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Fractals and L-systems

Abstract

Chaotic systems exhibit not only apparently random unpredictability, but also a degree of determinism, in that randomness remains confined within specific parameters. Biological systems manifest many of the features of chaotic systems, including the inherent repetition of self, fractal structure, and the existence of strange attractors.

In this presentation I will focus on fractals, the geometric shapes that are very complex and infinitely detailed. They are recursively defined and small sections of them are similar to large ones. One way to think of fractals for a function f(x) is to consider x, f(x), f(f(x)), f(f(f(x))), f(f(f(f(x)))), etc. Fractals are related to chaos because they are complex systems that have definite properties.

Fractal-like forms are easy to describe with an L-system (Lindenmayer system). L-system is a parallel rewriting system and a type of formal grammar. An L-system consists of an alphabet of symbols that can be used to make strings, a collection of production rules that expand each symbol into some larger string of symbols, an initial "axiom" string from which to begin construction, and a mechanism for translating the generated strings into geometric structures. L-systems were introduced and developed in 1968 by Aristid Lindenmayer, a Hungarian theoretical biologist and botanist at the University of Utrecht. Lindenmayer used L-systems to describe the behavior of plant cells and to model the growth processes of plant development. L-systems have also been used to model the morphology of a variety of organisms and finally can be also used to generate self-similar fractals such as iterated function systems.

The rules of the L-grammar are applied iteratively starting from the initial state. As many rules as possible are applied simultaneously, per iteration, this is the distinguishing feature between an L-system and a formal grammar generating a formal language. If production rules were to be applied only one at a time, one would quite simply generate a formal language, rather than a language described by an L-system. Thus, languages generated by an L-system are strict subsets of languages.

This presentation will be focused on L-systems that generate the fractal structures, namely on *context-free L-system*, where each production rule refers only to an individual symbol and not to its neighbors, and *context-sensitive L-systems*, where rule depends not only on a single symbol but also on its neighbors, in relation to formal language generated by formal grammar.