let . replace all brackets	118	85

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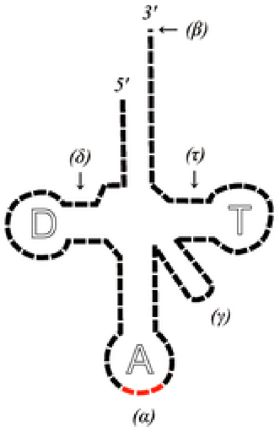

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c) Let make var. loop shorter	80	70

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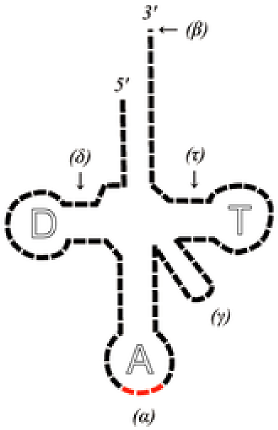

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d) Combination of b, c	109	70

Let have CFL $L = \{a^n b^n | \, n \geq 0\}$

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Approximating with a Finite-State Calculus (principle)

Construct a grammar G of L.

$$S \rightarrow aSb \quad [1]$$
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In G find all cycling rules ($S \rightarrow aS, \; S \rightarrow Sb$) by using right-hand-rule, seven formulae, and dotted rules.

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Final cover language

$$LA = \{a^+ b^+ \mid \varepsilon\}$$

Cons: We can find better approximation: $\{aa^+ bb^+ \mid ab \mid \varepsilon\}$

Syntax analysis

Let have context free grammar G1

$$
\begin{aligned}
E &\rightarrow (E) \\
  &\rightarrow E * E \\
  &\rightarrow E + E \\
  &\rightarrow i
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We don’t use infinite number of parenthesis.

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G1 can be covered with G2

$$
\begin{aligned}
A &\rightarrow (B & \bar{A} &\rightarrow +A & B &\rightarrow (C & \bar{B} &\rightarrow +B & C &\rightarrow \ldots \\
&\rightarrow i\bar{A} & &\rightarrow *A & &\rightarrow i)\bar{A} & &\rightarrow *B & & \\
& & &\rightarrow \varepsilon & &\rightarrow i\bar{B} & & & &
\end{aligned}
$$

Cons: Breaks syntax tree – no priorities given.

Covering L1 with FSM

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Then we can construct approximate language such as:

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We can construct FSM that accept approximated language.

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Experiment 8-bit numbers

Maximal number is 255. Then $\sqrt{255} = 16$. *LA* covers all primes to 16. Thus *LA* covers all primes to 255.

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Experiment 16-bit numbers

Maximal number is 65 535 and there is 6 542 primes. ***LA* covers them all**. It also contains 6 032 numbers which are not primes. The rest is rejected.

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Experiment 8-bit numbers *100% accuracy*

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Experiment 16-bit numbers *90% accuracy*

Maximal number is 65 535 and there is 6 542 primes. ***LA* covers them all**. It also contains 6 032 numbers which are not primes. The rest is rejects correctly.

Theorem

We can build FSM that covers **every** language.

(Even the languages beyond *L0*.)

FIT

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Proof

Let have $\Sigma = \{a_0, a_1, \dots, a_n\}$.

Then we can build FSM from RE: $(a_0^* a_1^* \dots a_n^*) *$

This FSM accepts all strings – **accepts everything**.

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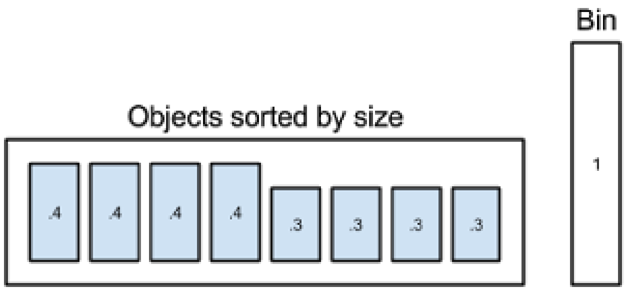
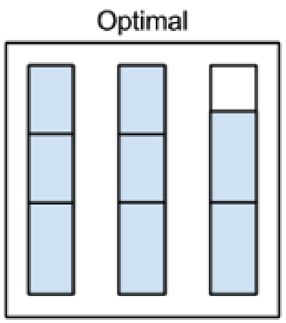
This FSM accepts all strings – **accepts everything**.

However the error language can be really large.

Bin-packing problem

- Pack all the stuff into as few bins as possible.
- NP complete problem





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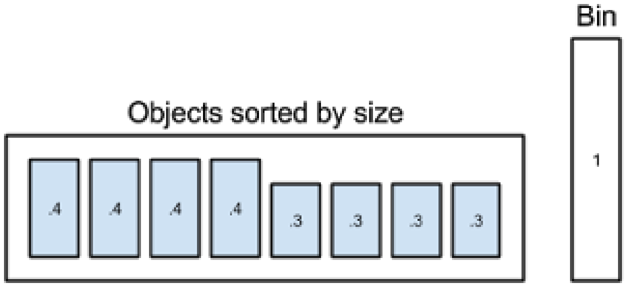
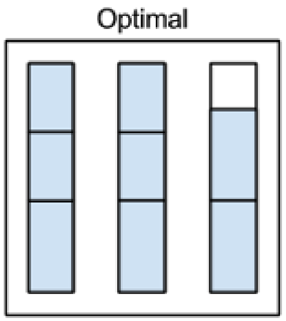
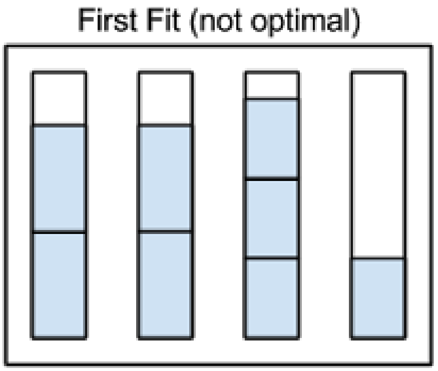




Approximation – principle

- Put an item into the first bin where is space.



Approximation

- is natural
- makes our brains and machines faster
- can be used in text search (tRNA genes in DNA)

Less powerful machines can be used for approximation of more complex problems

- FSM can cover CFL $\{a^n b^n \mid n \geq 0\}$
- FSM can cover CSL $\{a^n \mid n \text{ is prime}\}$
- FSM can cover all languages (but it is not wise)
- DTM solving NP-complete problem

Thank You For Your Attention !

References

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